Mr. S.M.Ulam,
Los Alamos Scientific Lab.,
New Mexico, USA.

    Dear Mr. Ulam:

    We send you our paper in which we are investigating the problem close to one in your report at 4-th Berkeley Symp. on Math. Stat. and Probab. (Proc., vol.3, 315). It would be very interesting to discuss with you the problem, especially the case of many degrees of freedom. Have you any new results?

    Please, send enclosed preprints to Mr.K.W.Fofd and Mr.J.M.Hammersley because we don't know their adress.

        With our best regards
            Sincerely yours

                G.M.Zaslavsky
                B.V.Chirikov

          Novosibirsk State University,
          Novosibirsk,72, USSR.

12/VI 64.

Mr.S.M.Ulam,
Los Alamos Scientific Lab,
New Mexico, USA.

Dear Mr.Ulam:

I write to you in connection with two J.Ford's papers
(J. Math. Phys.,$\underline{2}$, 387, 1961/a/; ib., $\underline{4}$, 1293, 1963 /b/) in
which the author is trying to abandon ergodicity. I disagree
completely with this "groundwork for futher study of irreversib-
le processes"/b/. I disagree with Ford's explanation of your
failure with energy sharing, too (E.Fermi, J.Pasta, S.Ulam, Stu-
dies of Nonlinear Problems, Los Alamos Sci. Lab. Rept. LA1940 /c/).
In my opinion the lack of energy sharing results not from an un-
fortuitous choice of uncoupled frequencies ω_K (about $\left(\frac{2N}{\pi}\right)\left(3\alpha\right)^{1/3} \approx$
≈ 40 least frequencies satisfy Ford's resonance conditions
(3)/b/) but from the special, very weak coupling between neigh-
bouring modes as it's clear from the expressions (18,30) of
Ford's paper /a/ (small multipliar $\left[\sin\frac{\pi(r+k)}{N} - \sin\frac{\pi r}{N} - \sin\frac{\pi k}{N}\right]$).

In this connection it would be curious for me to get a
copy of the paper /c/ with your numerical calculations especially
for the cubic nonlinearity in the force ($\ddot{x}_i = \ldots \beta\left[(x_{i+1}-x_i)^3 -$
$- (x_i - x_{i-1})^3\right]$) mentioned in your book ("A collectio of Mathemati-
cal Problems"). In the paper I sent you recently the criterion
for stochosticity (i.e. ergodicity, mixing and finite entropy in
Kolmogorov's sense (ДАН СССР /Proc. Acad. Sci. USSR/, $\underline{119}$, 861,
1958) was obtained. I think it's possible to derive similar
criterion for the cubic nonlinearity mentioned above.

With my best regards
Sincerely yours

B.Chirikov,
Novosibirsk University,
Novosibirsk,72, USSR.

29/viii 64.

Mr. S.M.Ulam,
Los Alamos Scientific Lab,
Los Alamos, New Mexico,
          U S A

          Dear Mr. Ulam:

          Thank you very much for your interesting letter and
preprint which I received recently. I just have sent you two co-
pies of your book translated into Russion. I enclosed some pic-
tures of our academic town, too.

          I think that existence of a "super period" mentioned
in your letter is natural and is connected with small denomina-
tors, i.e. with almost-resonances of high orders in nonlinear
system. I believe there are many such increasing super periods
with decreasing amplitudes. As far as I understand now your and
J.Tuck's calculations and recent work of E.Jackson (J. Math. Phys.,
4, 686, 1963) as well confirm the Kolmogorov's result about sta-
bility of almost periodic motion under small perturbations. How-
ever one fails to apply the exact theorems to the calculation
mentioned because the systems in question are linear in zero ap-
proximation. A very interesting paper of Arnold (ДAH CCCP, 156,
9, 1964/Proc.Acad.Sci.USSR/) appeared recently which deals
with such stability.

          I have read with great interest Zabusky's paper men-
tioned by you on shock wave formation in nonlinear string but I
think it bear no relation to modes stability.

          I would like very much to get a book of A.Einstein,
"Ideas and Opinions". If it won't be much trouble for you, please,
send me a copy.

                    With my best regards
                         Sincerely yours

     21 /IX 64.

                              B.Chirikov

Prof. S.M.Ulam
Department of Mathematics
University of Colorado
Boulder, Colorado

Dear Prof. Ulam:

At last I have heared from you! How are you? and what
are your plans for this year? Have you received my preprints in-
cluding one on "Numerical Experiments with a Nonlinear Chain"
originated by your work? There are a lot of questions to discuss
about this problem. Are you still interested in this? We could
start the discussion by mail.

Meanwhile I have finished the book about stochasticity.
It must appear soon in rotoprint edition and I shall send you a
copy. I am going to visit at CERN this May and June and to per-
form some numerical experiments on the stability of particle mo-
tion in proton storage rings.

Best wishes to you and your wife,

Sincerely,

21/I 69.

Boris Chirikov

P.S. I take the opportunity to congratulate you with this fasci-
nating Apollo 8 flight which really has oppened a new era of
space travel!

September 15, 1969

Professor Joseph Ford
Georgia Institute of Technology
School of Physics
Atlanta, Georgia   30332

Dear Professor Ford:

Thank you very much for your letter and for the preprint which
I started to study and which I found fascinating. A very impressive
theory is being built.   Ever since my calculations with Fermi and
Pasta (it is almost sixteen years ago I notice with horror) I have
been wondering which systems become "ergodic" and at what rate.

I would like very much to visit you in Atlanta (also would like
both Chirikov and yourself to visit Boulder which, especially in May
and in June, is very pleasant.) Perhaps we could arrange both, as
you say, at our leisure.

With best regards,

Sincerely yours,

S. M. Ulam

SMU/cmc

P.S. Dr. Conrad Longmire in Los Alamos has obtained interesting
results this summer on the problem which I have investigated some
years ago of a ball with small mass between two heavy walls which
oscillate periodically. The question was at what rate this mass
will acquire high velocity. The results which I had showed were
that this goes much more slowly than one would expect from the
precepts of statistical mechanics. Hammersley considered this
problem statistically in one of the Berkeley symposia on probability,
at which I gave my paper. Longmire's results make it plausible.

UNIVERSITY OF FLORIDA

DEPARTMENT OF MATHEMATICS
201 Walker Hall

Gainesville, Florida 32611
Telephone (code 904) 392-0281

February 12, 1976

Professor Joseph Ford
School of Physics
Georgia Institute of Technology
Atlanta, GA   30332

Dear Professor Ford,

Many thanks for your letter and your interesting reprint. Unfortunately, I do not have any more reprints of the articles you inquire about. Some of these have however, been reprinted in a volume published in 1974 (actually appeared early in 1975) by MIT Press under my name with the title "Sets, Numbers, and Universes". It is a thick volume of selected papers of mine. Perhaps your library has this volume and you could get a xerox copy of it. There is a long article there, jointly with P.R. Stein, "Non-Linear Transformation Studies on Electronic Computers", of some 80 pages or so. There is also a reprint of my article with Fermi and another one with J. R. Pasta. Still another one, "On Some Statistical Properties of Dynamical Systems."

I am sorry but I have given away the few copies of this book which I received from the publisher. With my very best regards,

Sincerely,

S. Ulam

SMU/jm

EQUAL EMPLOYMENT OPPORTUNITY/AFFIRMATIVE ACTION EMPLOYER

IN REPLY
REFER TO: T-DOT
MAIL STOP: 210   August 2, 1977

Professor Joseph Ford
Georgia Institute of Technology
Atlanta, Georgia 30332

Dear Professor Ford:

Many thanks for your most interesting note relating to Ergodic Theory. What you write me is amazing; I would have guessed less than 1/10 of the number of pariticpants and many fewer meetings.

I am interested in receiving the information. As a matter of fact, I have written, myself, several papers in this area. One, I think, rather important paper was with J. Oxtoby in the Annals of Mathematics in November 1941, I believe, where we proved that ergodic transformations are the rule in measure preserving transformation on manifolds. Together with Pasta and the late Enrico Fermi, we have shown some peculiar non-linear problems leading to seemingly non-ergodic flows. I know that you have made contributions to that and there is now a vast literature on this and allied problems. There are some other papers of mine on connected subjects and I will send you some of the references.

Many thanks again.

Sincerely,

Stan Ulam

SU/rep

cc: S. Ulam

Al