Solid State NMR Quantum Information Processing

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Could we implement a multi-round protocol of quantum error correction?

Need to go to solid state
Advantages:
- Stronger couplings (dipolar)
- Slower decoherence
- Higher polarization
How to reach higher polarization?

Cooling techniques:

- Lowering Temperature
- Increasing Energy Gap
- Polarization Transfer
- Optical Pumping
- Dynamic Nuclear Polarization
- Para-Hydrogen (Jones et al.)
- Algorithmic Cooling
Malonic acid

P-1 space group:
magnetic equivalence

$^{13}$C NMR spectrum
\[ \mathcal{H}(t) = \mathcal{H}_C + \mathcal{H}_H + \mathcal{H}_{CH} + \mathcal{H}_{RF}(t), \]
\[ \mathcal{H}_C = \mathcal{H}_{CZ} + \mathcal{H}_{CD}^{\text{intra}} + \mathcal{H}_{CD}^{\text{inter}}, \]
\[ \mathcal{H}_{CZ} = \sum_{j=1}^{3} \frac{\nu_j}{2} Z^j \quad ; \quad \mathcal{H}_{CD}^{\text{intra}} = \sum_{m<n\leq3} \frac{d_{mn}}{4} (2Z^m Z^n - Y^m Y^n - X^m X^n), \]
\[ d_{mn} = \gamma_C \hbar \frac{1 - 3\cos^2(\theta_{mn})}{2r_{mn}^3}, \]

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_m$</th>
<th>$H_{m1}$</th>
<th>$H_{m2}$</th>
<th>$T_2^* \text{(ms)}$</th>
<th>$T_1 \text{(s)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>5.893</td>
<td>0.227</td>
<td>0.935</td>
<td>-1.5</td>
<td>2.0</td>
<td>2.4</td>
</tr>
<tr>
<td>C2</td>
<td>1.057</td>
<td>1.070</td>
<td>1.4</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$C_m$</td>
<td>-3.445</td>
<td>-18.7</td>
<td>-0.9</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Decoherence time

The large part of the decoherence is due to dipolar coupling with nearby spins. We can decoupled $H$ and increase dilution of the labelled molecules.
In solid state, \( t_{\text{one qubit gate}} \sim t_{\text{two qubit gate}} \), so how do we control the system?

Answer: Construct a modulated RF waveform that generates the desired evolution \([3]\), i.e. minimize \( F \) by modifying \( H_{rf}(t) \) such that:

\[
F = \sum_{\mu} p_{\mu} | \text{Tr} [U_{\text{des}}^\dagger U_{\text{cal}}^\mu] / N |^2
\]

with

\[
U_{\text{cal}}^\mu = U(t) = e^{-i \int_0^t (H_{Bo}^\mu + H_{\text{int}} + H_{rf}^\mu(t)) dt}
\]

using simplex methods.
Example: Control-Not-Not

Fidelity is 98%, average RF amplitude is 9.4 KHz (magnitude of $^{13}C$ Hamiltonian is 7.3KHz), fidelity $> 90\%$ over 1KHz range.
Feedback from the coil

Before feedback:

After feedback:
The decoherence time $T_{C_1}^2 = 8.66\text{ms}$, $T_{C_2}^2 = 9.07\text{ms}$, $T_{C_m}^2 = 5.37\text{ms}$, $T_{3Q}^2 = 2.37\text{ms}$.
Algorithmic cooling

Sorensen [5], Schulman and Vazirani [4]

We have seen that we can cool a subset of spins by swapping states. For example, with 3 spins, implementing a gate that swaps $|011\rangle \leftrightarrow |100\rangle$ will increase the order of the first spin at the expense of the last two. We could concatenate this process to reach polarization of order 1.

$$
\rho \sim e^{-\beta H} \sim \frac{1}{2^n} (\mathbb{I} - \beta \omega (Z_1 + Z_2 + Z_3) + \ldots)
$$

$$
\rho_{\text{thermal}}^{d} \approx \frac{\beta \omega}{8} \begin{pmatrix}
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -3
\end{pmatrix}
\iff
\rho_{\text{pol}}^{d} \approx \frac{\beta \omega}{8} \begin{pmatrix}
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -3
\end{pmatrix}
$$

$$
\tilde{\rho}_{\text{pol}}^{d} = \text{Tr}_{2,3} \rho_{\text{pol}}^{d} \approx \frac{3}{4} \beta \omega \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
$$

We could concatenate this process to reach polarization of $O(1)$, but this would take a lot of resources ($\sim 1/\beta^2$).
Algorithmic cooling with heat bath

Heat bath, polarization $P$

$\tilde{\epsilon}_b \gg 2^{-n}$
$\rightarrow \tilde{\epsilon}_1^{max} = 1$

$\tilde{\epsilon}_b < 2^{-n}$
$\rightarrow \tilde{\epsilon}_1^{max} = \tilde{\epsilon}_b \times 2^{n-2}$
Algorithmic cooling with heat bath

Manipulate spins that are coupled to a heat bath. The first six steps of (Schulman, Mor and Weinstein, PRL94, 2005)
Experimental results

Next step: DNP
DNP results on Tempo

Thermal and DNP Polarization Enhancement

- $^{13}$C /$^1$H cross-polarization

- 60:40 glycerol:water sample with 40mmol TEMPO radical

- Natural abundance $^{13}$C in glycerol/water glass

- ~50X signal enhancement compared to room temperature

- Would have 420X enhancement if full thermal equilibrium at low temperature is reached ($T_1 \sim 550$ s)

- 2 scans

- 4K + DNP

- 4K

- 100K

- 30 scans

- Frequency (Hz)

- Intensity (a.u.)
References


