Arc-Community Detection via Triangular Random Walks

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Social networks & Communities

- Complex networks exhibit a **finer-grained internal structure**
- Community = **densely connected** set of nodes
- Community detection = partition that optimizes some **quality function**
- **BUT:** rarely a node is part of a **single community**!
- ⇒ **Overlapping communities**

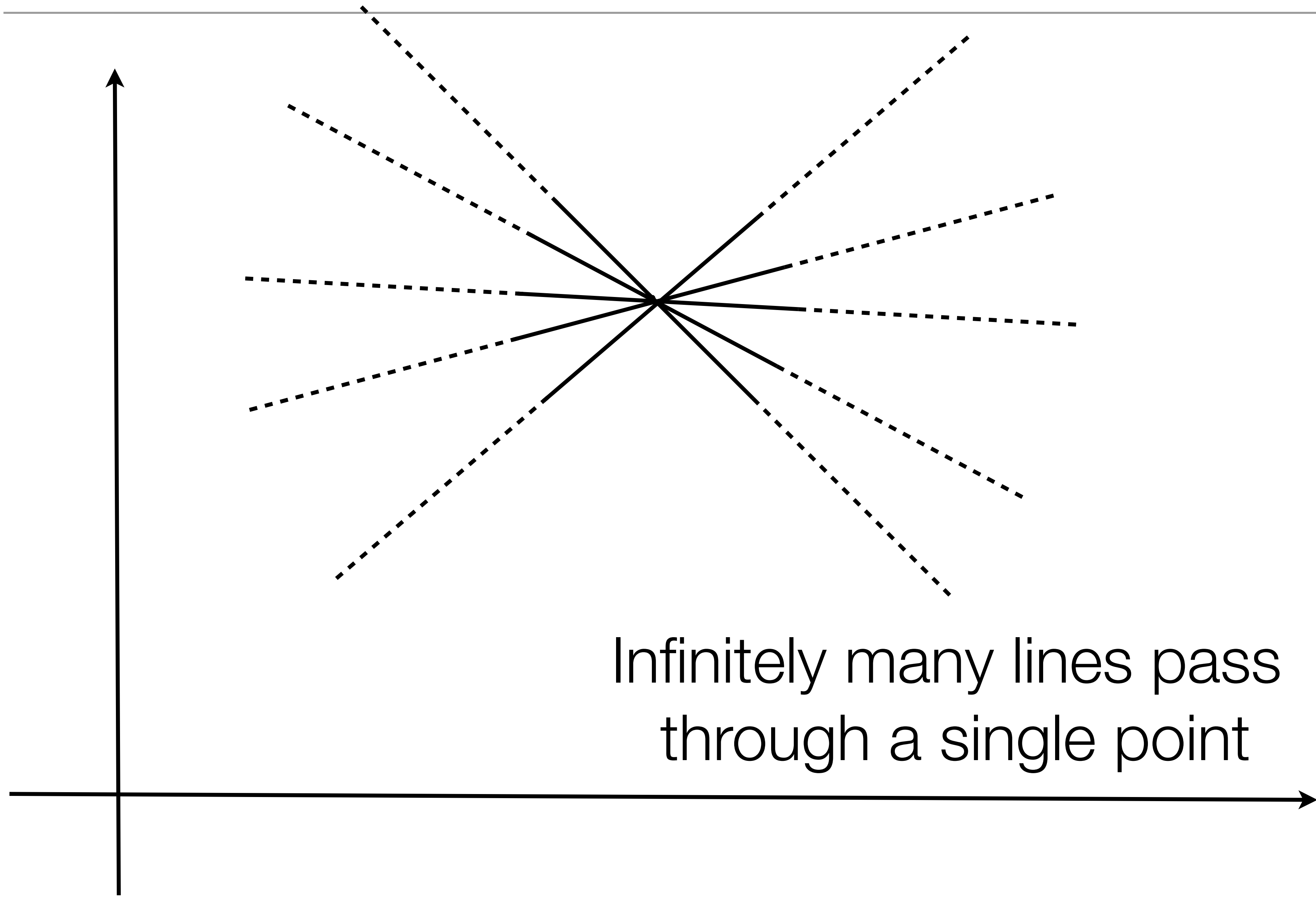
Plan of the talk

- From node-communities to arc-communities?
- Standard vs. Triangular Random Walks
- Using Triangular Random Walks for clustering, through
  - off-the-shelf clustering of the weighted line graph
  - direct implicit clustering (ALP)
- Experiments

Overlapping node clustering vs. arc clustering

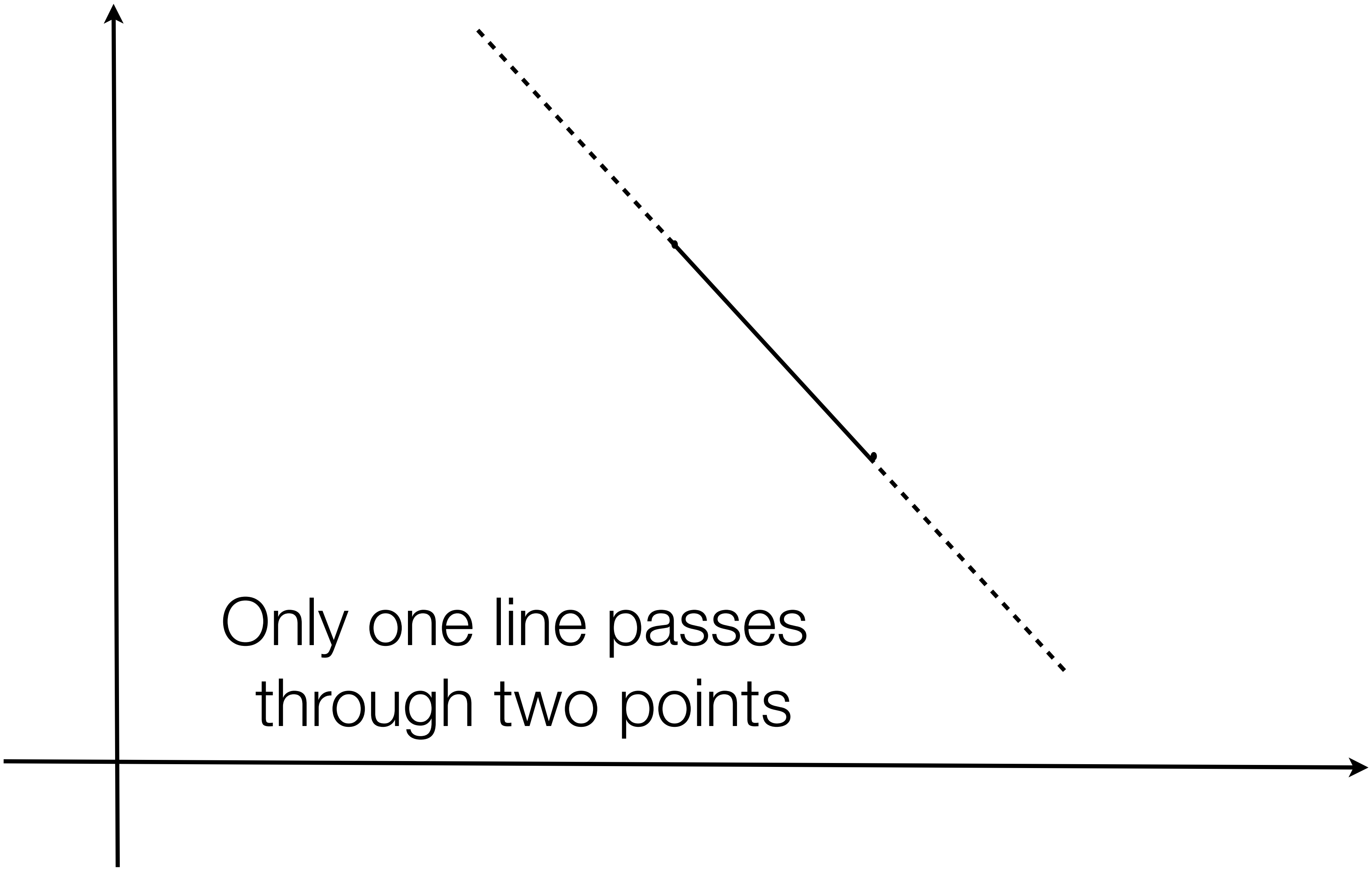
- Most algorithms: considering overlapping communities think of **overlap** as a possibly frequent phenomenon, but stick to the idea that **most** nodes are well inside a community
- In a large number of scenarioes: belonging to more groups is a **rule** more than an exception
- In a social network, every user has different personas, belonging to different communities...
- ...On the other hand, a **friendship relation has usually only one reason**!
- ⇒ **Arc clustering**

Arc-clustering: a metaphorical motivation



Infinitely many lines pass through a single point

Arc-clustering: a metaphorical motivation



Related work - Community detection

- Community detection (possibly with overlaps): too many to mention! [Kernighan & Lin, 1970; Girvan & Newman, 2002; Baumes *et al.*, 2005; Palla *et al.,* 2005; Mishra *et al.,* 2008; Blondel *et al.*, 2008]

- Good surveys / comparisons / analysis: Lancichinetti & Fortunato, 2009; Leskovec *et al.,* 2010; Abrahao *et al.,* 2012

- The latter, in particular, concludes essentially that:

    - different algorithms discover different communities

    - baseline (BFS) performs better than most algorithms (!)

Related work - Link communities

- Lehman, Ahn, Bagrow: *Link communities reveal multiscale complexity in networks*. Nature, 2010.
- Kim & Jeong. *The map equation for link community*. 2011.
- Evans & Lambiotte. *Line graphs, link partitions, and overlapping communities*. Phys. Rev. E, 2009.
- The latter uses *line graphs* (like we do), but in their undirected version

Random walks (RW) on a graph

- *Standard random walk*: a sequence of r.v.

$$X_0, X_1, \ldots$$

such that

$$P[X_{t+1} = y | X_t = x] = \begin{cases} 1/d^+(x) & \text{if } x \to y \\ 0 & \text{otherwise} \end{cases}$$

- The surfer moves around, choosing every time an arc to follow uniformly at random

Random walks with restart (RWR) on a graph

- *Random walk with restart*: a sequence of r.v.

$$X_0, X_1, \ldots$$

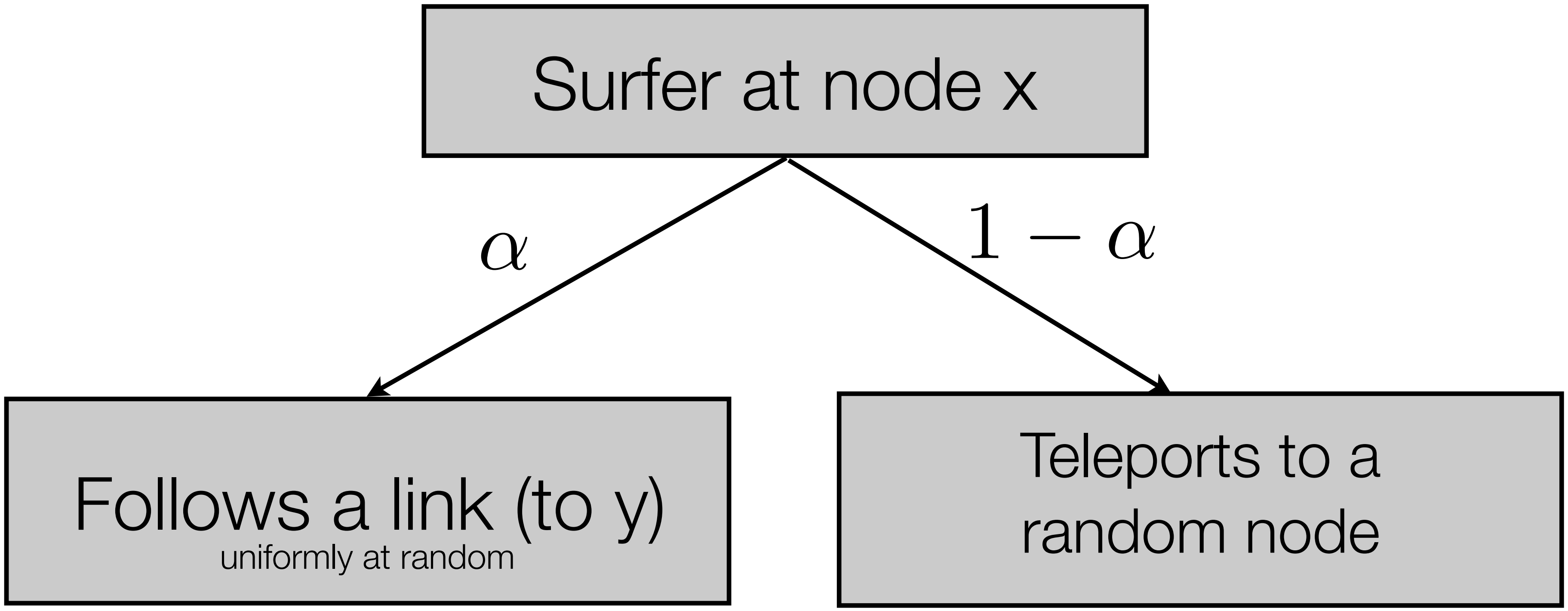
such that

$$P[X_{t+1} = y | X_t = x] = \begin{cases} \alpha/d^+(x) + (1-\alpha)/n & \text{if } x \to y \\ 1 - \alpha/n & \text{otherwise} \end{cases}$$

- The surfer every time, with probability α follows a random arc...

- ...otherwise, teleports to a random location

A graphic explanation of RWR



Why random walk with restart?

- Teleporting guarantees that there is a **unique stationary distribution**
- This is *not* true for standard RW, unless the graph is strongly connected and aperiodic
- Note that the stationary distribution will depend on the **damping factor** as well
- The stationary distribution of RWR is **PageRank**

From nodes to arcs

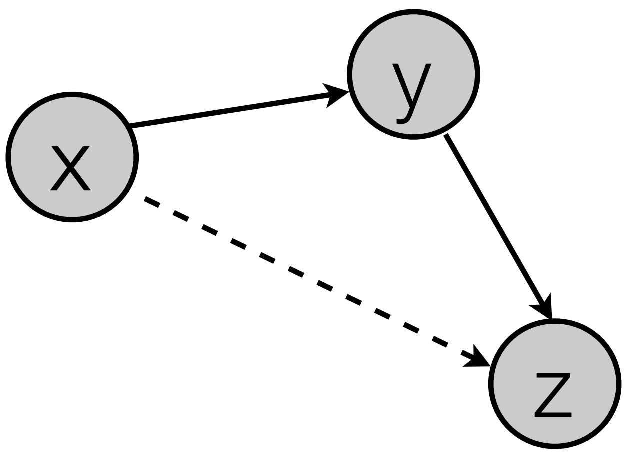
- The stationary distribution of RWR associates a probability v_x to every node
- Implicitly, it also associates a probability (frequency) to every arc $x \to y$:

$$P[X_t = x, X_{t+1} = y] =$$
$$P[X_{t+1} = y | X_t = x] P[X_t = x] =$$
$$v_x(\alpha/d^+(x) + (1-\alpha)/n)$$

Triangular random walks (TRW) on a graph

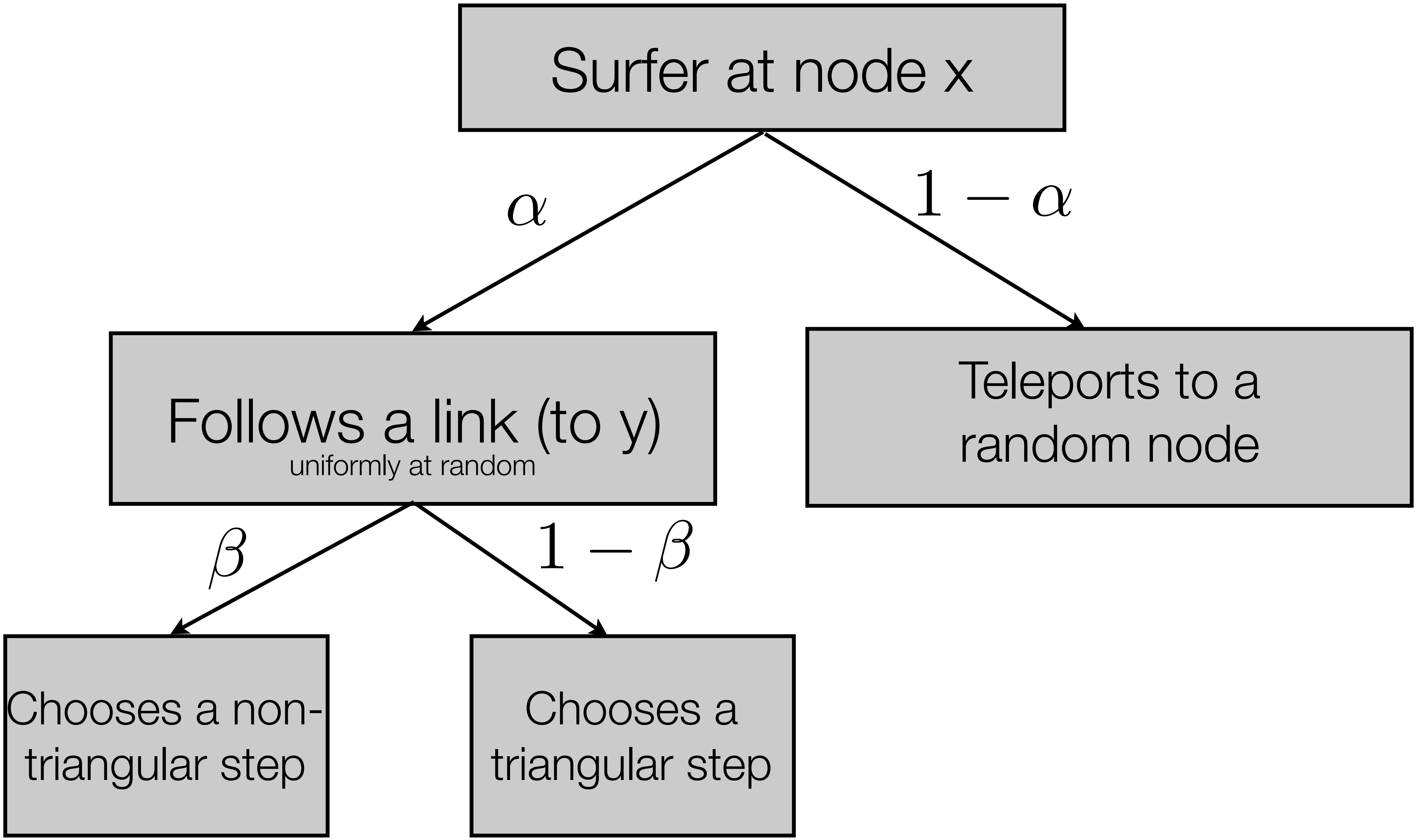
- A TRW is more easily explained *dynamically*

- A surfer goes from x to y and then to z



- Was there a way to go *directly* from x to z? If so the move y->z is called **triangular step** (because it closes a triangle)

A graphic explanation of TRW



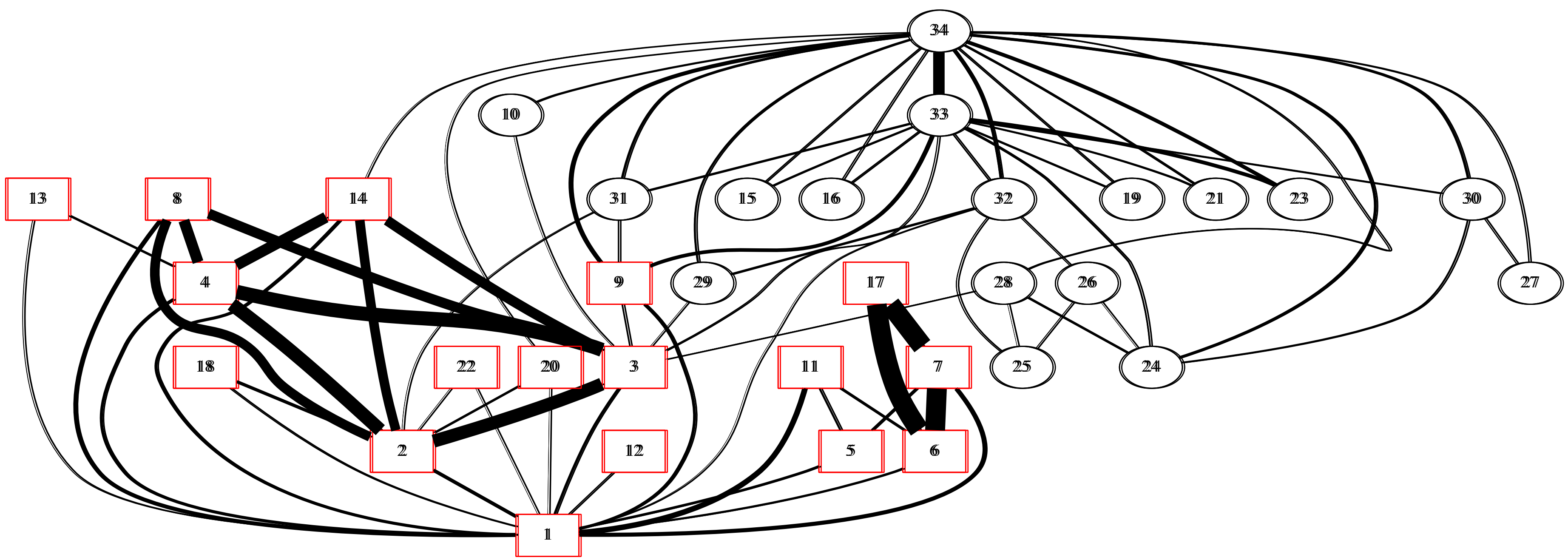
TRW: interpretation of the parameters

- α tells you how frequently one follows a link (instead of teleporting)
- β tells you how frequently one chooses non-triangles (instead of triangles)
- No-teleportation is obtained when $\alpha \to 1$
- There is no choice of β that reduces TRW to RWR
- One possibility would be to change the definition of a TRW so that β is the ratio between the probability of non-triangles and the probability of triangles...
- ...then one would recover RWR from TRW when $\beta \to 1$

The idea behind TRW

- Triangular random walks tend to insist differently on **triangles** than on non-triangles...
- ...you can decide how much more (or less) using β as a knob
- The idea is to **confine the surfer as long as possible** within a community
- Note that when β is close to zero, we virtually never choose non-triangular steps...
- ...in such a scenario, the only way out of dense communities is **by teleportation**

An experiment: Zachary’s Karate Club



TRW, $\beta = 0.01$

TRW & Markov chains

- A standard random walk is memoryless: your state at time t+1 just depends on your state at time t
- A TRW is **a Markov chain of order 2**: your state at time t+1 depends on your state at time t *plus* your state at time t-1
- Can we turn it into a *standard Markov chain*?

Line graphs

- Given a graph G=(V,E), let's define its **(directed) line graph**
- L(G)=(E,L(E)) where there is an *arc* between every node of the form (x,y) and every node of the form (y,z)
- *Theorem:* A TRW on G is a standard RWR on a (weighted version of) L(G)
- Weights depend on the choice of β
- Those weights will be denoted by w_T
- "T" is mnemonic for "triangular"

Second-order weights

- One can compute the stationary distribution (=PageRank) on L(G) using w_T as weights...
- This is a distribution on the nodes of L(G) (=arcs of G)
- Recall the Karate Club example
- Also induces (as usual) a distribution on its *arcs* (=pairs of consecutive arcs of G)
- This can be seen as another form of weight, denoted by w_S
- "S" for "Second-order" (or "Stationary")

Triangular Arc Clustering
(1) Using an off-the-shelf algorithm

- Given G...
- a) compute L(G)
- b) weight it (using either w_T or w_S)
- c) use *any* node-clustering algorithm on L(G) that is sensible to weights

Cons and pros of this solution

- **CONs:** The main limit of this solution is **graph size**
- L(G) is larger than G
- If G has $\approx Ck^{-\gamma}$ nodes of degree *k*...
- ...L(G) has $\approx C^2 k^{-2\gamma}$ nodes of degree k
- **PROs:** You can use *any* off-the-shelf standard node-clustering algorithm
- Moreover, L(G) turns out to be very easy to compress...
- ...and PageRank converges extremely fast on it

Triangular Arc Clustering
(2) A direct approach (ALP)

- There is no real need to compute L(G) **explicitly!**

- One can take a node-clustering algorithm of her will, and have it manipulate L(G) **implicitly**

- We did so for *Label Propagation* [Raghavan *et al.*, 2007]

Triangular Arc Clustering
(2) A direct approach (ALP)

- The advantage of LP [Raghavan *et al.*, 2007] with respect to other algorithms is that:
  - it provides a good compromise between **quality** and **speed**
  - efficiently **parallelizable** and suitable for **distributed** implementations
  - due to its diffusive nature it is very easy to adapt it to run implicitly on the line graph
- Recently shown that *naturally clustered graphs* are correctly decomposed by LP [Kothapalli *et al.*, 2012]

Quality measure

- Given a measure σ of *arc similarity...*
- *...*and an *arc clustering* λ
- The PRI (Probabilistic Rand Index) is

$$PRI(\lambda, \sigma) = \sum_{\lambda(xy)=\lambda(x'y')} \sigma(xy, x'y') - \sum_{\lambda(xy)\neq\lambda(x'y')} \sigma(xy, x'y')$$

Quality measure

- Computing PRI exactly on large graphs is out of question!
- Instead, we sample arcs according to some distribution Ψ

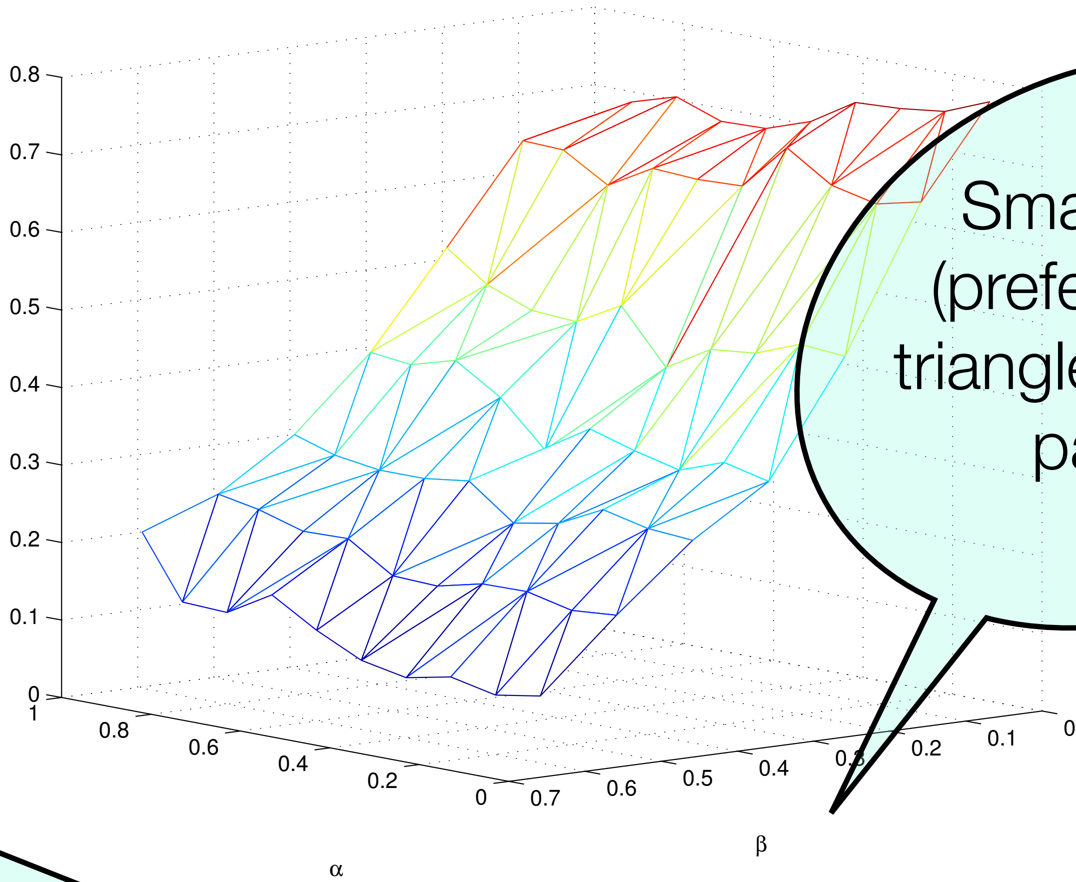
$$E_\Psi\left[(-1)^{\lambda(xy)\neq\lambda(x'y')}\sigma(xy)\right]$$

- If Ψ is uniform, the value is an unbiased estimator for PRI
- We experiment with: uniform (u), node-uniform (n), node-degree (d)

A) Parameter tuning

- We tuned the parameters α and β using different networks
- Consistent results
- We present them on DBLP
  - edge-similarity: TF-IDF of paper titles

A) Parameter tuning



Small betas (preference to triangles) always pay off

Weak dependency on alpha

But for small betas the quality decreases as alpha increases

B) Quality and computation time

DBLP (6,707,236 arcs)

ALP	#clust	PRI u	PRI n	PRI d	time
TRW	613203	**0.74**	0.71	0.75	32s
st. TRW	592562	0.72	**0.75**	**0.75**	32s
RWR	48025	0.02	0.16	0.18	24s
st. RWR	38498	0.02	0.08	0.03	22s
-	38498	0.02	0.08	0.03	22s

B) Quality and computation time

DBLP (6,707,236 arcs)

Louvain	#clust	PRI u	PRI n	PRI d	time
TRW	1493	0.01	0.69	0.53	494s
st. TRW	2116	0.02	**0.71**	**0.53**	456s
RWR	2301	0.01	0.44	0.39	1080s
st. RWR	232	0.01	0.43	0.39	1028s
-	250	0.01	0.16	0.15	316s

Suffers of excessive fragmentation

B) Quality and computation time

DBLP (6,707,236 arcs)

	#clust	PRI u	PRI n	PRI d	time
Evans	200	0.01	0.58	0.44	46min
LINK	1415245	0.28	0.31	0.51	50h
Infomap	62680	0.05	0.27	0.29	874s
Louvain (nodes)	6442	0.01	0.28	0.28	13s

Best competitor: LINK (but slooooow)

B) Quality and computation time

- ALP offers best compromise between quality and computation time
- **Triangular weights** outperform all the others
- **Stationary triangular weights** slightly outperform “normal” ones
- Same behavior on all datasets (not shown here)

Summary

- We introduced a new type of random walk that treats triangles in a preferential way
- We used it to enhance existing community-detection algorithms
- We applied it through off-the-shelf algorithm to the line graph, as well as by implementing an algorithm that never computes the line graph explicitly
- Experiments show that the results obtained have high quality

Future work

- Work out a closed formula for **triangular stationary distribution**
- Apply the triangular weighting to **other problems** (e.g., information spread, influence maximization etc.)
- See if triangular weighting can help explaining better the **structure of social networks**
- See if it is possible to improve existing **models** of social networks

Thanks!

Questions?