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COMMENT

On self-avoiding walks in critical dimensions

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Abstract. A new interpretation of the numerical data for self-avoiding walks in critical dimensions is suggested on the basis of a different renormalization scheme for the random walk with a long-term correlation.

In a recent paper [1] Grassberger *et al* presented very rich numerical data on the self-avoiding walks in critical dimensions $d = 4$. They clearly demonstrated the logarithmic dependence predicted by the standard renormalization theory. However, the critical exponent of this dependence, $\alpha = 0.31$, found by the fitting of numerical data to the asymptotic relation ($\ln N \to \infty$)

$$\langle R_N^2 \rangle = RN \left[\ln \left(\frac{N}{A} \right) \right]^{\alpha} \tag{1}$$

differs substantially from the theoretical value $\alpha = \frac{1}{4}$. The authors [1] resolve this apparent contradiction by including the first correction term of the renormalization theory

$$\langle R_N^2 \rangle = rN \left[\ln \left(\frac{N}{a} \right) \right]^{1/4} \left[1 - \frac{17 \ln(4 \ln(N/a)) + 31}{64 \ln(N/a)} \right] \tag{2}$$

with fitting parameters $r = 1.331$ and $a = 0.1237$ in the range $N = 20$–4000.

Additional numerical data up to $N = 10^7$ are mentioned (but not given) in [1], and the local exponent is said to decrease down to $\alpha \approx 0.285$. Meanwhile, using three-parameter equation (1) it is possible to fit *all* the numerical data with a single value of $\alpha = 0.298$, and $R = 1.10$, $A = 1.22$ provided the additional data ($N = 4000$–10^7) match (2) with the same r and a. The fitting accuracy is $|\delta q|/q < 1.3 \times 10^{-3}$ where $q(N) = \langle R_N^2 \rangle$. So, the comparison with the theory is a tricky task indeed [1].

The main purpose of this comment is to point out a different approach to the problem based upon another renormalization theory for the random walk with a long-term correlation [2, 3]. The main idea of this approach is in that the self-avoiding of a trajectory with a *finite* width results in a correlation proportional to the ratio of the trajectory length N to the volume occupied by the diffusion [3]:

$$C(N) = b \frac{N}{q^{d/2}} = \frac{d^2 q(N)}{2d N^2} \tag{3}$$

where b stands for some numerical factor and where the latter equality is the standard correlation/dispersion relation if the correlation integral (diffusion rate) diverges.

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The power-law solution to equation (3)

$$q(N) = RN^{2\nu} \qquad \nu = \frac{3}{2+d} \qquad R^{1+d/2} = \frac{b(2+d)^2}{3(4-d)} \tag{4}$$

leads to the well known Flory formula for ν which is valid for $d < 4$. In the critical case $d = 4$ the asymptotic solution has the form (1) with $R = (6b)^{1/3}$ and $\alpha = \frac{1}{3}$. That the latter value is well in agreement with the numerical data is the main result of this comment. The remaining parameter A is determined by a non-asymptotic correction term similar to that in (2).

References

[1] Grassberger P, Hegger R and Schäfer L 1994 *J. Phys. A: Math. Gen.* **27** 7265
[2] Chirikov B V and Shepelyansky D L 1984 *Physica* **13D** 395
[3] Chirikov B V 1991 *Chaos Solitons Fractals* **1** 79