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Book Review

Long-Time Prediction in Dynamics. Edited by C. W. Horton, Jr., L. E. Reichl, and V. G. Szebehely. John Wiley & Sons, New York, New York, 1983, xv + 496 pp., $85.00 (cloth).

The reviewed book, a collection of papers presented at an International Workshop held in March 1981 in Lakeway, Texas, is mainly devoted to various aspects of a peculiar phenomenon, the *dynamical chaos* (or stochasticity), i.e., the random (without quotes!) motion of completely deterministic (= dynamical) systems in classical mechanics. Perhaps, it would have been more natural to entitle the book something like "Long-Time *Unpredictability* in Dynamics."

Until very recently dynamical chaos had appeared so "strange" and a few then known examples of the chaos seemed "pathological." By now, hundreds of examples are known in dynamical systems ranging from "simple" mechanics to chemistry and biology—hence a great and ever-increasing interest in this phenomenon. The reviewed collection of papers, written by leading experts, contributes a significant insight into this interesting and important domain of classical mechanics.

Besides numerous applications over a broad spectrum of problems on motion stability, chaos bears upon some fundamental questions, such as the nature of randomness and of the statistical laws as well as its relation to the dynamical laws of physics. The book would therefore be of particular interest to readers of *Foundations of Physics* as well. The present review does emphasize just this latter point.

To begin with, all the authors adhere to the idea of the primality of the dynamical laws, which are considered to be more fundamental than the statistical laws that arise out of the former under particular conditions. This is indeed a natural viewpoint, though not the only one possible [see, e.g., J. A. Wheeler, *Am. J. Phys.* **51**, 398 (1983)]. Besides, one should bear in mind the possibility of a *secondary dynamics* (e.g., Prigogine's dissipative structures, or the so-called "synergetics"), that is, the dynamical laws arising out of statistical ones.

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From the logical viewpoint, the principal paper in the book is that due to J. Ford, as it presents an up-to-date understanding of the nature of randomness and of statistical laws (Sections 4 and 5). It is based upon the so-called algorithmic theory of dynamical systems, developed essentially by the Kolmogorov school. A fundamental concept of this theory is the mathematically rigorous and physically satisfactory notion of random individual trajectories. According to the Alekseev–Brudno theorem, the necessary and sufficient condition for such a randomness is positivity of the dynamical entropy ($h>0$), i.e., a strong (exponential) local instability of motion. *No* additional *Ansätze* are required for a statistical description. It should be mentioned, however, that the conjectures of a more general nature in Section 6 are disputable.

Even though the new explanation of randomness seems to be of a fundamental importance, it has not yet been developed into a sound theory of statistical mechanics. Therefore, it was well justified (and not from the historical viewpoint only) to include the papers by J. Lebowitz, by B. Misra and I. Prigogine, by H. Grad, and by S. Goldstein, in which a much more elaborate classical explanation of the statistical laws is essentially discussed (with some reservations, perhaps, for the second paper of a transient philosophy). The classical explanation applies to the limit of indefinitely many degrees of freedom, and it requires some additional, though natural, statistical *Ansätze*, such as certain restrictions upon the initial distribution function or Boltzmann's "initial chaos." As we know now, neither is necessary under the condition $h>0$. However, the classical explanation acquires today a new and unexpected relation to the problem of *quantum chaos*, that is, the evolution of the $\psi(t)$ function of a classically chaotic quantum system. In this case, only a temporary imitation of true randomness, or a *transient chaos*, is possible. But the same is true for the completely integrable, particularly linear, systems in classical mechanics. Such an example (linear waves in inhomogeneous plasma) is considered by E. Ott. Here one just needs the classical explanation! The nature of the imitation is also demonstrated via a simple example by A. Salat and J. Tataronis. In the latter model there is no real chaos, of course, as $h=0$ (for a finite motion). Yet, some trajectories do appear fairly irregular. This emphasizes the importance of the rigorous criterion for randomness ($h>0$).

The rest of the book is devoted to various particular aspects of chaotic dynamics as well as to some of its applications.

In papers of R. Helleman, of R. MacKay, and of D. Escande the critical phenomena, typical for the transition from regular to chaotic motion, are considered. The first two papers deal with Feigenbaum's period-doubling bifurcations whose importance for chaotic dynamics appears to be rather exaggerated, at least as to Hamiltonian systems. A more universal approach is presented by D. Escande, following the ideas of J. Greene (whose paper in the book is devoted to another, more special question).

M. Lieberman and J. Tennyson consider a rather peculiar regime of chaotic motion, the so-called Arnold diffusion, as well as a related modulational diffusion. An application of the former to a particular problem of the beam–beam interaction is discussed by T. Bountis, C. Eminhizer, and R. Helleman. A general review of various experimental data on the stability of the colliding beams as well as some theoretical estimates are provided by S. Kheifets.

The effects of an external noise upon a dynamical system are described in the papers by A. Rechester *et al.* and by J. Tennyson. C. Grebogi and A. Kaufman and D. Dublin and J. Krommes discuss some methodical problems in the theory of dynamical systems. Celestial mechanics is addressed in the papers by V. Szebehely and by R. Vicente, with examples of chaotic motion for the restricted three-body problem in the first paper, while the theory of plasma turbulence is the subject of papers by C. Horton and by K. Molvig *et al.*

The case opposite to chaotic motion, namely, the dynamics of stable nonlinear entities, the solitons, is described by Y. Ichikawa for the Alfven waves in plasma, and by J. Hyman *et al.* for the Davydov solitions in α-helix protein.

At the end of his paper, V. Szebehely poses a question: "... if celestial mechanics is not a deterministic science (and aparently it is not), is there any meaning in our attempts to predict the future of the solar system or of our galaxy?" Perhaps, this is a good place to conclude my review with a brief remark that the *long-time unpredictability* (rather than indeterminacy) does not exclude the *short-time prediction* and even presupposes the latter, as the chaos we are talking about is the *dynamical chaos.* Just recall weather forecasting!

The theory of dynamical chaos is fairly young and rapidly developing. It is therefore less of a surprise that the related workshop and the collection of papers (as well as the present review!) are somewhat "chaotic," too. If, nevertheless, I managed to attract the reader's attention to this new and intriguing field of research in physics and mathematics and to prompt him to "rummage" through the book—where a lot of new insights and ideas, conjectures, and results are scattered around—I would regard my objective as realized.

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