

# Centrality Prediction in Temporally Evolving Networks\*

Róbert Pálovics   Ferenc Béres   Nelly Litvak   Frederick Ayala-Gómez   András A. Benczúr

## 1 Abstract

In networks with very fast dynamics, such as Twitter mentions and retweets, predicting links and the emerging centrality of nodes is a challenging task. In contrast to existing methods that either consider static networks or a sequence of static snapshots, in this paper we give predictive models and centrality measures, both of which can be dynamically updated after the addition of each new edge.

We propose a variant of matrix factorization for link prediction and compare the results with the online version of matrix factorization. To analyze the centrality measures, we propose online evaluation of Harmonic Centrality, PageRank, and Katz. To demonstrate our results, we use collections of topic specific Tweets.

## 2 Introduction

The research of complex networks and large graphs generated a wide variety of stochastic graph models that try to capture the properties of these complex systems [7, 11, 16, 25, 24]. Most of the well-known models can describe a static graph extracted from a real-world dataset. They are capable of generating an ensemble of graphs, in which all graph instances are similar in terms of specific statistics to the original one. For example, models that capture the power-law degree distribution of real-world networks such as the Albert-Barabasi one are dynamic but do not attempt to model the actual temporal evolution of large graphs. Our goal is to give temporal stochastic graph model for the temporal dynamics of these complex systems.

Our models address the link prediction problem introduced by Liben-Nowell and Kleinberg [29], in a *temporal* setting. More specifically, we try to predict accurately each new link in the graph at the time when it is created in the network. This experimental setting is similar to our method introduced for recommender systems [32]. In Section 6.1 we explain this setup in case of dynamic graphs. For baseline algorithm, we apply online matrix factorization [23, 34, 35] on temporal network data (see Section 4).

Various node centrality measures capture the “importance” of a node by using the structural properties of the graph [10]. While these metrics are widely investigated, few is known about the evolution of graph centrality in temporal graphs. In our work, we investigate the applicability of node centrality metrics in temporal graphs by examining their temporal behavior and computational complexity. We also use these metrics as side features in our matrix factorization models.

In our experiments we use the data set of [2] that consists of the messages and the corresponding user network of the Occupy movement.

As our main result, we demonstrate that methods of matrix factorization by online learning are capable of improving predictions for the future centrality of nodes. Surprisingly, we find no direct relation between the quality of predicting the links and the derived quality of predicting centrality. As a byproduct, we define time aware variants of certain centrality measures, however our main goal is the prediction and not the definition of centrality metrics as those in e.g. [28, 17]. For centrality metrics, we use those in [29, 10] with appropriate time aware modifications.

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## 2.1 Related results

Social influence in Web based networks is investigated in several results: Bakshy et al. [5] model social contagion in the Second Life virtual world. Ghosh and Lerman [18] compares network measures for predicting the number of votes for Digg posts, who even give an empirical comparison of information contagion on Digg vs. Twitter [27]. In [19, 20], long discussion based cascades built from comments are investigated in four social networks, Slashdot (technology news), Barrapunto (Spanish Slashdot), Meneame (Spanish Digg) and Wikipedia. They propose models for cascade growth and estimate model parameters but give no size predictions.

A number of related studies have largely descriptive focus, unlike our quantitative prediction goals. In [12] high correlation is observed between indegree, retweet and mention influence, while outdegree (the number of tweets sent by the user) is found to be heavily spammed. [26] reports similar findings on the relation among follower, mention and retweet influence. Several more results describe the specific means of information spread on Facebook [6, 3, 8]. In the first paper on the data set that we also use for our experiments [2], the authors investigate how emotions appear in Twitter.

Myers and Leskovec [30] showed that the Twitter network is highly dynamic with about 9% of all connections changing in a month. Thus, in order to infer central nodes, the factors driving the dynamics of this social network must be considered. They focused on local bursts in the user-follower network to identify key events or bursts in the information flow. They consider both follow and unfollow bursts.

Chierichetti et al. [14] propose a robust model for the real-time identification of key events. They examined tweet and retweet production/consumption patterns around these incidents. The experiments showed that there is a heartbeat phenomenon in the balance of primary and secondary information spreading. When an important event unfolds, the users are busy with tweeting about it, as they try to report everything. Thus, nobody has time to retweet these messages. Whereas after the event, there is a huge amount of tweets to be retweeted. So in this case, the secondary information spreading dominates the network. The authors used this phenomenon to obtain a simple classifier which, by only evaluating the tweet/retweet volume could detect these events.

The results of Bakshy et al. [4] attempt to predict the influential users of a Twitter user-follower graph by generating diffusion cascades. At first, they extract influential vertices with regression merely based on network features. Their main problem is the minority of cascades with significant size. Although they improved their results with content information about the cascades, the problem remained open.

Cheng et al. [13, 21] predict retweet count based on network features. Petrovic et al. [33] introduce time sensitive modeling by using the PA algorithm of [15], which is an online solution to the linear regression problem. They only predict if a tweet will be retweeted at all.

Rodriguez et al. [36] gave an algorithm for inferring the structure of temporal diffusion networks. They examined an interesting aspect of centrality for many real-time events.

The direct starting point of our work is the first comprehensive overview of methods for time agnostic link prediction is given in [29]. Most of the methods used in [29] are listed in Section 5.

As one of the first time aware link prediction methods, Tylenda et al. [38] propose a maximum entropy model with weights inversely proportional to the age of the edges, however their method is trained on a single, though timestamped, snapshot and evaluated on the future in a batch. Similar to our evaluation methodology described in Section 6.1, they use DCG, however they do not consider a time aware DCG evaluation as first proposed in our work [32].

Closest to our work, [28, 17] defines a new time aware centrality measure, which they evaluate only on yearly snapshots of scientific citation networks. Our main contribution is the use of online learning methods for fine granularity evaluation of centrality measures similar to those in time aware centrality research results.

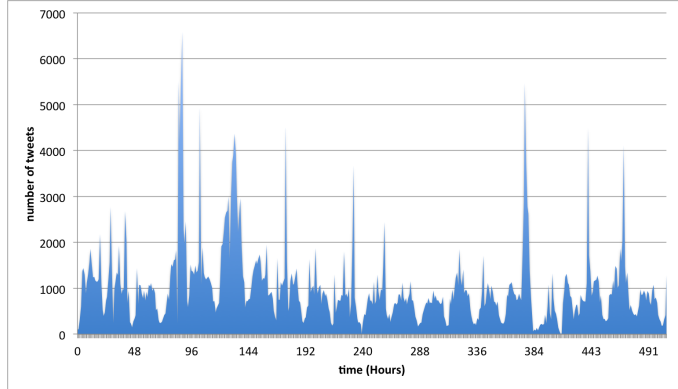


Figure 1: Temporal density of tweeting activity.

Table 1: Size of the tweet time series.

Number of users	371,401
Number of tweets	1,947,234
Number of retweets	1,272,443

### 3 Datasets

The dataset was collected by Aragón et al. [2] using the Twitter API that we extended by a crawl of the user network. Our data set hence consists of two parts:

- *Tweet dataset*: tweet text and user metadata on the Occupy Wall Street movement<sup>1</sup>.
- *Follower network*: The list of followers of users who posted at least one message in the tweet dataset.

Table 1 shows the number of users and tweets in the dataset. One can see that a large part of the collected tweets are retweets. Table 2 contains the size of the crawled social networks. Note that the average in- and outdegree is relatively high. Fig. 1 shows the temporal density of tweeting activity.

For each tweet, our data contains

- tweet and user ID,
- timestamp of creation,
- hashtags used in the tweet, and
- the tweet text content.

In case of a retweet, we have all these information not only on the actual tweet, but also on the original *root tweet* that had been retweeted. We define the root tweet as the first occurrence of a given tweet.

<sup>1</sup>[http://en.wikipedia.org/wiki/Occupy\\_Wall\\_Street](http://en.wikipedia.org/wiki/Occupy_Wall_Street)

Table 2: Size of the follower network.

Number of users	330,677
Number of edges	16,585,837
Average in/out degree	37

## 4 Dynamic adjacency matrix factorization

Batch modeling algorithms may iterate several times over the graph until convergence. In our temporal setting, the model needs to be retrained after each new event and hence reiterations over the earlier parts of the data is ruled out.

In this section, we give an online factorization method for the graph adjacency matrix. Matrix factorization yields a low-rank approximation of the adjacency matrix with entries for non-edges filled with values that we consider an indication for the edge to appear. Links for a given node are predicted by taking the largest values in the corresponding row or column. In our algorithm, we allow a single iteration over the training data only, and this single iteration processes the events in the order of time. We use each record in the dataset as a positive training instance and generate negative training instances by selecting random items for each positive record. We use the regularized matrix factorization method of [37], and use the  $k$ -factor model for prediction.

Temporal modeling methods seem more restricted than those that may iterate over the data set several times and one would expect inferior quality by the online methods. Online methods however have the advantage of giving much more emphasis on recent events that we empirically verify in our research.

## 5 Centrality measures

### 5.1 Negative $\beta$ -measure

Let  $d^+(v)$  denote the outdegree of vertex  $v$ . Negative  $\beta$ -measure is defined by

$$\sum_{y \rightarrow x} \frac{1}{d^+(y)}.$$

and it can be considered as *Markovian indegree*.

### 5.2 Closeness

The closeness of node  $x$  is defined by

$$\frac{1}{\sum_y d(y, x)},$$

where  $d(y, x)$  denotes the distance of  $x$  from  $y$  in the directed network. While indegree and negative  $\beta$ -measure were relying on the local graph structure, closeness is defined by the global graph structure. Thus, it is more costly to compute.

It is important to remark that the graph must be strongly connected. Without this condition the result will be a null score for all  $x$  node, that cannot coreach the whole graph. Nevertheless there have been many propositions on how to mend this troublesome quality of closeness. But the most straightforward idea is to exclude infinite distances

$$\frac{1}{\sum_{d(y,x) < \infty} d(y, x)}.$$

### 5.3 Lin's index

One of the ideas that tried to repair the definition of closeness for graphs with infinite distances was Nan Lin's. Lin's index defines the score of node  $x$  with a nonempty coreachable set as

$$\frac{|\{y | d(y, x) < \infty\}|^2}{\sum_{d(y,x) < \infty} d(y, x)}.$$

Nodes with an empty coreachable set have centrality 1 by definition.

This change in the definition means that closeness is not the inverse of a sum of distances, but rather the inverse of the average distance. One of the results of this modification is that closeness is normalized across the graph.

## 5.4 Harmonic centrality

Paolo Boldi and Sebastiano Vigna in [10] gave another solution on how to eliminate the problem of non-finite distances between nodes. The main idea is to use harmonic mean instead of arithmetic averaging. The reason why harmonic mean is involved is that it conveniently deal with  $\infty$  distances, as  $\frac{1}{\infty} = 0$ . The definition for the harmonic centrality of node  $x$  is

$$\sum_{x \neq y} \frac{1}{d(y, x)} = \sum_{d(y, x) < \infty, x \neq y} \frac{1}{d(y, x)}, \quad (1)$$

which is the reciprocal of the denormalized harmonic mean of distances. In [10] the authors found that harmonic centrality is strongly correlated to closeness in simple networks. Moreover, this definition also accounts for nodes  $y$  that cannot reach  $x$ . Thus, this measure can also be used in cases when the given graph is not strongly connected.

## 5.5 Katz index

Katz defined his index through summation of all paths coming into a node  $x$ . In order to get a finite score, he introduced an *attenuation factor*  $\beta$  with which a weight could be calculated for the paths. The Katz index can be expressed as

$$k = \mathbf{1} \cdot \sum_{i=0}^{\infty} \beta^i A^i, \quad (2)$$

which is equivalent to

$$k = \mathbf{1} \cdot (1 - \beta A)^{-1}, \quad (3)$$

where  $\mathbf{1}$  is the vector with uniformly 1 coordinates. Furthermore, by Brauer's theorem on the displacement of eigenvalues, the Katz index is the left dominant eigenvector of a perturbed matrix

$$\beta \lambda \cdot A + (1 - \beta \lambda) \cdot e^T \cdot \mathbf{1},$$

where  $e$  is a right dominant eigenvector of  $A$  such that  $\mathbf{1}e^T = \lambda$ . Hubbell [22] proposed a generalization for the Katz index, in which some preference vector  $v$  is used instead of  $\mathbf{1}$ . In other words, the paths can be weighted individually depending on their starting node. The normalized limit of the Katz index when  $\beta \rightarrow \frac{1}{\lambda}$  is the dominant eigenvector.

## 5.6 PageRank

Recently, PageRank is one of the most frequently discussed and cited spectral measure in use, mainly because of its alleged use in the Google ranking algorithm. PageRank [31] is defined by the unique vector  $p$  satisfying equation

$$p = \alpha \cdot p \bar{A} + (1 - \alpha)v, \quad (4)$$

where  $\bar{A}$  is derived from the adjacency matrix  $A$  with the same  $l_1$ -normalization, that was used in the formulation of Seeley's index and the negative  $\beta$ -measure. PageRank has two additional parameter. A *damping factor*  $\alpha \in [0, 1)$ , and a *preference vector*  $v$ . The only constraint for  $v$  is that it must be a distribution.

However, it is important to note that  $p$  is not necessarily a probability distribution if  $A$  has null rows. There has been several propositions on how to make  $\bar{A}$  stochastic. A common solution is to replace every

null row with the preference vector  $v$ . Another popular idea is to add loop arcs to all nodes with zero outdegree (dangling nodes).

Equation 4 is solvable even without any patching, as after reorganizing the formula we get

$$p = (1 - \alpha)v(1 - \alpha\bar{A})^{-1}. \quad (5)$$

Moreover, another equation can be formulated for PageRank

$$p = (1 - \alpha)v \sum_{i=0}^{\infty} \alpha^i \bar{A}^i, \quad (6)$$

which shows that the Katz index and PageRank differ only by a constant factor and by the  $l_1$  normalization applied to the adjacency matrix. If  $A$  has no null rows, or  $\bar{A}$  has been patched to be stochastic, PageRank can be equivalently defined as the stationary distribution of the Markov chain whose transition matrix is

$$\alpha\bar{A} + (1 - \alpha)\mathbf{1}^T v.$$

## 5.7 Betweenness

Let  $\sigma_{yz}$  denote the number of shortest paths going from  $y$  to  $z$ . A subset of these paths also passes through node  $x$ , and suppose their number is  $\sigma_{yz}(x)$ . The betweenness measure of node  $x$  is defined by

$$\sum_{y,z \neq x, \sigma_{yz} \neq 0} \frac{\sigma_{yz}(x)}{\sigma_{yz}}$$

The definition tries to capture the intuition that if a significantly large fraction of shortest paths passes through  $x$ , then  $x$  is an important junction point of the graph. Moreover, Boldi et al. in [9] showed that removing nodes with high betweenness score results in an instant network disruption.

# 6 Experiments

## 6.1 Experimental setting and evaluation metrics

In the dynamic link prediction task, we have to rank the best  $K$  links for the given node at the given time instance. Our dataset contains records  $\langle u, v, t \rangle$  of links between users  $u$  and  $v$  that appear at time  $t$ . Our goal is to recommend new links for user  $u$  at time  $t$  with the constraint that there is only a single link that appears at the given time  $t$ . This means that we have to maximize the rank of the given link in the actual predicted list of links. A time sensitive or online link prediction system should retrain its model after each and every training record  $\langle u, v, t \rangle$ . We have to generate new top- $K$  recommendation list for *every* single record. The online top- $K$  task is hence different from the standard recommender evaluation settings, since there is always a single neighbor only in the ground truth and the goal is to aggregate the rank of these single neighbors over the entire testing period. For our task, we need carefully selected quality metrics that we describe next. We use our full dataset both for training and testing. We iterate on the records one by one in temporal order. For a given record  $\langle u, v, t \rangle$ , we allow the recommender algorithm to use full of the data *before*  $t$  in question for training and require a ranked top list of possible neighbors as output. We evaluate the given single actual neighbor  $v$  in question against the recommended top list of length  $K$ .

For measuring the accuracy of predicting a new link, we face the difficulty that only a single correct answer exists at the given time and the next edge arrives to be tested against an updated model. We propose DCG [38, 32], a modified version of NDCG, the preferred model for batch top- $K$  recommendation [1]. DCG is a slowly decreasing function of the rank and hence measures how close the actual new link appears in the top list.

To sum up, in our experiments we use this experimental setting and evaluation. We iterate over the edge list of a given graph in temporal order. One record in our dataset is a timestamped edge between two users in the graph,  $\langle u, v, t \rangle$ . Instead of items, we recommend for users new neighbors. For each  $\langle u, v, t \rangle$ , we evaluate our top- $K$  recommendation by using DCG as evaluation metric. Finally, we compute temporal averages of the DCG scores.

## 6.2 Accuracy of link prediction

In Fig. 2, we show daily average link prediction quality by using online matrix factorization defined in Section 4. We give results for different learning rates and conclude that there is a large variance but in general, very low learning rates around 0.05 perform the best.

## 6.3 Accuracy of centrality prediction

In Fig. 3, we show daily average centrality prediction quality by computing various centrality measures over the graph augmented by the edges predicted by online matrix factorization as in Section 4. We give results for different learning rates. Unlike for link prediction, we observe stable performance across different metrics improving up to a learning rate of 0.08 and declining beyond.

Also note that for in-degree, Beta and PageRank we are able to improve over the prediction given by the previous state of the graph as baseline. We plan to evaluate different weighted combinations of centrality values on past and predicted future graphs.

# 7 Conclusion and Further Work

In this paper, we analyze the dynamic network data as a stream of nodes and edges. To predict link formation, the regularized matrix factorization model is proposed. Different centrality measures are used with online computation over the graph stream to identify the evolution of centrality. As the main lesson learned, we show how recent results in recommender systems can be deployed for the analysis of complex networks.

Matrix factorization algorithm may use so-called side information associated with the rows and columns of the matrix. We plan to use centrality measures as side information associated with the nodes. We may use directed centrality with different values for rows and columns of the same node. We plan to compare the following metrics in the temporal setting of dynamic networks based on [10]: Harmonic Centrality, PageRank, HITS and SALSA.

In addition, we would like to test our methods on a variety of other data sets from Twitter, Last.fm, scientific citation networks and more.

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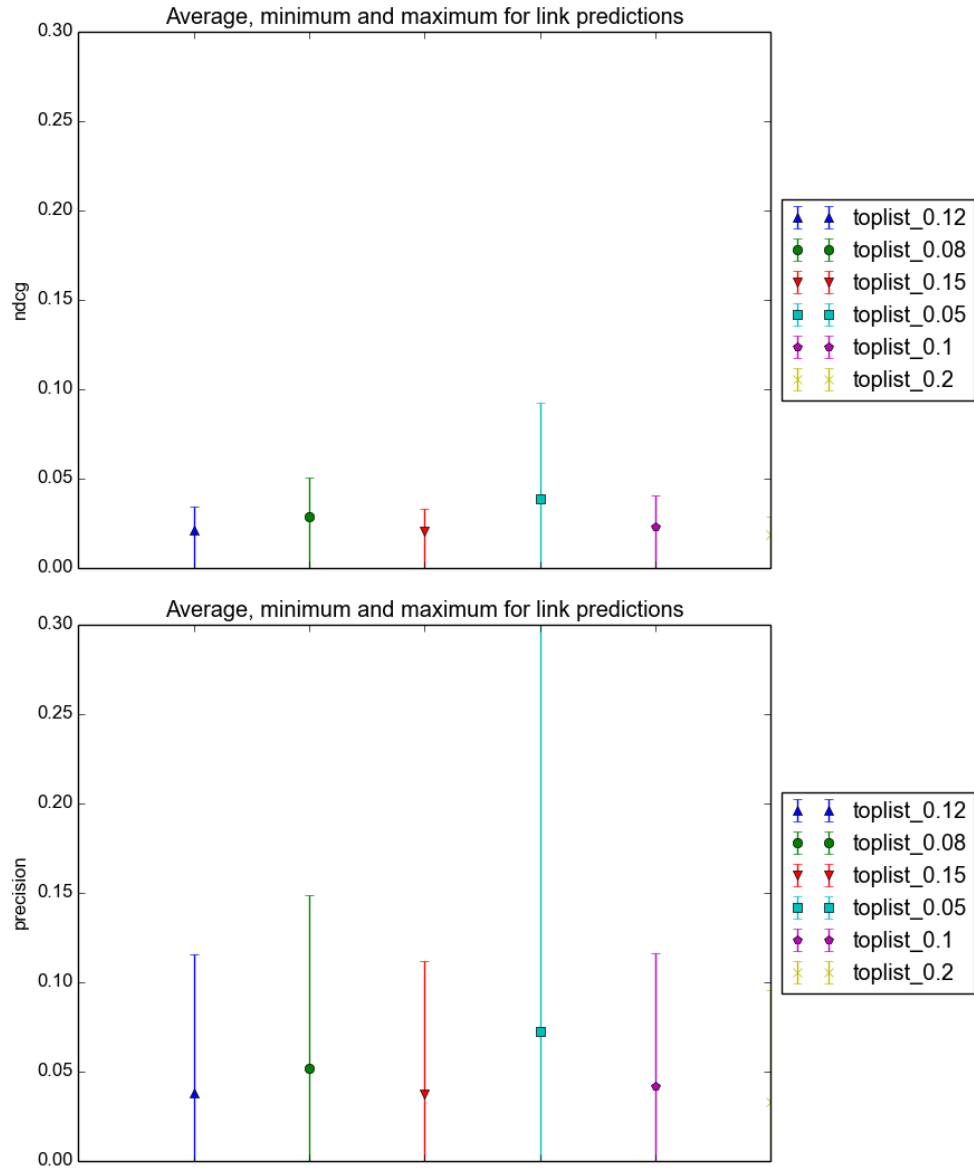


Figure 2: Quality of link prediction, NDCG (**top**) and precision (**bottom**).



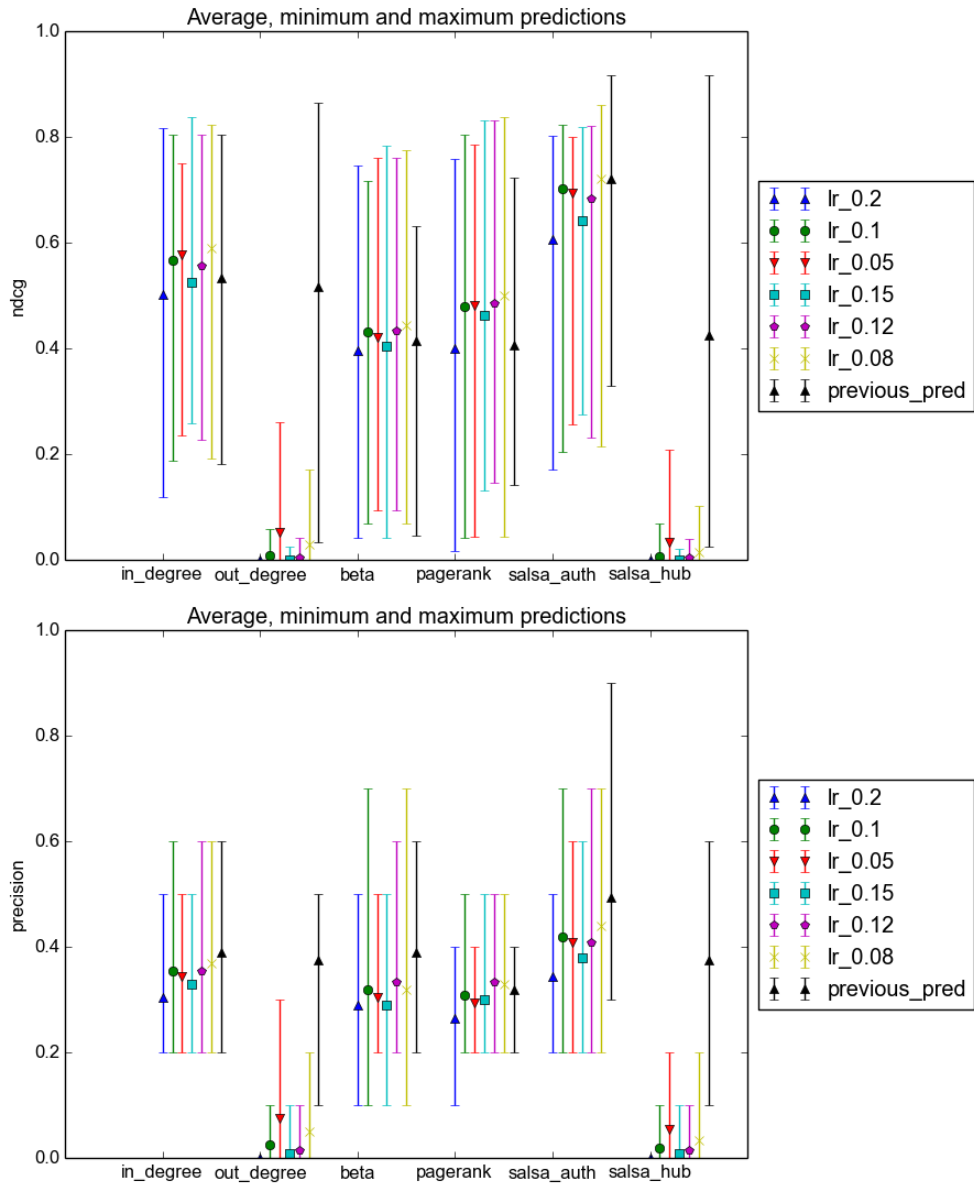


Figure 3: Quality of centrality prediction, NDCG (**top**) and precision (**bottom**).

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