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PageRank model of opinion formation on Ulam networks

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ABSTRACT

We consider a PageRank model of opinion formation on Ulam networks, generated by the intermittency map and the typical Chirikov map. The Ulam networks generated by these maps have certain similarities with such scale-free networks as the World Wide Web (WWW), showing an algebraic decay of the PageRank probability. We find that the opinion formation process on Ulam networks has certain similarities but also distinct features comparing to the WWW. We attribute these distinctions to internal differences in network structure of the Ulam and WWW networks. We also analyze the process of opinion formation in the frame of generalized Sznajd model which protects opinion of small communities.

1. Introduction

The understanding of mechanisms of opinion formation in the modern society is at the heart of a newly emerged research field, known as sociophysics [1]. A number of voter models has been developed during the last few decades for understanding of nontrivial features of opinion formation in a society (see Refs. [2–6] for details). However, these models are generally considered on abstract regular lattices, which are very different from a scale-free structure of modern social networks with hundreds of millions of users. In particular, such social networks as LiveJournal [7], Facebook [8] or Twitter [9] allow to have a rapid information exchange over a large fraction of network users and to share social events, making an essential contribution to the mass opinion formation. These social networks have a growing influence on the social and political life.

A straightforward way of taking into account the main features of such networks was recently proposed in Ref. [10]: the opinion on each given node of a scale-free network is assumed to be formed by opinions of its linked neighbors, weighted with their PageRank probability. The latter quantity is interpreted as a probability of finding a random surfer on a given node [11,12]. Obviously, this approach introduces the notion of importance of a node, naturally reproducing the real society, where each person has its degree of authority. Mathematically the PageRank is defined as the right eigenvector with unit eigenvalue of Google matrix of a given network [12]. Although the PageRank algorithm was initially proposed for an efficient ranking of web pages [11], it turned out to be useful for the analysis of broad class of real networks including e.g. scientific journal rating, neuronal and world trade networks, etc. [13–16]. The rules of Google matrix construction for a given directed network are described in [11,12,15].

In the present work we study the PageRank Opinion Formation (PROF) model, proposed in [10], on another family of directed networks, known as Ulam networks. The Ulam method, introduced in Ref. [17], was initially proposed for constructing a matrix approximant for a Perron–Frobenius operator of dynamical systems (we note that the Google matrix also falls in the same class of operators). The Ulam conjecture [17] was shown to be true for various types of generic fully chaotic maps on an interval [18–21]. Recent studies have shown that this method naturally generates a class of directed networks, which properties have certain similarities with the WWW directed networks [22,23]. Thus the Ulam networks demonstrate a sensitivity to the damping parameter α of the corresponding Google matrix and a power law decay of its PageRank. Here we are interested in two particular examples: the typical Chirikov map with dissipation and the one-dimensional intermittency map. The first one, introduced in Ref. [24] for a description of continuous chaotic flows, has been studied in [22,25]. The second one is generated from intermittency maps, studied in systems exhibiting intermittency phenomenon, featuring anomalous diffusion and transport [26–30]. We note that a similar approach, directly related to the Ulam method and based on network

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representation of coarse-grained maps, can be used for the investigation of predictability and information aspects of a system [31]. In addition, coarse graining and associated symbolic dynamics, local and global spectra analysis is also at the heart of prediction, error estimates and monitoring of nonlinear complex systems [32,33].

In this work we analyze the properties of PROF model on the Ulam networks and study the influence of network elite on opinion formation process. We also consider the Sznajd model [34], generalized for scale-free networks following [10]. This model incorporates the effect of groups, consisting of voters of the same opinion following the trade union slogan *united we stand, divided we fall*. We note that the above models share some similarities with a recently analyzed model of continuous opinion dynamics [35]. In particular, the diffusion effects introduced there can be associated with the damping factor of the Google matrix, to be discussed below.

In the rest, the Letter is organized as follows: in the next section we give a brief description of the Ulam method and PROF model and present our numerical results. In Section 3 we combine the PROF and Sznajd models and analyze their properties on Ulam networks. The discussion of the results is given in Section 4.

2. The PROF model and Ulam networks

We start with a brief outline of the Ulam method for dynamical maps following the description given in [22,23]. As the first model we use the one-dimensional (1d) intermittency map described in [23]:

$$\bar{x} = f(x) = \begin{cases} x + (2x)^{z_1}/2, \\ \quad \text{for } 0 \leqslant x < 1/2, \\ (2x - 1 - (1-x)^{z_2} + 1/2^{z_2})/(1 + 1/2^{z_2}), \\ \quad \text{for } 1/2 \leqslant x < 1, \end{cases} \tag{1}$$

where $\bar{x}$ notes the new value of variable x. The Ulam network generated by this map is constructed in the following way: the whole interval $0 < x < 1$ is divided to N equal cells and N_c trajectories (randomly distributed inside a cell) are iterated on one map iteration from cell j, to obtain matrix elements for transitions to cell i: $S_{ij} = N_i(j)/N_c$, where $N_i(j)$ is the number of trajectories arrived from cell j to cell i. From the matrix S_{ij}, one constructs the Google matrix $\mathbf{G}$, defined as:

$$\mathbf{G} = \alpha \mathbf{S} + (1-\alpha)\mathbf{E}/N, \tag{2}$$

where $E_{ij} = 1$ and α is the damping factor. We use a probability normalization of the eigenstate $|\psi_1\rangle$ (with a unit eigenvalue) of the matrix (2), which results in the PageRank P_j of the network (see [23] for a detailed description of its properties). We also arrange all N nodes in monotonic decreasing order of the PageRank probability. In what follows we set the damping factor of the Google matrix of the intermittency map (1) to $\alpha = 1$. We also fix the parameters of (1) to $z_1 = 2$ and $z_2 = 0.2$. This choice gives a power law decay of the PageRank (sorted in descending order): $P_j \propto 1/j$ [23].

We construct the PROF model for the Google matrix of the intermittency map (1) in the following way. We associate each node of the network with a spin variable σ_i, taking values $+1$ (red color) or -1 (blue color). Afterwards, we compute the quantity Σ_i over all directly linked neighbors j of a node i:

$$\Sigma_i = a \sum_j P^{+}_{j,in} + b \sum_j P^{+}_{j,out} - a \sum_j P^{-}_{j,in} - b \sum_j P^{-}_{j,out}, \tag{3}$$

where $P_{j,in}$ and $P_{j,out}$ denote the PageRank probability P_j of a node j pointing to node i (incoming link) and a node j to which node i points to (outgoing link). The two parameters a and b are used to tune the importance of incoming and outgoing links with the imposed relation $a + b = 1$ ($0 < a, b < 1$). The values P^+ and P^- correspond to red and blue nodes respectively. On one iteration the value of a spin σ_i is fixed to $+1$ (red) for $\Sigma_i > 0$ or -1 (blue) for $\Sigma_i < 0$. We note that the a and b parameters define the type of a society: for a large value a a person takes mainly the opinion of those electors who point to him/her (a tenacious society) and the opposite for large values of b (a conformist society).

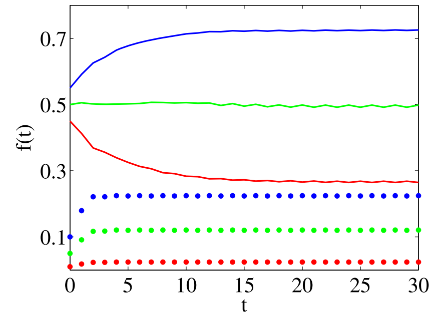

Fig. 1. Time evolution of the opinion, given by a fraction of red nodes $f(t)$, as a function of number of time iteration t ($a = b = 0.5$). Full curves correspond to different initial fractions $f_i = f(0)$ at a random realization: $f_i = 0.45$ (red); 0.5 (green); 0.55 (blue). The dotted curves stand for the initial state with the first N_{top} nodes of the highest PageRank probability being red: $N_{top} = 100$ (red); $N_{top} = 500$ (green); $N_{top} = 1000$ (blue). The total matrix size is $N = 10^4$; $\alpha = 1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

In Fig. 1 we present the evolution of the fraction of red nodes $f(t)$ ($f(t) = N_{red}/N$) versus the iteration time t. We distinguish two important cases, namely, when initially opinions are randomly distributed over the network, and when the first N_{top} nodes of the highest PageRank probability are of the same opinion, e.g. of a red color. For a random distribution the system converges to its final state after $t_c \approx 25$ iterations for $a = b = 0.5$. Iterations are defined as in [10].

In Fig. 1 we show the time evolution of opinion for the initial state where the society elite, corresponding to the top nodes N_{top} of highest PageRank probability, has the same opinion (dotted curves). In this case the elite can impose its opinion to a faction of society which is by a factor 2–3 larger than the initial fraction. However, in comparison with the social or university networks considered in [10] this increase is less significant that is due to a smaller number of linked nodes for the Ulam network of intermittency map.

For a comprehensive analyzes of the dependence of the final fraction of red nodes f_f on the initial state f_i, we consider below the evolution of $f(t)$ for a large number of N_r initial (random) distributions of red nodes (Fig. 2). We find that there is a certain critical value f_c such, that initial fractions f_i of red nodes completely die out if $f_i < f_c$, or become dominant for $f_i > 1 - f_c$. For $a = 0.2$ the value of f_c is $f_c \approx 0.45$, while for $a = 0.65$ we have $f_c \approx 0.35$. In contrast to results obtained in [10] we find that the system has no bistability for $a < 0.7$: the final state is fixed for a concrete homogeneous initial distribution of opinions. However, for a dominating tenacious society at $a > 0.7$ there is a small probability that a small initial fraction of red nodes leads to a complete domination of red color for values of $f_i > f_c$ (see Fig. 2 left bottom panel). For the case of $a = 0.8$, we have $f_c \approx 0.3$. Obviously, the results are symmetric with respect to a change of red and blue colors.

We also analyze how the final state depends on the number of the elite members N_{top} with the highest PageRank of the same opinion (Fig. 3). We see that for any type of a society (any a) there exists a value of N^c_{top} such that the elite can convince the whole society, if $N_{top} > N^c_{top}$. Note that the value of N^c_{top} depends on the

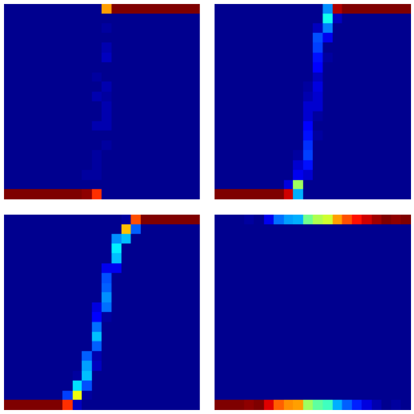

Fig. 2. Density plot of probability W_f to find a final red fraction f_f, shown in y-axis, in dependence on an initial red fraction f_i, shown in x-axis; data are shown inside the unit square $0 < f_i, f_f < 1$. The values of W_f are defined as a relative number of realizations found inside each of 20×20 cells, which cover the whole unit square. Here $N_r = 10^3$ realizations of randomly distributed red and blue colors are used to obtain W_f values (with convergence time up to $t = 150$). Here $a = 0.2$ (left top panel), 0.5 (left bottom panel), 0.65 (right top panel), 0.8 (right bottom panel); $N = 10^4$. The probability W_f is proportional to color changing from zero (blue) to unity (brown). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

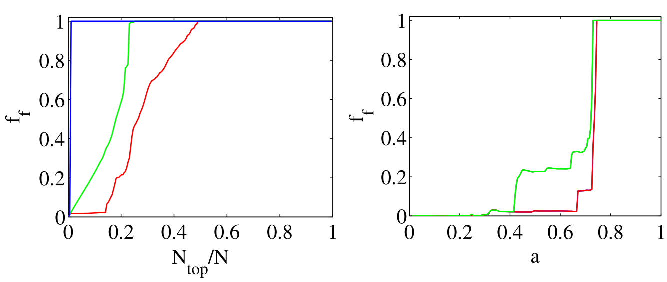

Fig. 3. Left panel: final fraction of red nodes f_f versus N_{top}/N, for $a = 0.4$ (red), 0.6 (green), 0.8 (blue). Right panel: dependence of the final fraction of red nodes f_f on the parameter a, for initial state with different number of the first N_{top} nodes of the highest PageRank being red: $N_{top} = 100$ (red); 1000 (green). Here $N = 10^4$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

tenacious parameter a. The larger the tenacious parameter is, the smaller number of the elite members of a same opinion can bring the system to unanimity.

3. The generalized PROF–Sznajd model

In this section we consider the properties of the combination of PROF and Sznajd models [34]. The Sznajd model features the idea of groups of a society and thus incorporates a well-known principle “United we stand, divided we fall”. A thorough analyzes of the problem on regular lattice networks can be found in Ref. [36]. The present generalization (which results in the PROF–Sznajd model) is applicable to scale-free and Ulam networks. We define the notion of group of nodes at each discrete time step τ following Ref. [10]:

1. we pick randomly a node i in the network and consider the state of the $N_g - 1$ highest PageRank nodes pointing to it;
2. if the node i and all other $N_g - 1$ nodes have the same color (same spin orientation), these N_g nodes form a group, whose effective PageRank value is the sum of all the member values $P_g = \sum_j^{N_g} P_j$. If it is not the case, we leave the nodes unchanged and perform the next time step;
3. consider all the nodes pointing to any member of the group and check all these nodes n directly linked to the group: if an individual node PageRank value P_n is less than the defined above P_g, the node joins the group by taking the same color (polarization) as the group nodes and increase P_g by the value of P_n; if it is not the case, a node is left unchanged.

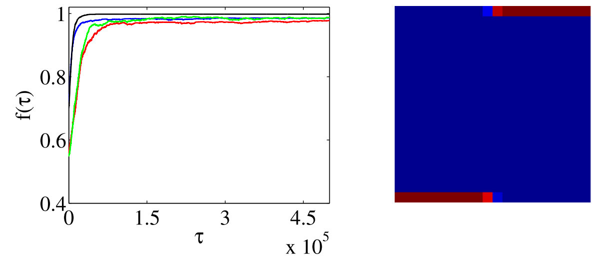

Fig. 4. Left panel: time evolution of the fraction of red nodes $f(\tau)$ of the PROF–Sznajd model, with different initial fractions of red nodes and the group size N_g (at one random realization each): $f_i = 0.55$, $N_g = 3$ (red); $f_i = 0.55$, $N_g = 4$ (green); $f_i = 0.7$, $N_g = 3$ (blue); $f_i = 0.7$, $N_g = 4$ (black). Right panel: the same as in Fig. 2, but for the PROF-Sznajd model with group size $N_g = 3$, with convergence time up to $\tau = 5 \cdot 10^5$; colors are as in Fig. 2. Here $N = 10^4$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

In Fig. 4 we present a typical behavior of the PROF–Sznajd model on Ulam network generated by the intermittency map. Firstly, we find that the convergence time is longer than that of the PROF model, which is the generic feature of the Sznajd model. The system converges to its final state after a time τ_c of the order of $\tau_c \sim 10N$. Note that there are still some fluctuations in the steady state regime, which were absent in the conventional PROF model. Another observation concerns the group size N_g: we find that the size of the group does not affect much the properties of the model: there is a small decrease in the resistivity of minorities with the group size increase (of around 2% with a change from $N_g = 3$ to $N_g = 4$). Furthermore, the network practically does not have nodes with more than four incoming links, hence, we find that considering a group size with $N_g > 5$ loses its sense.

The right panel of Fig. 4 shows a density plot of probability W_f, constructed in a similar to Fig. 2 way. We see, that the rate of surviving of small fractions of (red) nodes is drastically small (we address this result to the poor incoming link structure of the Ulam network). The initial states are suppressed if $f_i \lesssim 0.45$. But for $0.45 < f_i < 0.5$ ($0.5 < f_i < 0.55$) there is a small probability of approximately 8% that the fraction will become dominant (be suppressed). Outside of this small range of f_i we don’t find any regions of bistability: the final state of the system is fixed.

For the PROF–Sznajd model we are additionally interested in the Ulam network, generated by another dynamical map, the typical Chirikov map with dissipation:

$$\begin{cases} y_{t+1} = \eta y_t + k \sin(x_t + \theta_t), \\ x_{t+1} = x_t + y_{t+1}. \end{cases} \tag{4}$$

Here the dynamical variables x and y are taken at integer moments of time t. Also x has a meaning of phase variable and y is a conjugated momentum or action. For a detailed description of this dynamical system, see Ref. [22]. The map region is $0 \leqslant x < 2\pi$ and $-\pi \leqslant y < \pi$, with 2π-periodic boundary conditions. The phases $\theta_t = \theta_{t+T}$ are T random phases periodically repeated along time t. Here we consider the T10 case with $T = 10$, analyzed in Ref. [22]. The values of parameters are set to $\eta = 0.99$, $k = 0.22$. The list of 10 values of θ_t phases can be found in the Appendix of Ref. [22]. For the construction of the Ulam network we divide the phase space to $n_x \times n_y$ cells ($n_x = n_y = 100$). Afterwards, N_c trajectories are propagated from each given cell j during T map iterations to obtain elements of the adjacency matrix S_{ij} for transitions to cell i

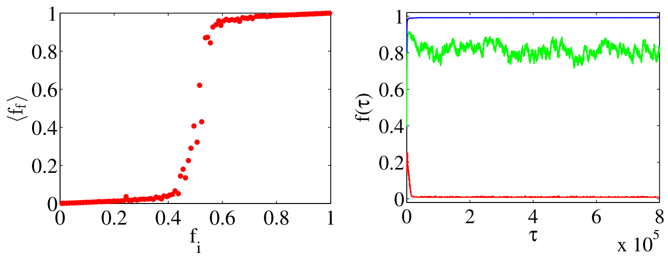

Fig. 5. Left panel: The average of the final fraction of red nodes $\langle f_f \rangle$ versus the initial fraction f_i for the PROF–Sznajd model of T10 model of the typical Chirikov map ($\alpha = 0.95$, $n_x = n_y = 100$, $N = 10^4$). Here, $N_r = 10^3$ realizations with a convergence time up to $\tau = 3 \cdot 10^5$ are used to obtain the average $\langle f_f \rangle$ (the group size is $N_g = 3$). Right panel: time evolution of the fraction of red nodes $f(\tau)$ for the same model, for the initial state with the first N_{top} nodes of the highest PageRank being red: $N_{top} = 1500$ (red); $N_{top} = 4000$ (green); $N_{top} = 8000$ (blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

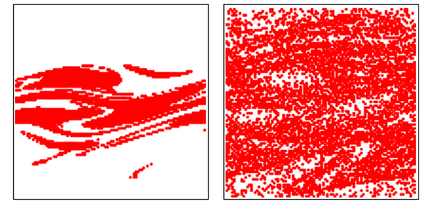

Fig. 6. Coordinate distribution of red nodes in (y, x) phase space of the PROF–Sznajd T10 model ($\alpha = 0.95$, $n_x = n_y = 100$, $N = 10^4$); the phase plane is shown in $2\pi \times 2\pi$ square. Left panel shows the initial state with $N_{top} = 2200$ nodes of the highest PageRank being red and $f_i = 0.22$; right panel corresponds to the final state with $f_f = 0.5758$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

(in the same manner as for the mapping (1)). The total matrix size is $N = 10^4$.

For this network we find a higher strength of resistivity of minorities, since it has a richer link structure. In Fig. 5 we plot the average of the final fraction of red nodes f_f versus the initial fraction f_i. We see here that minor opinions die out if $f_i \lesssim 0.3$. The damping factor of the Google matrix here is set to $\alpha = 0.95$, which gives a power law decay of the PageRank with a slope of 0.48 (see Ref. [22]). We also looked at the f_f versus f_i behavior for other values of the damping factor. As mentioned above, the Google matrix properties of Ulam networks are sensitive to the values of α. Nevertheless, our calculations showed, that for $0.95 < \alpha < 1$, qualitative behavior of the PROF-Sznajd model remains similar to that of Fig. 5. On the other hand, as pointed out in Ref. [10], the increase of the slope of the power law decay of the PageRank should result in a bistable behavior of the PROF and PROF-Sznajd models on social and university networks. However, this argument does not hold true for Ulam networks: although the slope of the PageRank increases with growth of α (e.g. for $\alpha = 0.98$ we have $P_j \propto 1/j^{0.7}$, while for $\alpha = 0.99$ we have $P_j \propto 1/j^{0.9}$), bistability does not emerge. Thus we conclude that a presence of bistability behavior is associated not only with the slope of the PageRank decay, but also with the intrinsic structure of the network itself.

For the PROF–Sznajd T10 model we find that the elite of the society cannot convince any elector, if its fraction is initially relatively small. In particular, the first N_{top} nodes of the highest PageRank with the same opinion are suppressed for $N_{top}/N \lesssim 0.2$. For $N_{top}/N \gtrsim 0.2$, the elite becomes capable to influence the opinion of other electors, but the convergence process as well as the final state starts exhibiting fluctuations of a significant amplitude. These fluctuations become smaller for higher values of N_{top} and almost disappear for $N_{top}/N \gtrsim 0.7$ where the society comes to unanimity.

Finally, we shortly describe the initial and final distributions of red nodes in the coordinate space. It is of interest to consider the case of initial state with N_{top} red nodes with the highest PageRank, since for random distributions the final and initial states are homogeneously distributed over phase plane. Fig. 6 shows the initial and final distributions for $N_{top} = 2200$. We find that the top elite nodes first tend to convince other members of the elite corresponding to the denser regions in the right panel of Fig. 6 with high values of the PageRank probability.

4. Discussion

In this work we analyzed the features of a recently proposed PageRank opinion formation model on two examples of Ulam networks. The Ulam networks generated by the discussed above one-dimensional intermittency and typical Chirikov maps exhibit some intrinsic properties similar to the WWW. This fact makes the analyzes relevant to the opinion formation process in real societies. We pointed out that the elite of a society does not have a considerable influence on the decision making process of the electors for an equal mixture of conformist and tenacious society. However, the influence of the elite becomes tangible for a dominating tenacious society. In contrast to the university networks analyzed in [10] we find practically no regions of bistability behavior for a random distribution of initial opinions. Only a dominating tenacious society shows some signs of bistability.

We also considered a generalization of the Sznajd model for Ulam networks (PROF–Sznajd model). We found here that the system still practically does not feature bistable regimes. On the basis of our studies we conclude that the PageRank decay exponent does not influence the bistability for the Ulam networks considered in this work. We argue that the chaotic maps considered generate strong stretching of small regions of phase space but do not generate significant number of loop returns. We think that this feature is different from university networks which are characterized by a significant number of loops. We presume that this internal feature of the Ulam networks is at the origin of significant difference in opinion formation on these two types of scale-free networks. The presented results can be useful for analysis of opinion formation on other types of scale-free directed networks.

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References

[1] S. Galam, Int. J. Mod. Phys. C 19 (2008) 409.
[2] S. Galam, J. Math. Psychol. 30 (1986) 426.
[3] T.M. Liggett, Stochastic Interacting Systems: Contact, Voter and Exclusion Processes, Springer, Berlin, 1999.
[4] S. Galam, Europhys. Lett. 70 (2005) 705.
[5] P.L. Krapivsky, S. Redner, E. Ben-Naim, A Kinetic View of Statistical Physics, Cambridge University Press, Cambridge, UK, 2010.
[6] C. Castellano, S. Fortunato, V. Loreto, Rev. Mod. Phys. 81 (2009) 591.
[7] Wikipedia, LiveJournal, http://en.wikipedia.org/wiki/LiveJournal.
[8] Wikipedia, Facebook, http://en.wikipedia.org/wiki/Facebook.
[9] Wikipedia, Twitter, http://en.wikipedia.org/wiki/Twitter.
[10] V. Kandiah, D.L. Shepelyansky, Physica A 391 (2012) 5779.
[11] S. Brin, L. Page, Comput. Netw. ISDN Syst. 30 (1998) 107.
[12] A.M. Langville, C.D. Meyer, Google's PageRank and Beyond: The Science of Search Engine Rankings, Princeton University Press, Princeton, 2006.
[13] F. Radicchi, S. Fortunato, B. Markines, A. Vespignani, Phys. Rev. E 80 (2009) 056103.
[14] J.D. West, T.C. Bergstrom, C.T. Bergstrom, Coll. Res. Libr. 71 (2010) 236.

[15] D.L. Shepelyansky, O.V. Zhirov, Phys. Lett. A 374 (2010) 3206.
[16] L. Ermann, D.L. Shepelyansky, Acta Phys. Pol. A 120 (6A) (2011) A158.
[17] S.M. Ulam, A Collection of Mathematical Problems, Interscience Tracts Pure Appl. Math., vol. 8, Interscience, New York, 1960, p. 73.
[18] Z. Kovacs, T. Tel, Phys. Rev. A 4641 (1989) 40.
[19] Z. Kaufmann, H. Lustfeld, J. Bene, Phys. Rev. E 1416 (1996) 53.
[20] G. Froyland, R. Murray, D. Terhesiu, Phys. Rev. E 76 (2007) 036702.
[21] D. Terhesiu, G. Froyland, Nonlinearity 21 (2008) 1953.
[22] D.L. Shepelyansky, O.V. Zhirov, Phys. Rev. E 81 (2010) 036213.
[23] L. Ermann, D.L. Shepelyansky, Phys. Rev. E 81 (2010) 03622.
[24] B.V. Chirikov, Research concerning the theory of nonlinear resonance and stochasticity: Preprint No. 267, Institute of Nuclear Physics, Novosibirsk, 1969; English translation: CERN Trans., Geneva, 1971, pp. 71–40.
[25] K.M. Frahm, D.L. Shepelyansky, Phys. Rev. E 80 (2009) 016210.
[26] Y. Pomeau, P. Manneville, Commun. Math. Phys. 74 (1980) 189.
[27] T. Geisel, J. Nierwetberg, A. Zacherl, Phys. Rev. Lett. 54 (1985) 616.
[28] A.S. Pikovsky, Phys. Rev. A 43 (1991) 3146.
[29] R. Artuso, C. Manchein, Phys. Rev. E 80 (2009) 036210.
[30] R. Artuso, G. Cristadoro, Phys. Rev. Lett. 90 (2003) 244101.
[31] A. Garciá Cantú Ros, G. Forti, G. Nicolis, Commun. Nonlinear Sci. Numer. Simul. 18 (2013) 2018.
[32] V. Basios, D. Mac Kernan, Int. J. Bifurc. Chaos 21 (2011) 3465.
[33] G. Nicolis, C. Nicolis, Foundations of Complex Systems: Nonlinear Dynamics, Statistical Physics, Information and Prediction, World Scientific, Singapore, 2007.
[34] K. Sznajd-Weron, J. Sznajd, Int. J. Mod. Phys. C 11 (2000) 1157.
[35] M. Pineda, R. Toral, E. Hernandez-Garcia, Eur. Phys. J. D 62 (2011) 109.
[36] C. Castellano, S. Fortunato, V. Loreto, Rev. Mod. Phys. 81 (2009) 591.