



Google matrix of the world trade network



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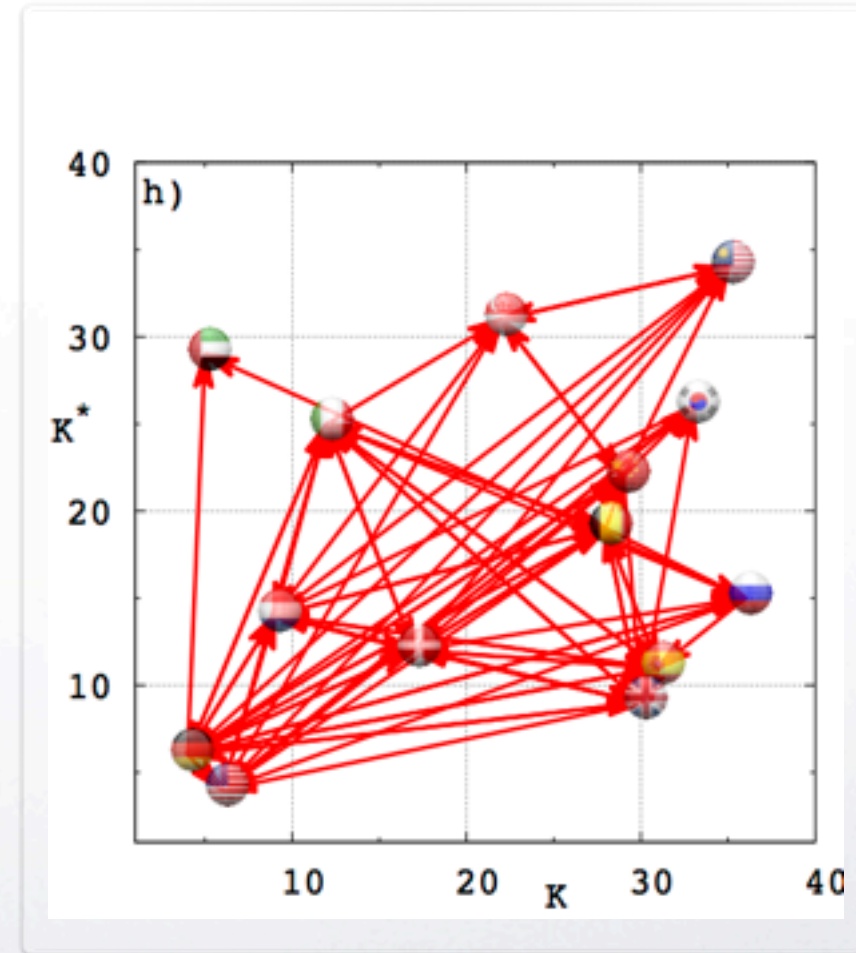


Outline

- I • Google Matrix of WTN
- 2D-rank of WTN
- WTN models

- II • Ecological Ranking (nestedness)

- III • Multi-Product Network
- Crisis model





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Google Matrix of the WTN



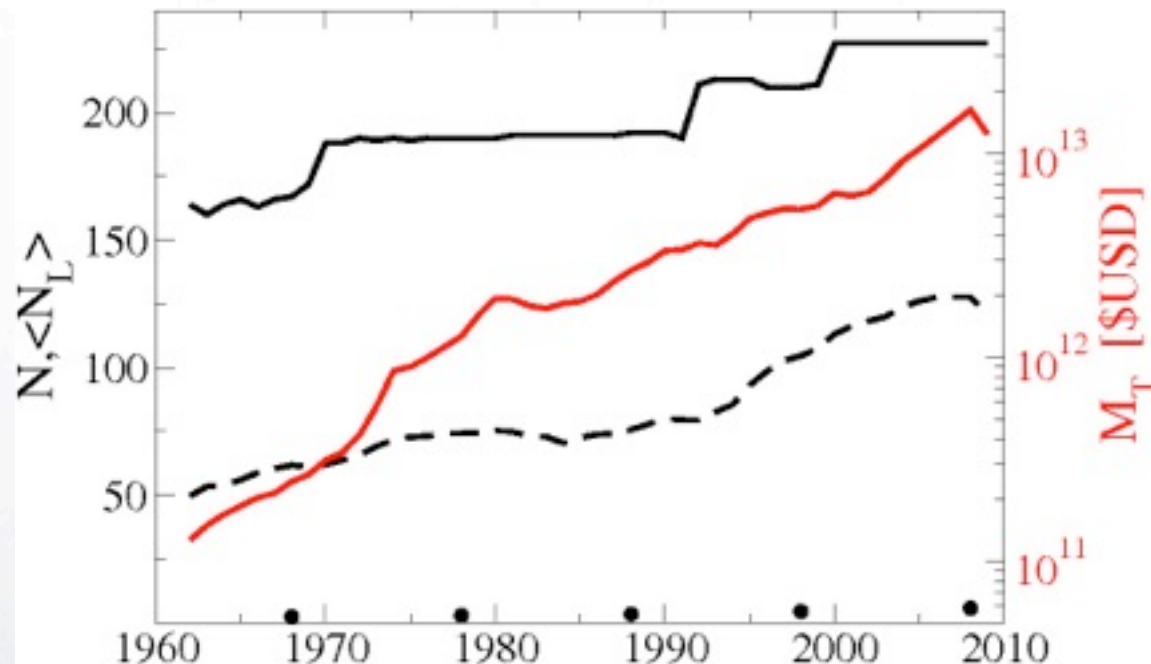
Google Matrix of the WTN

United Nation Commodities Trade Network

all countries of UN, from 1962 to 2011, all commodities or some specific products

Money Matrix

$$M_{i,j} = US\$(j \rightarrow i)$$



L. Ermann and D.L. Shepelyansky, APPA, Vol. 120, A-158 (2011), arXiv:1103.5027, <http://www.quantware.ups-tlse.fr/QWLIB/tradecheirank>



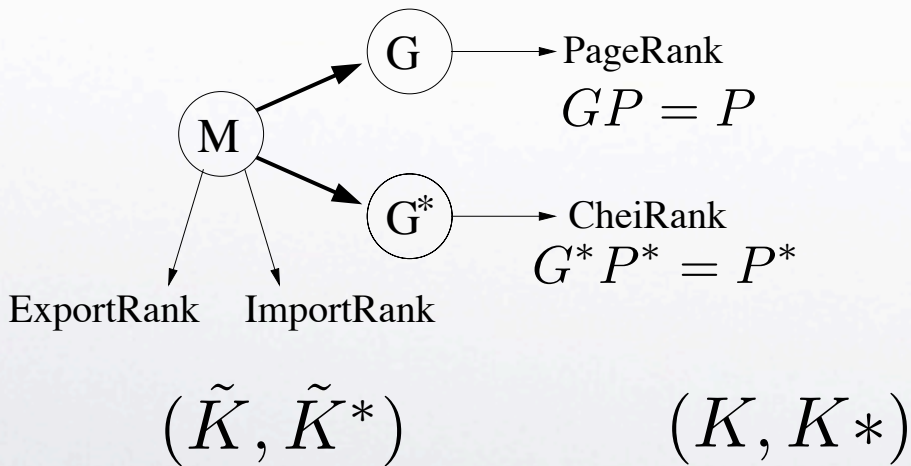
Google Matrix of the WTN

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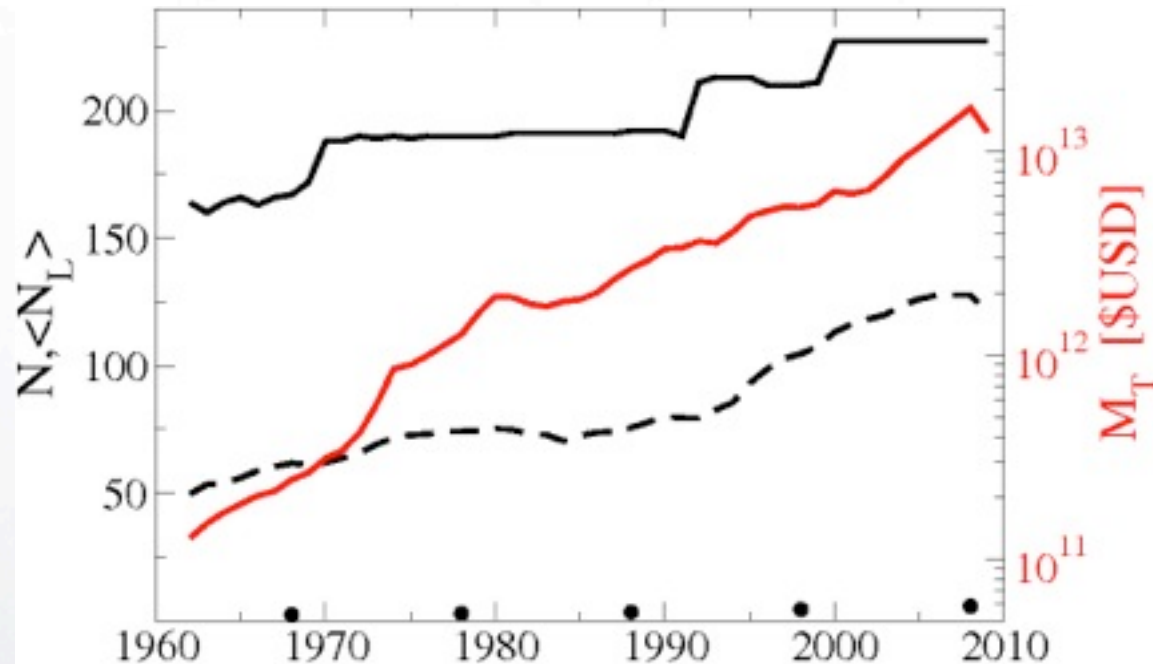
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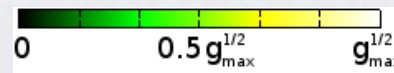
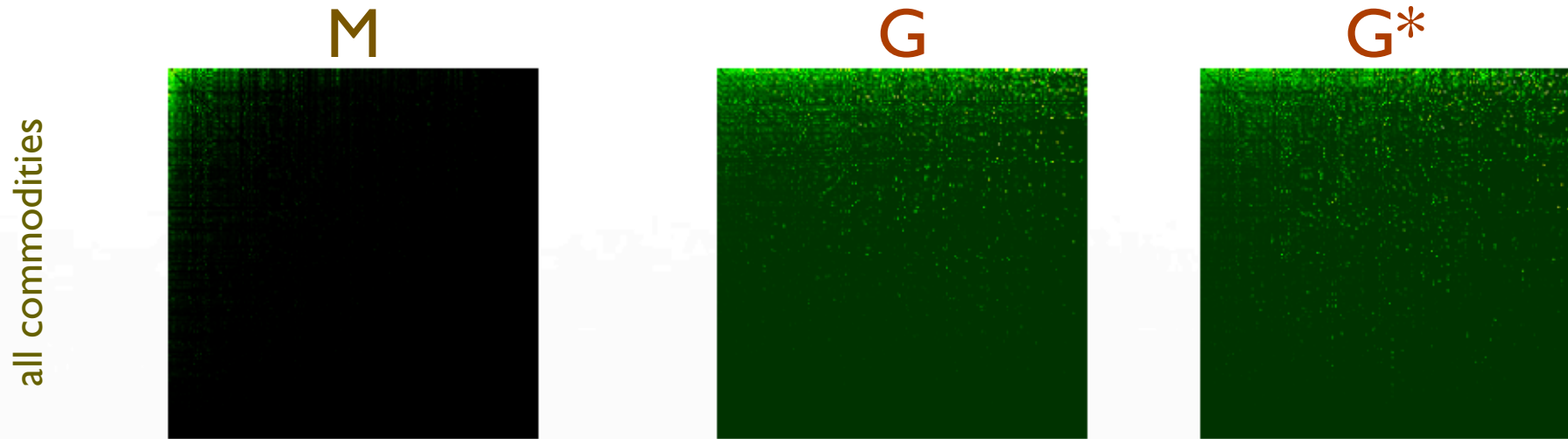
ImportRank, ExportRank PageRank, CheiRank



L. Ermann and D.L. Shepelyansky, APPA, Vol. 120, A-158 (2011), arXiv:1103.5027, <http://www.quantware.ups-tlse.fr/QWLIB/tradecheirank>

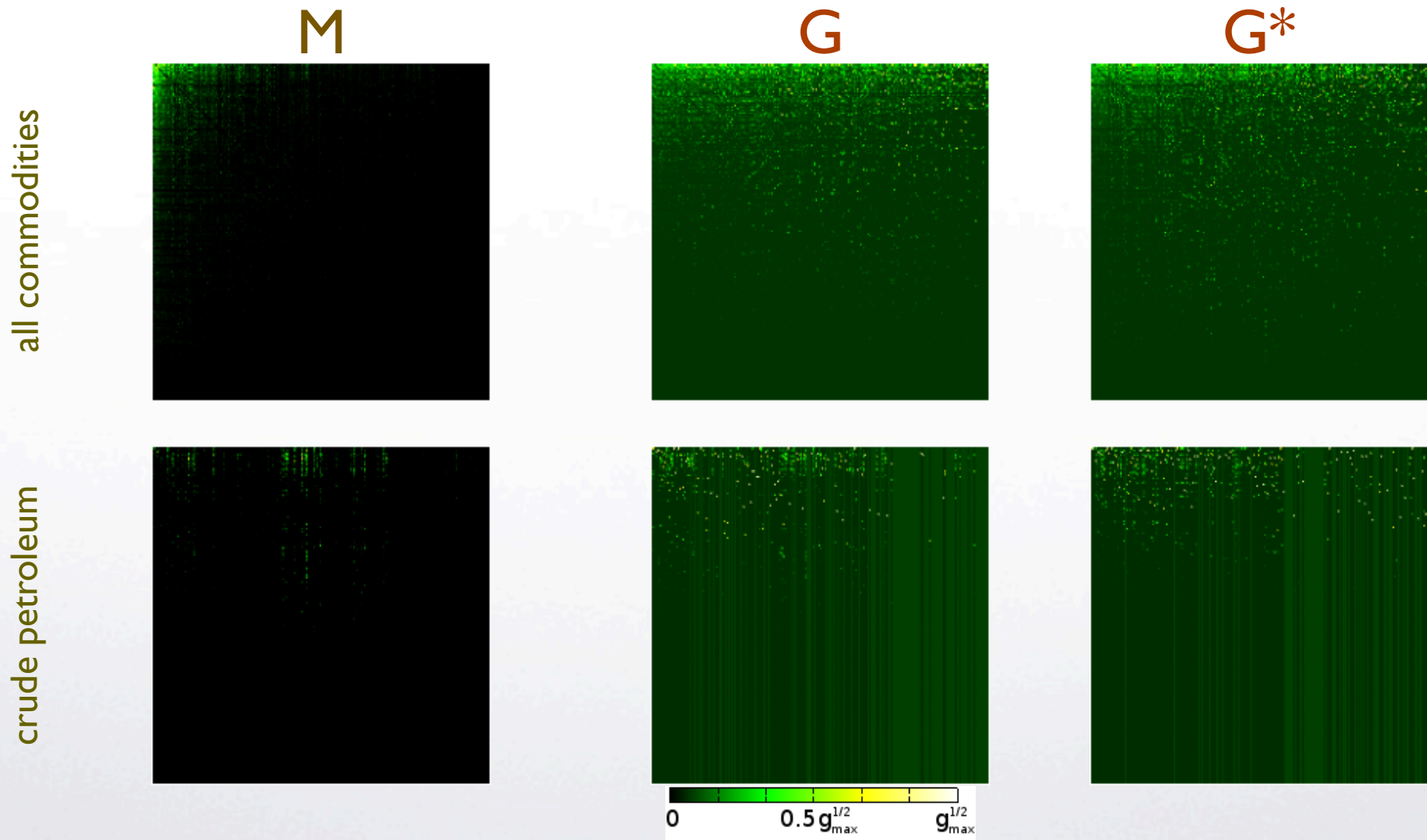


G and G^* matrices of the WTN (2008)





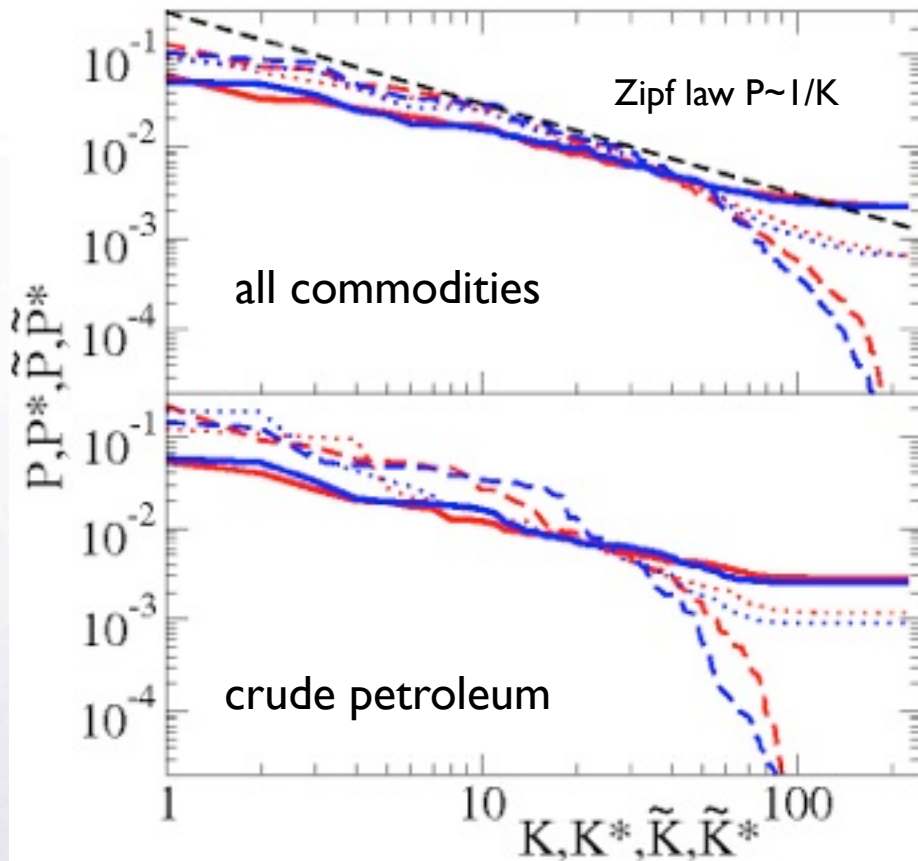
G and G^* matrices of the WTN (2008)





PageRank, CheiRank and Spectrum

PageRank, CheiRank, ImportRank, ExportRank $\alpha = 0.85$
 $\alpha = 0.5$

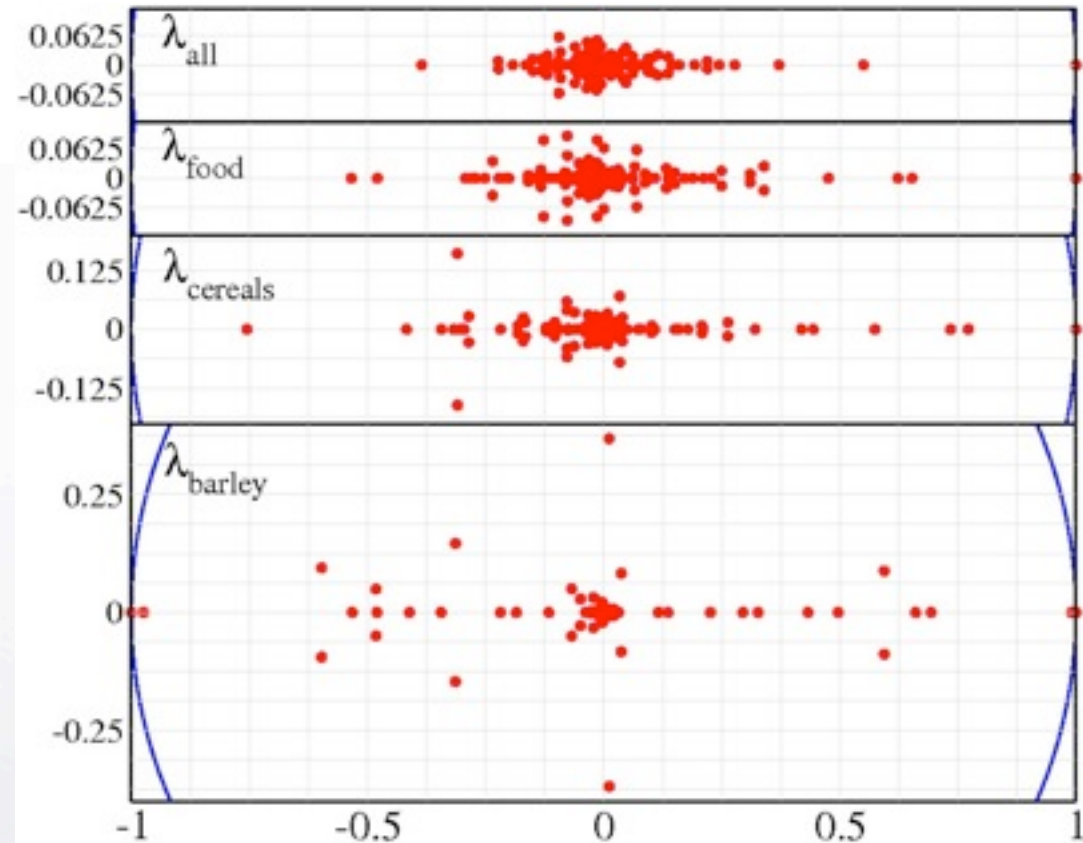
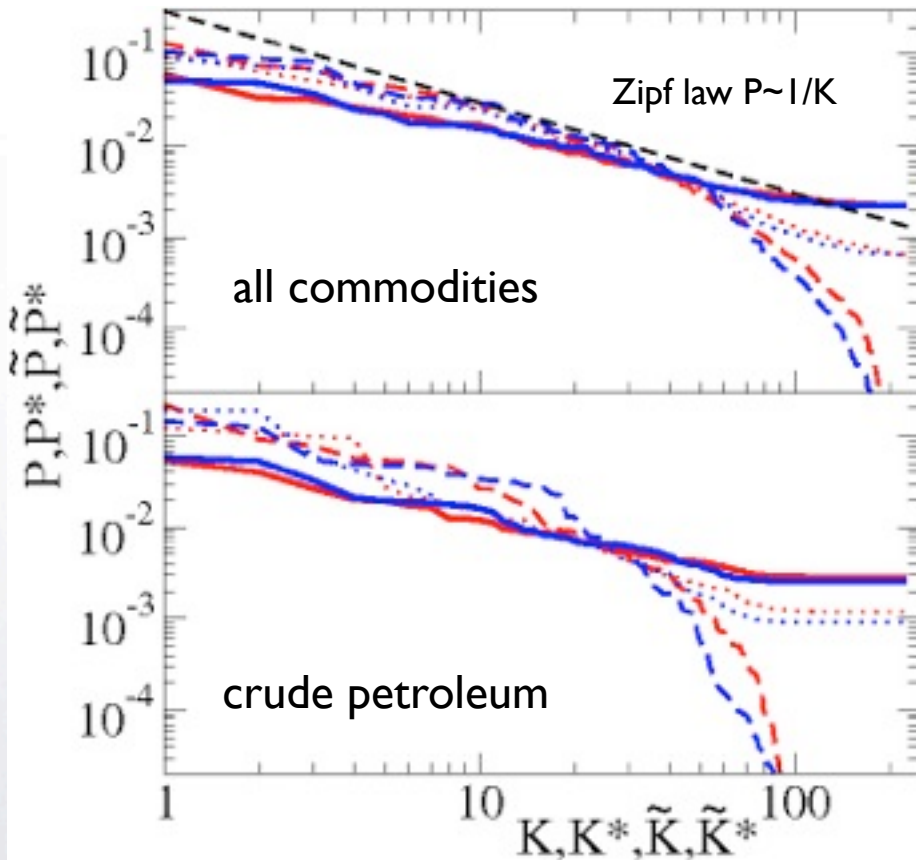




PageRank, CheiRank and Spectrum

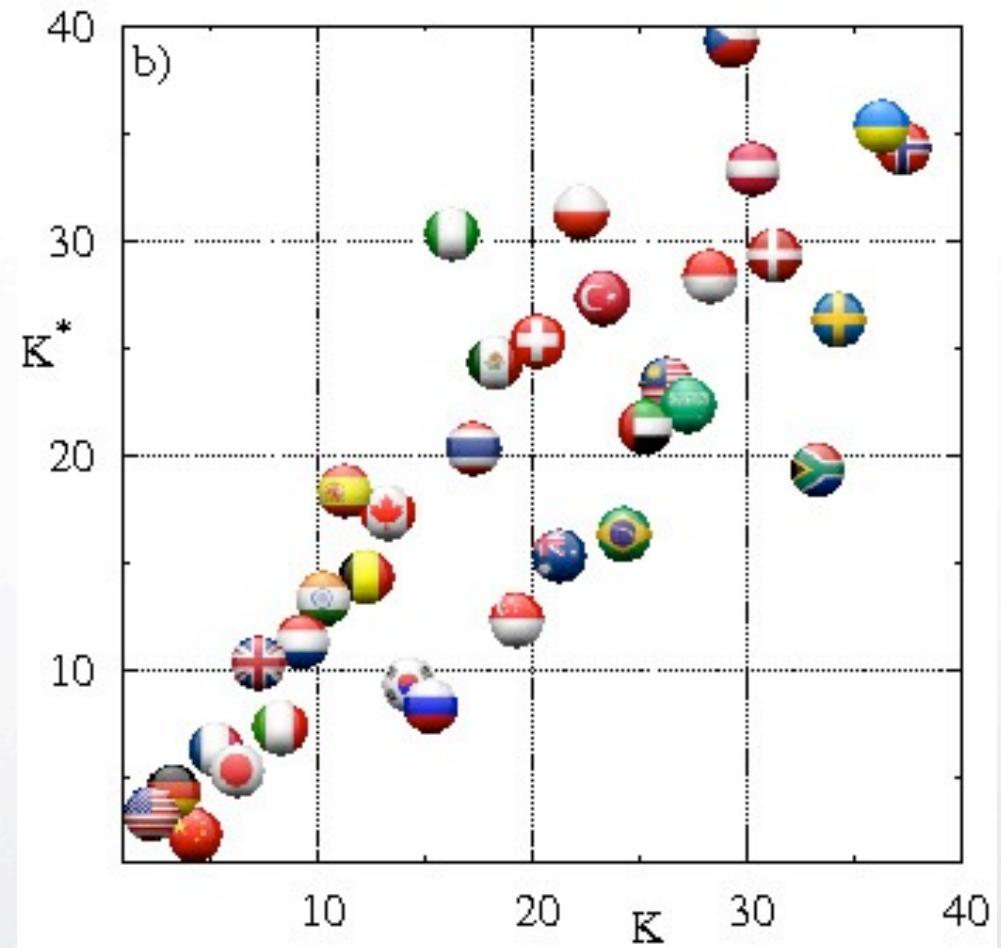
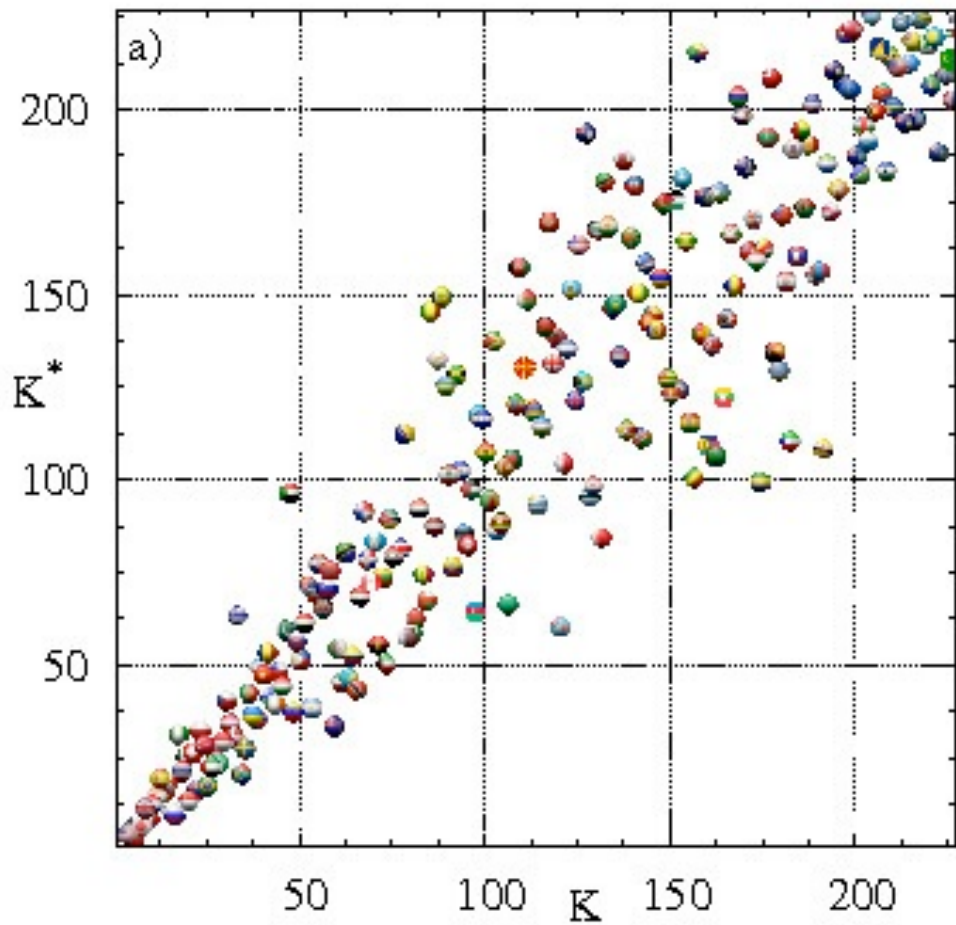
PageRank, CheiRank, ImportRank, ExportRank $\alpha = 0.85$
 $\alpha = 0.5$

Spectra $\alpha = 1$



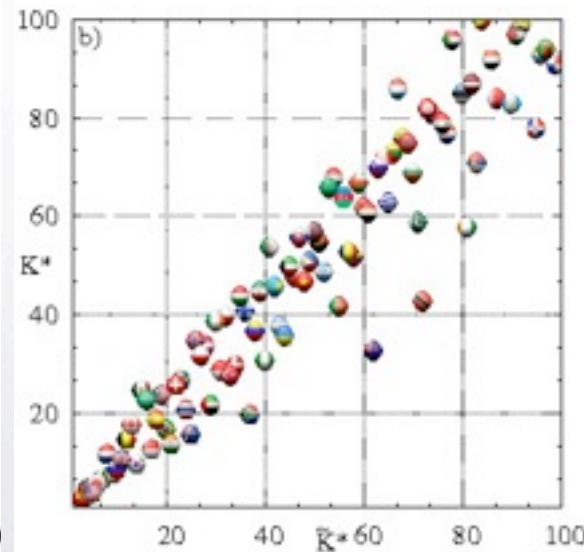
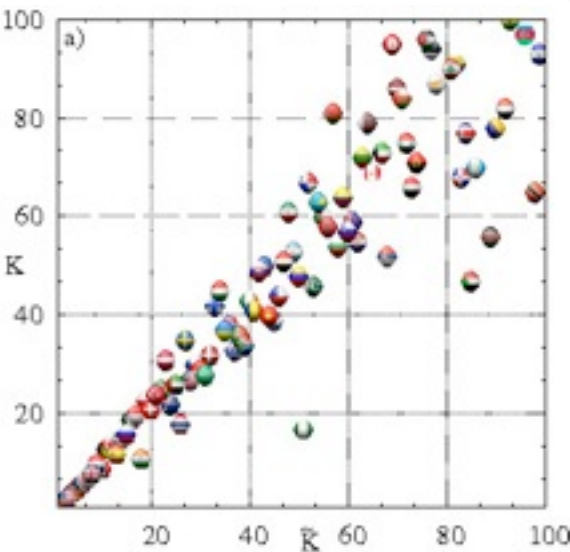
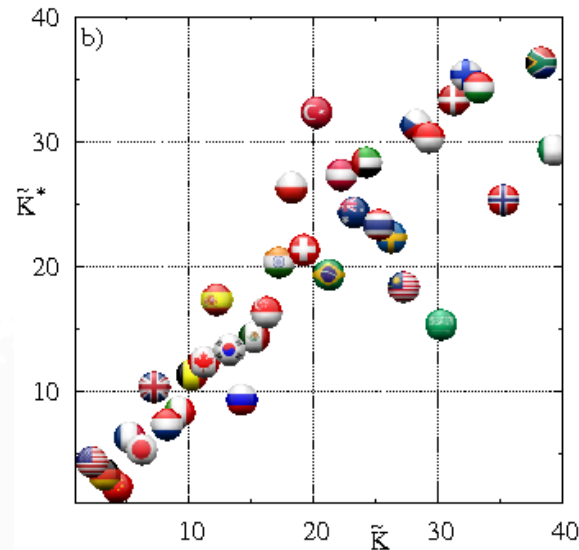
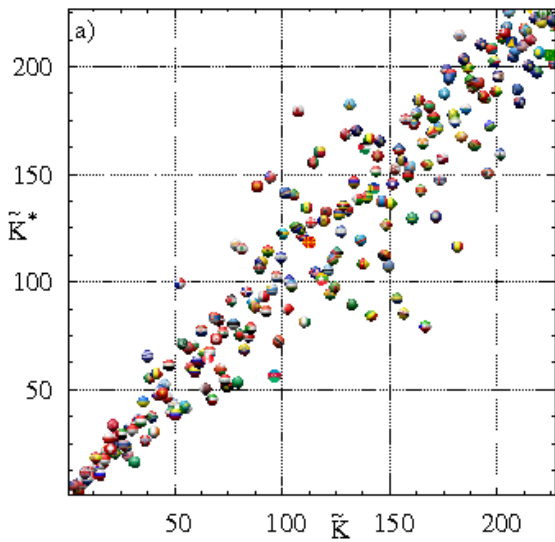


2d ranking “all commodities”



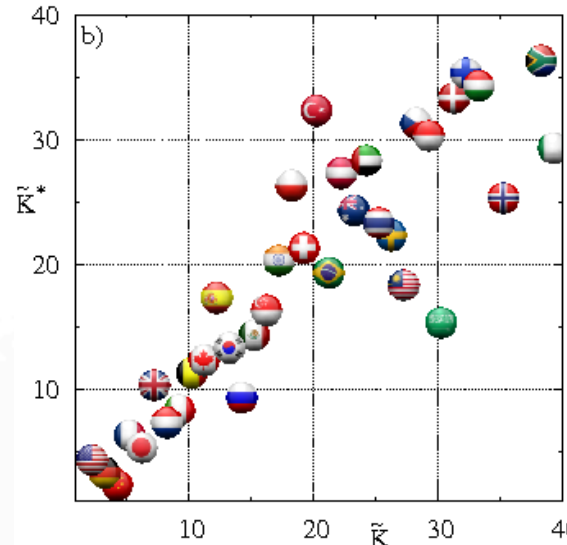
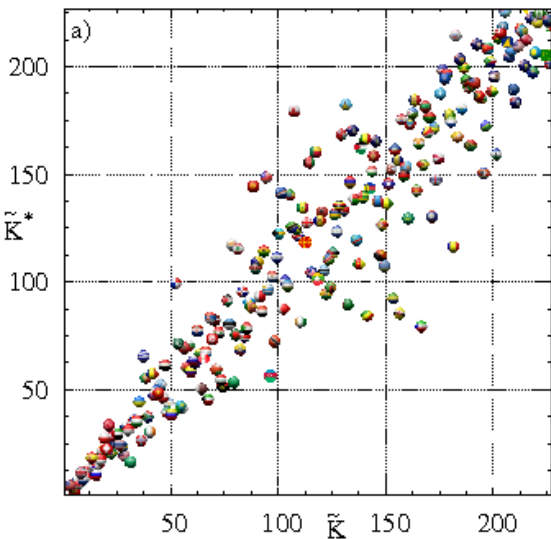


PageRank, CheiRank vs. ImportRank, ExportRank





PageRank, CheiRank vs. ImportRank, ExportRank



countries are treated on equal democratic ground
G-20 ~ 74%

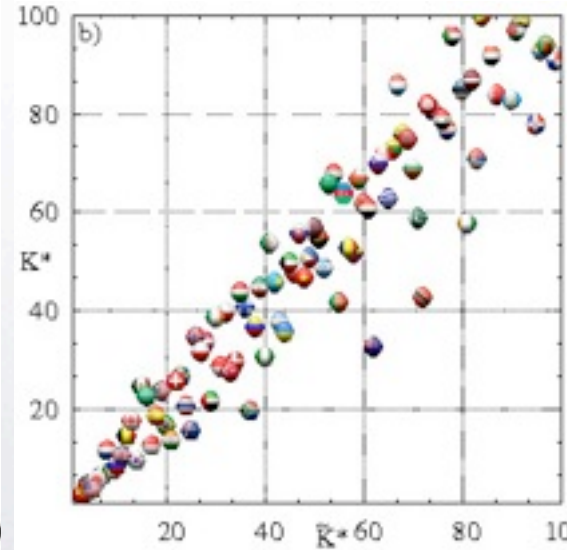
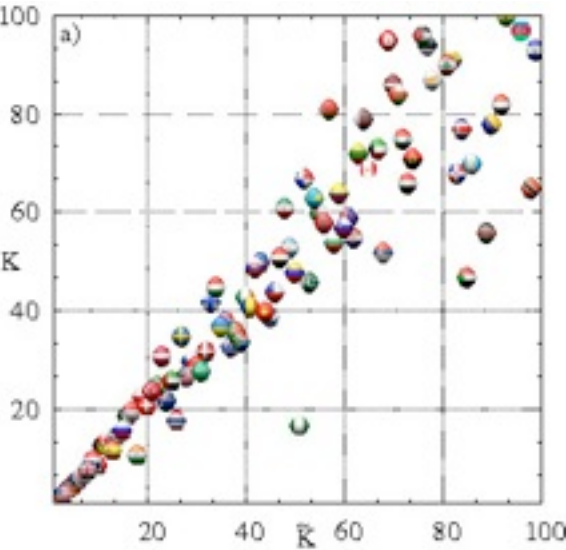
Ran	K	K^*	K_2	\tilde{K}	\tilde{K}^*
1	USA	China	USA	USA	China
2	Germany	USA	China	Germany	Germany
3	China	Germany	Germany	China	USA
4	France	Japan	Japan	France	Japan
5	Japan	France	France	Japan	France
6	UK	Italy	Italy	UK	Netherlands
7	Italy	Russian Fed.	UK	Netherlands	Italy
8	Netherlands	Rep. of Korea	Netherlands	Italy	Russian Fed.
9	India	UK	India	Belgium	UK
10	Spain	Netherlands	Rep. of Korea	Canada	Belgium
11	Belgium	Singapore	Belgium	Spain	Canada
12	Canada	India	Russian Fed.	Rep. of Korea	Rep. of Korea
13	Rep. of Korea	Belgium	Canada	Russian Fed.	Mexico
14	Russian Fed.	Australia	Spain	Mexico	Saudi Arabia
15	Nigeria	Brazil	Singapore	Singapore	Singapore
16	Thailand	Canada	Thailand	India	Spain
17	Mexico	Spain	Australia	Poland	Malaysia
18	Singapore	South Africa	Brazil	Switzerland	Brazil
19	Switzerland	Thailand	Mexico	Turkey	India
20	Australia	U. Arab Emir.	U. Arab Emir.	Brazil	Switzerland

$$\tilde{K}^* = 11 \longrightarrow K^* = 16$$

$$\tilde{K}^* = 13 \longrightarrow K^* > 20$$

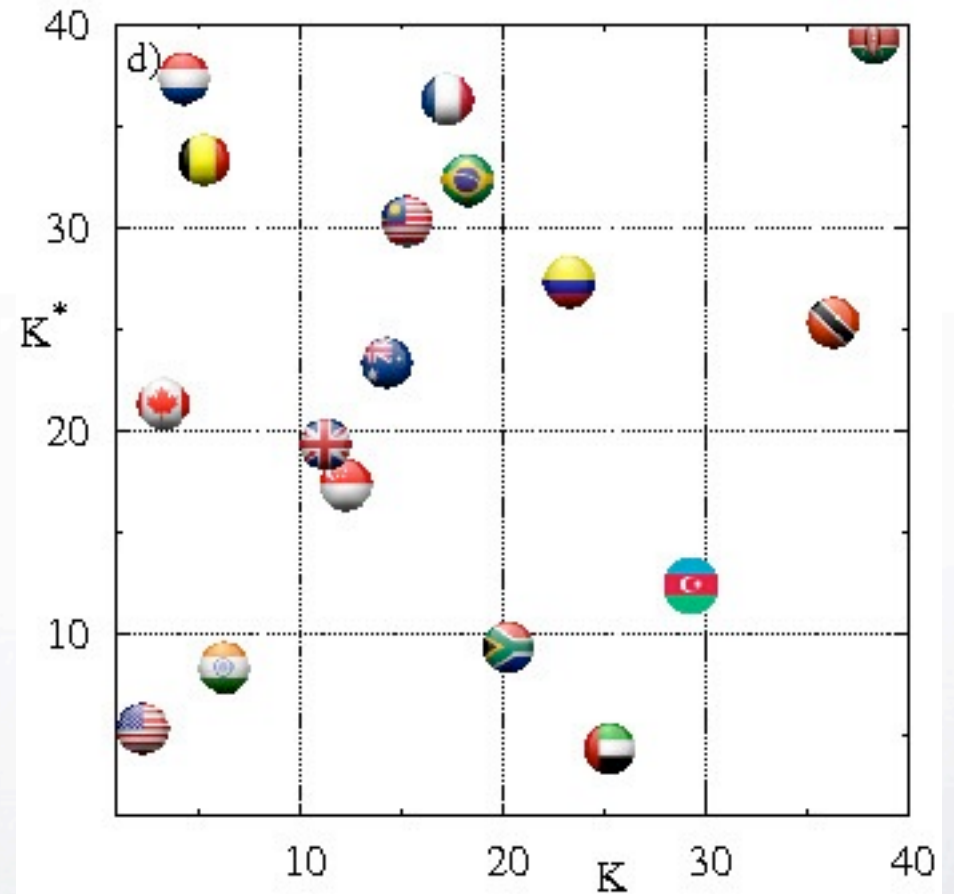
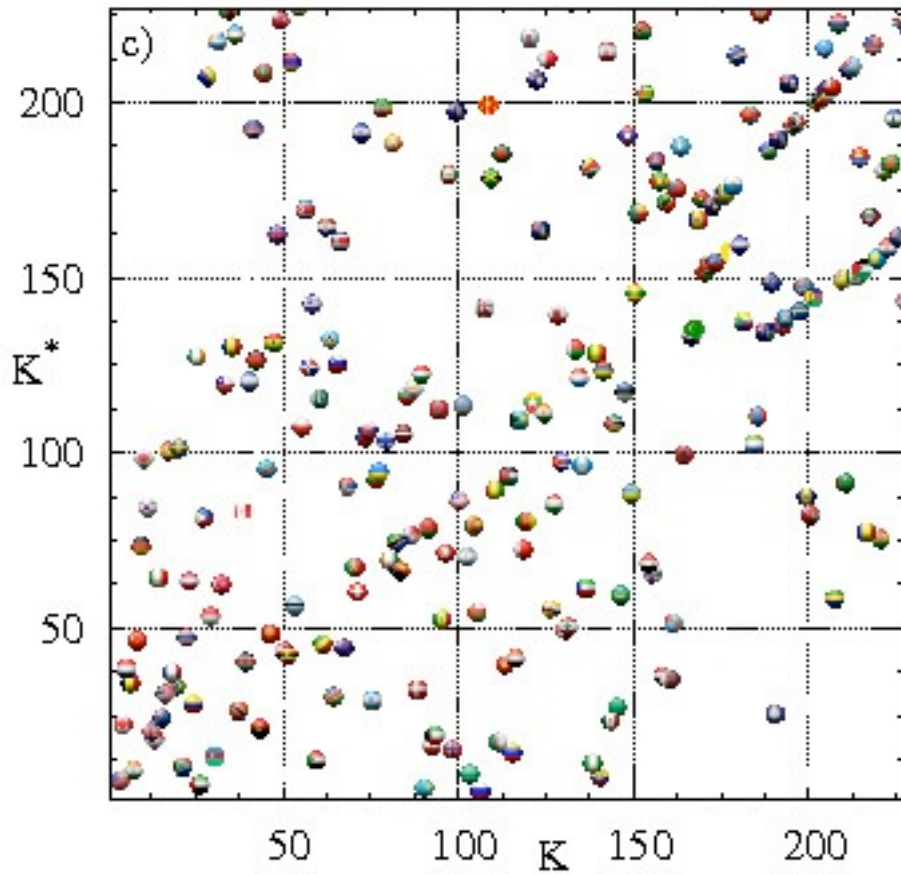
$$\tilde{K}^* = 15 \longrightarrow K^* = 11$$

$$\tilde{K}^* = 19 \longrightarrow K^* = 12$$





2d ranking: crude petroleum



Russia $\tilde{K}^* = 2 \rightarrow K^* = 1$

Iran $\tilde{K}^* = 5 \rightarrow K^* = 14$

Kazakhstan $\tilde{K}^* = 12 \rightarrow K^* = 2$

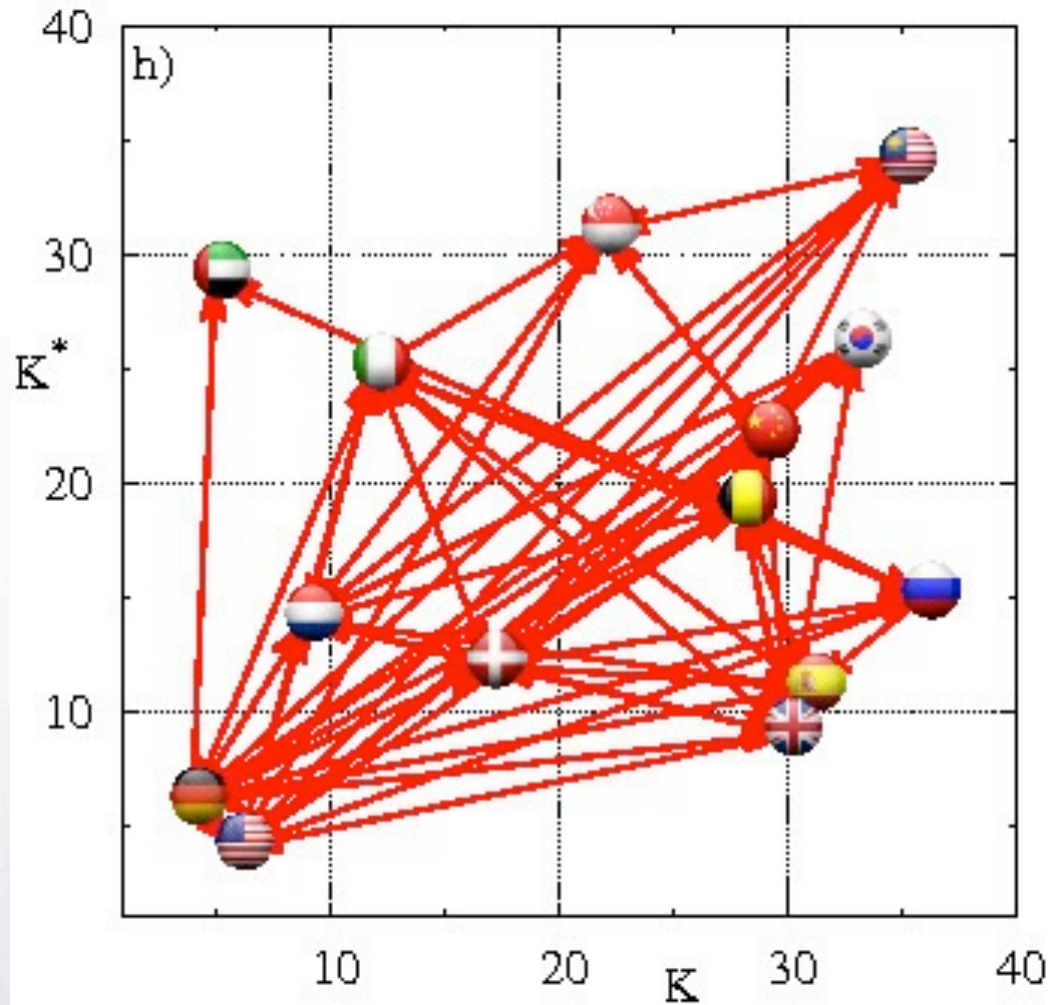
its trade network in this product is better and broader than the one of Saudi Arabia (1st)

its trade network is restricted to a small number of nearby countries.

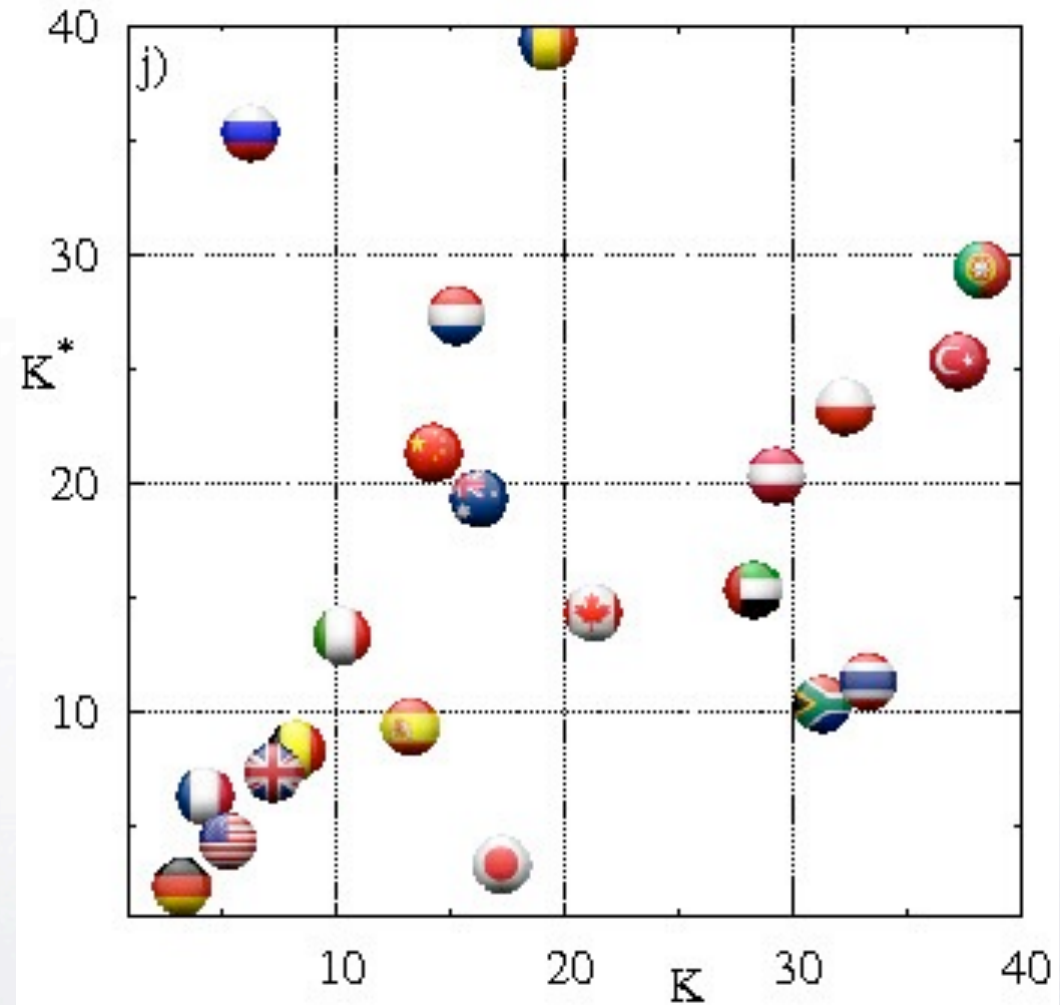
is practically the only country which sells crude petroleum to the CheiRank leader in this product Russia.



2d ranking: barley and cars



Ukraine (K^* 1st to 6th) USA (K^* 8th to 3rd)



France (K^* 7th to 3rd) Thailand (K^* 19th to 10th)



WTN model

- Gravity model of trade:

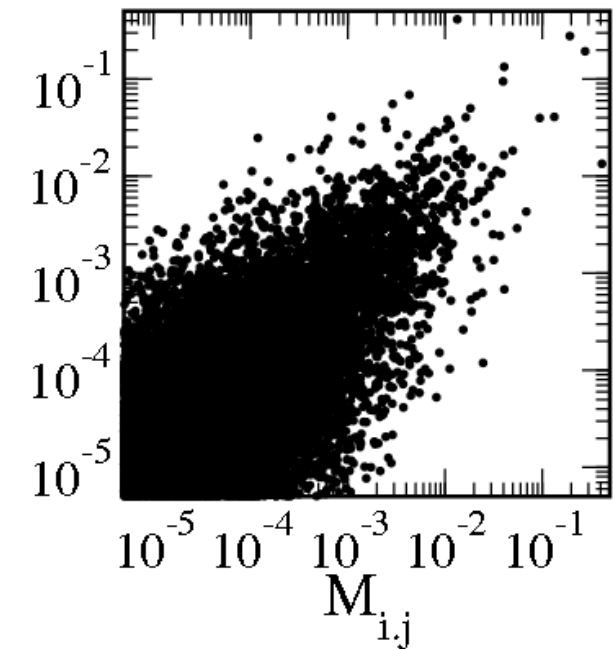
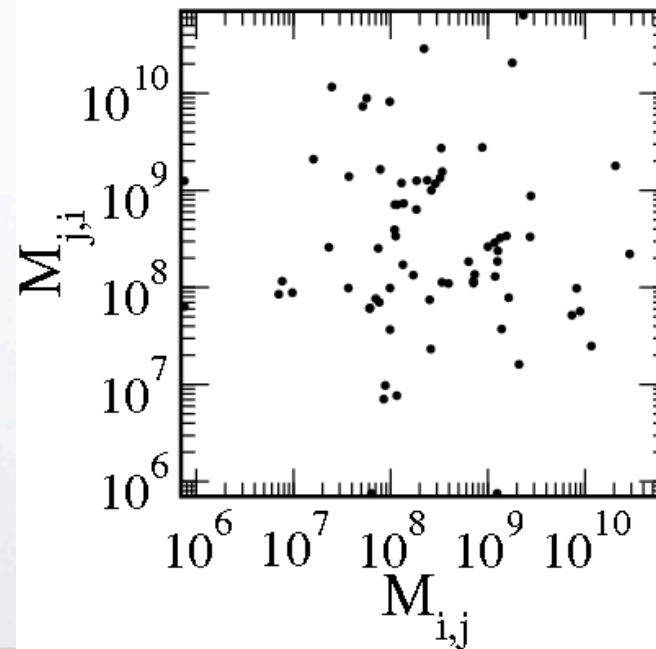
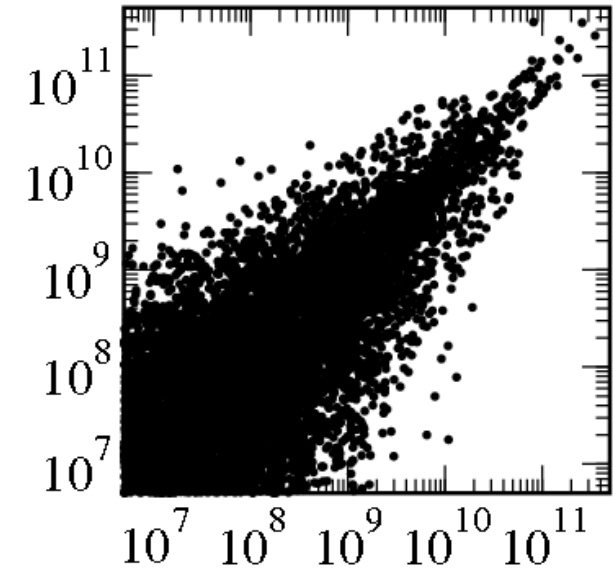
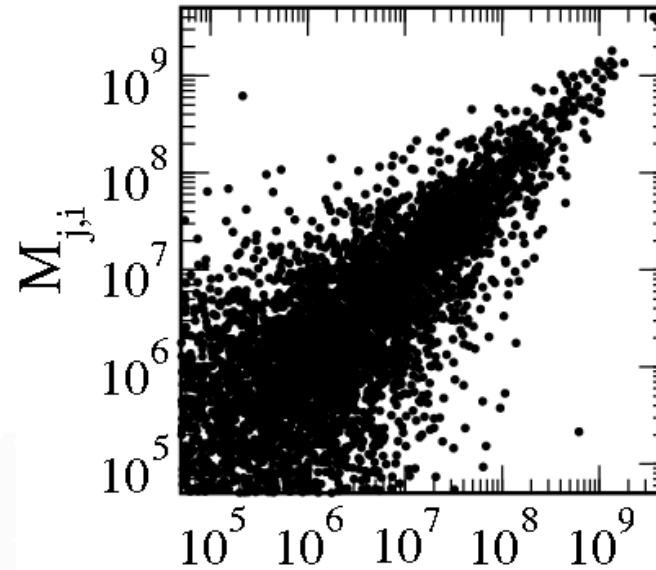
$$M_{i,j} = gm_i m_j / D_{i,j}$$

(symmetric)

- Random model

$$M_{i,j} = \epsilon_i \epsilon_j / ij \quad \epsilon_{i,j} \in [0, 1)$$

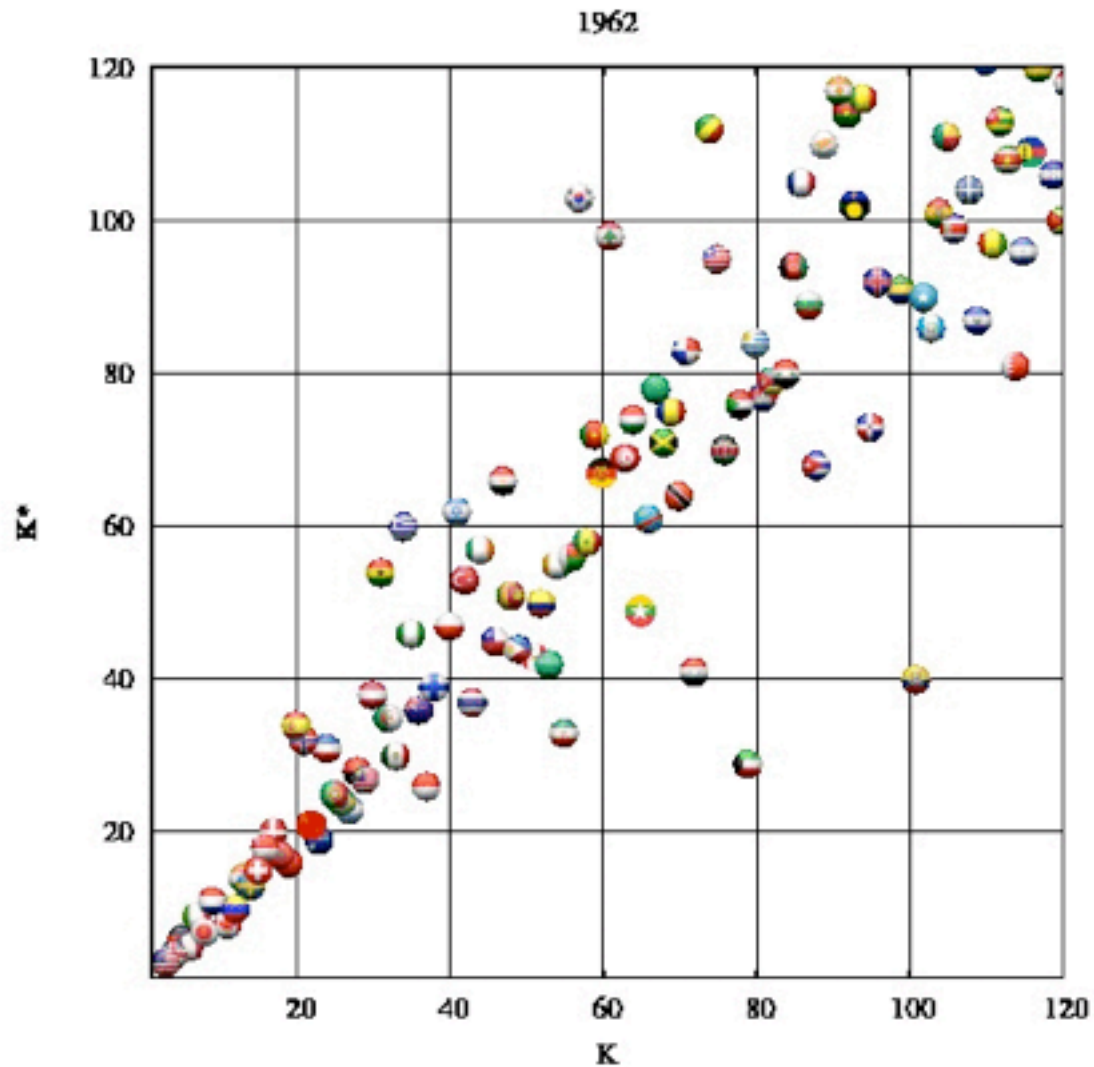
(preserves Zipf law)



t:: all commodities (1962, 2008);
b: crude petroleum (2008), random model

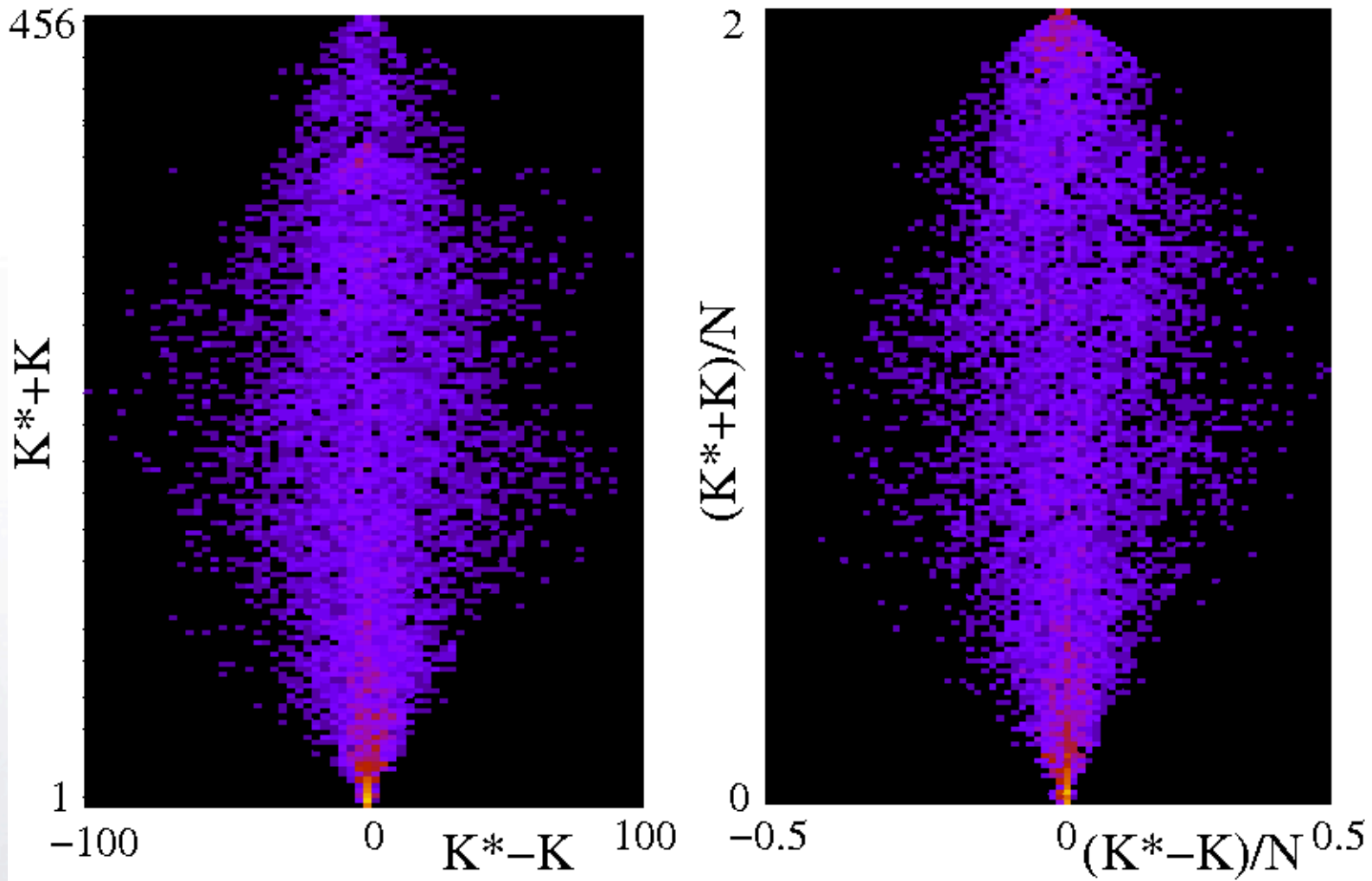


2d ranking evolution



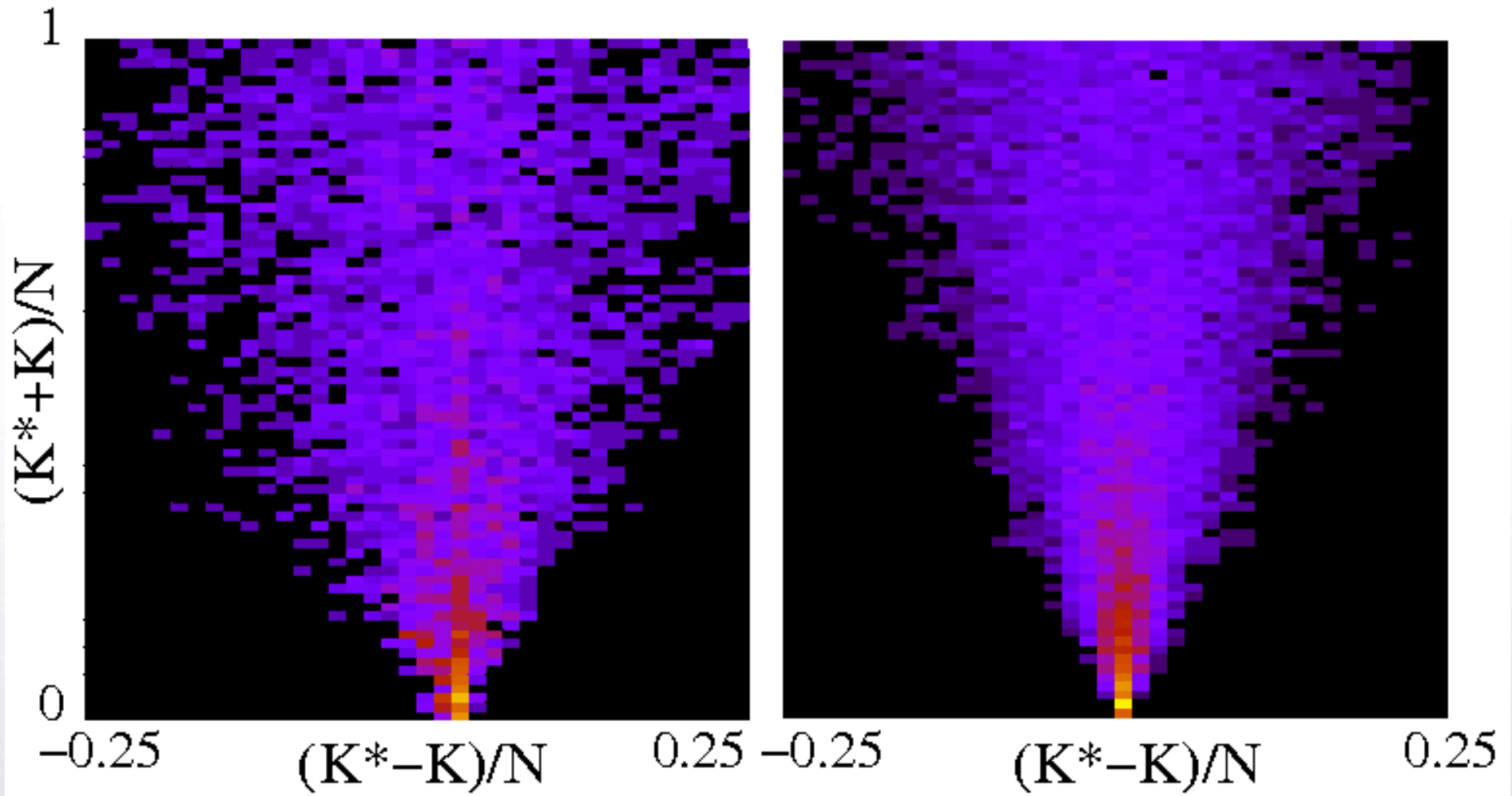


Model statistics





Model statistics





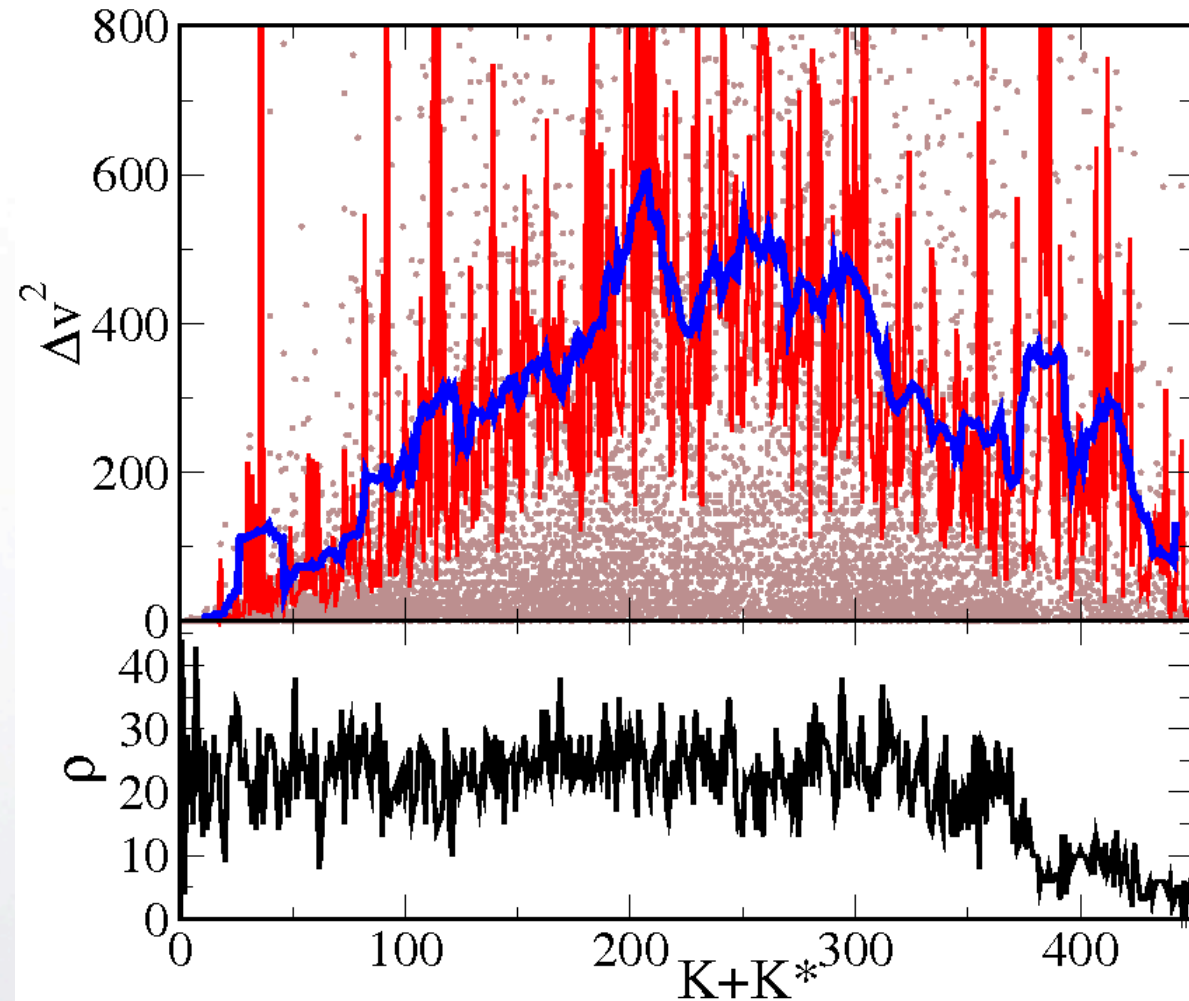
Time evolution and velocity K and K^* from 1962 to 2009

Velocity square vs. $K+K^*$

$$\Delta v^2 = [K(t) - K(t-1)]^2 + [K^*(t) - K^*(t-1)]^2$$

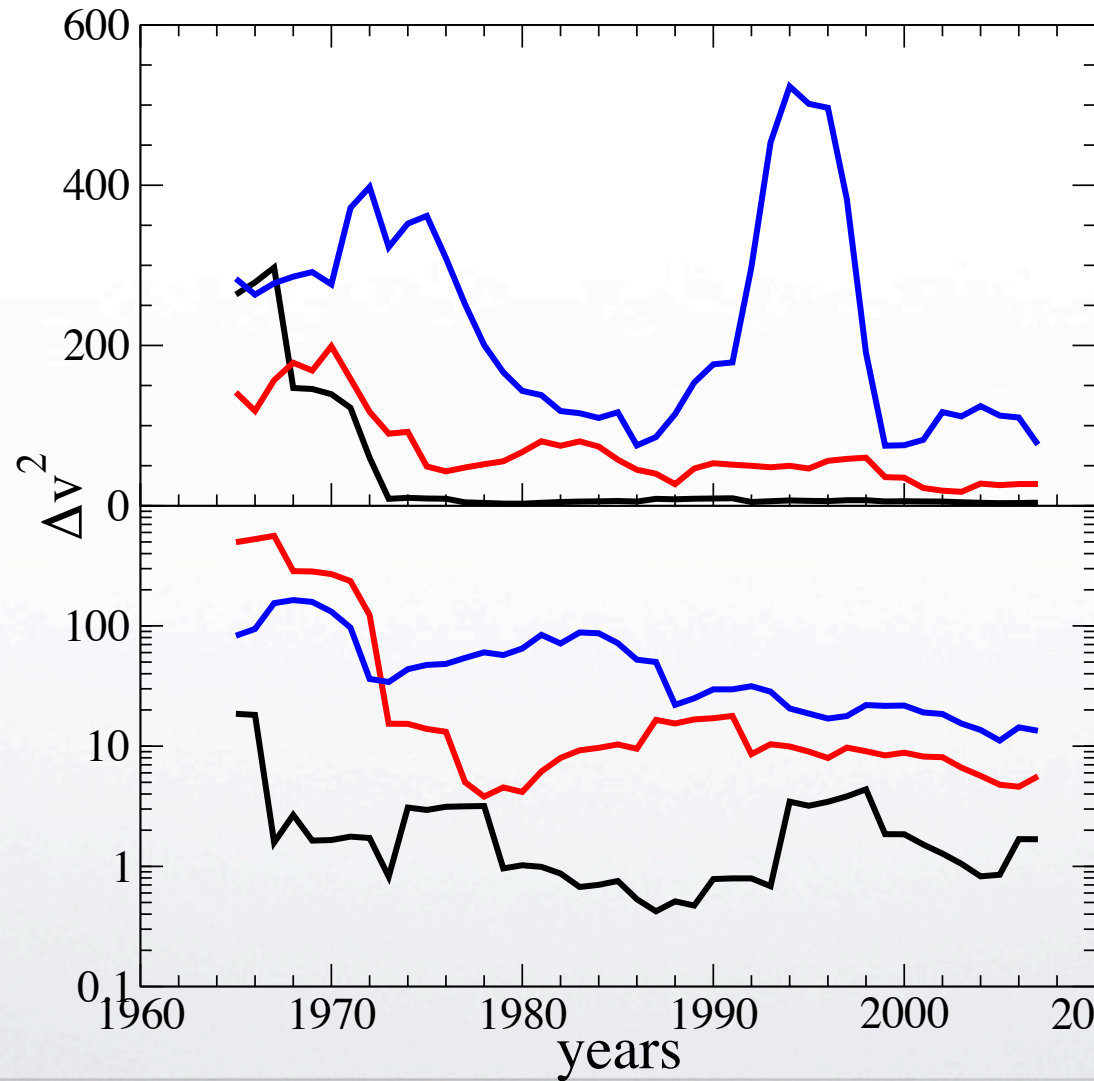
average per $K + K^*$

average in $[K + K^* - 10, K + K^* + 10]$





Velocity evolution

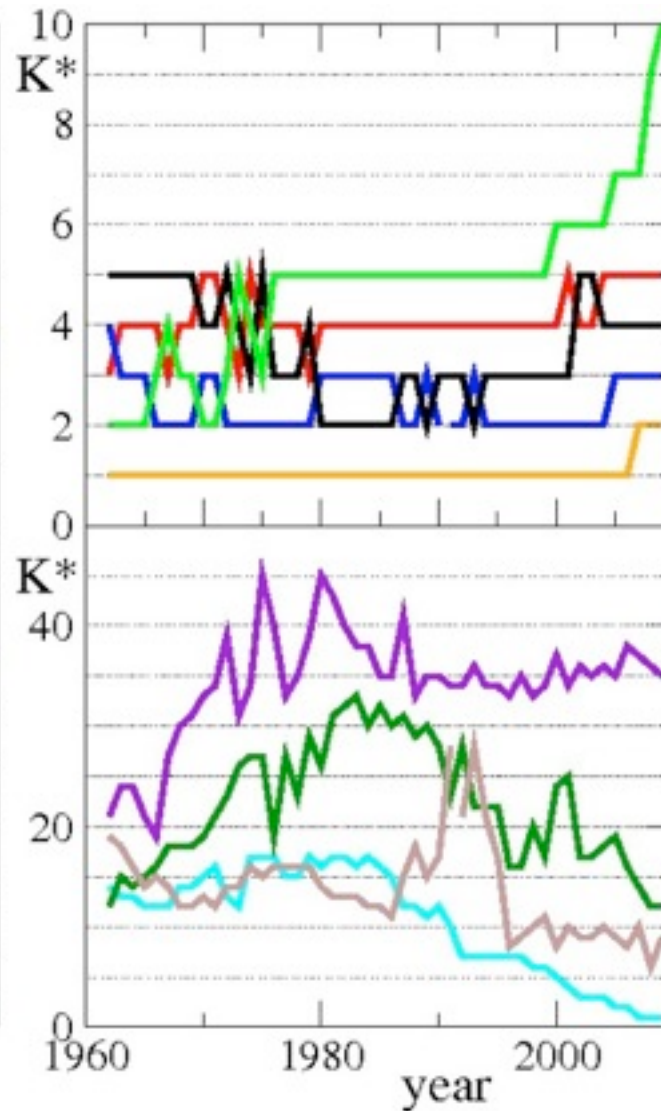
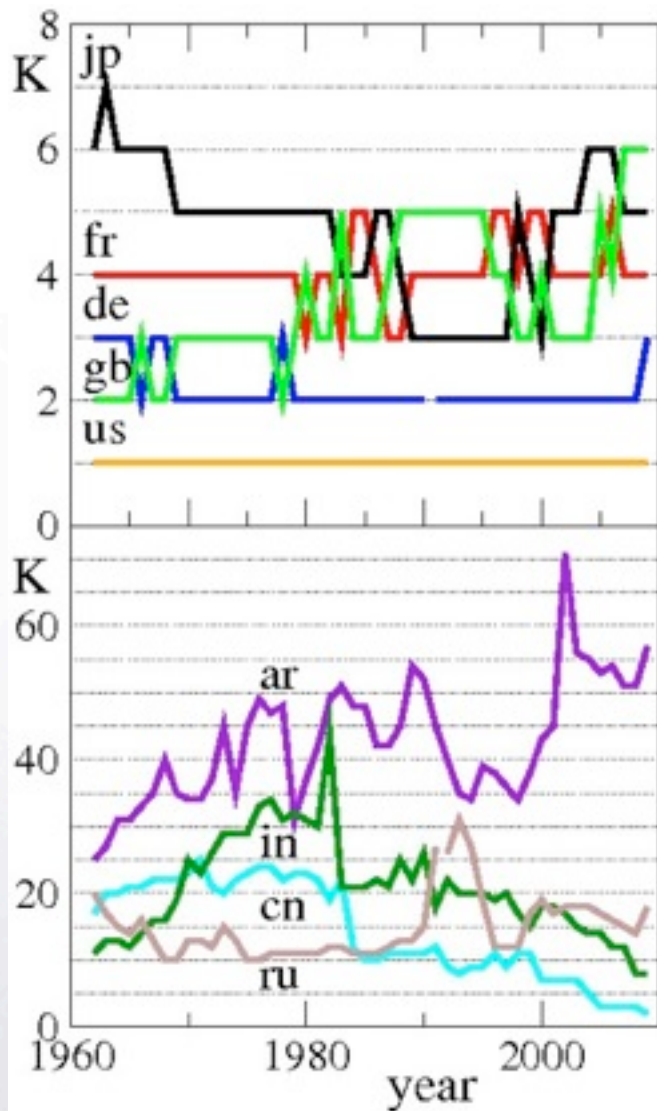


$K + K^* \in [1, 40]$
 $K + K^* \in [41, 80]$
 $K + K^* \in [81, 120]$

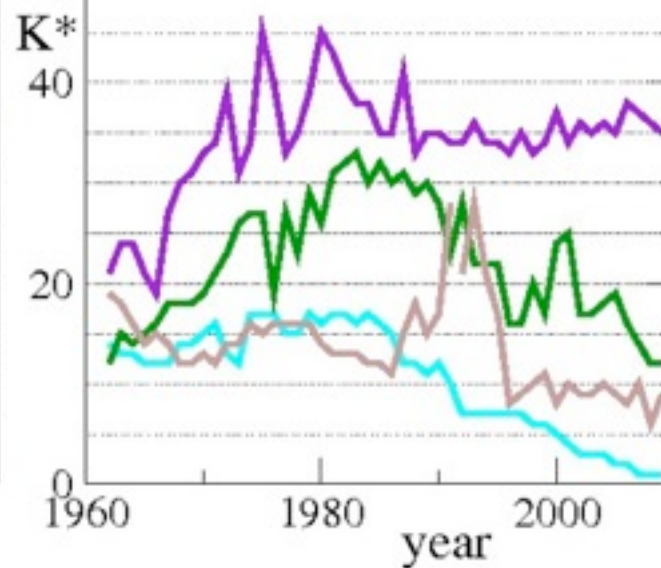
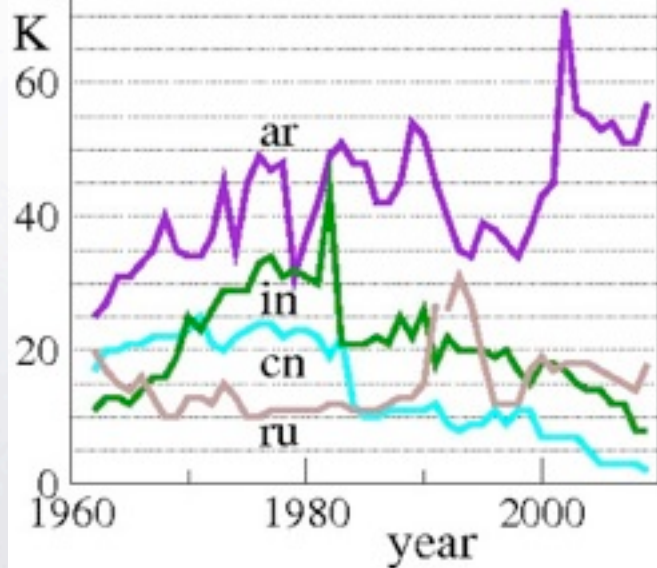
$K + K^* \in [1, 20]$
 $K + K^* \in [21, 40]$
 $K + K^* \in [41, 60]$



K and K* ranking evolution (some examples)



Japan
France
Fed. Rep. of Germany and Germany
Great Britain (sublimation?)
USA



Argentina
India
China (deposition)
USSR and Russian Fed.



II

Ecological Ranking (nestedness)



Nestedness

biogeography

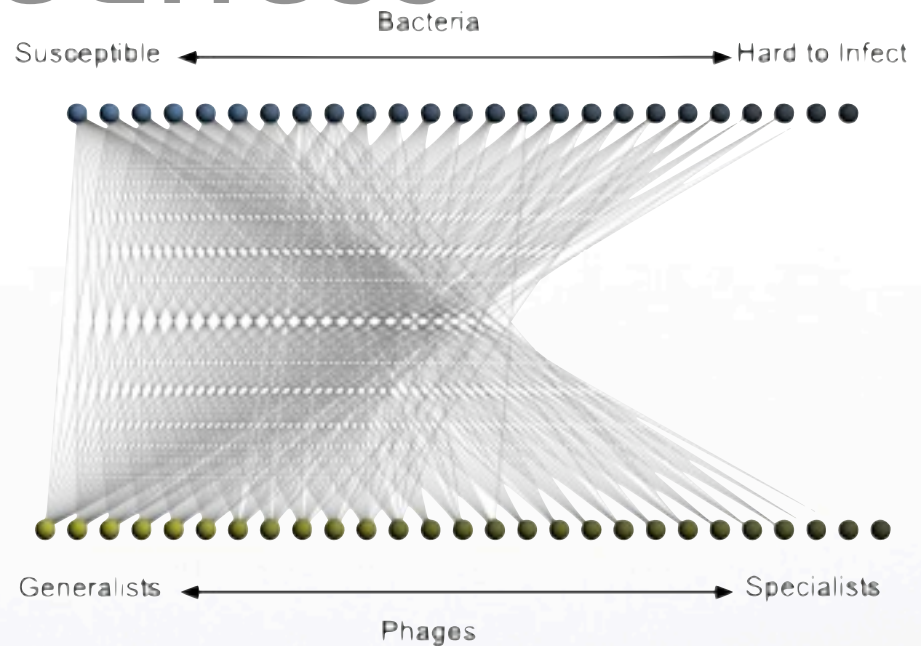
bipartite networks: species - sites (islands, plants, etc)

1937 Hulten

1957 Darlington

1975 Daubenmire

Causes: rates of extinction and colonialization
(at least 7 mechanisms)





Nestedness

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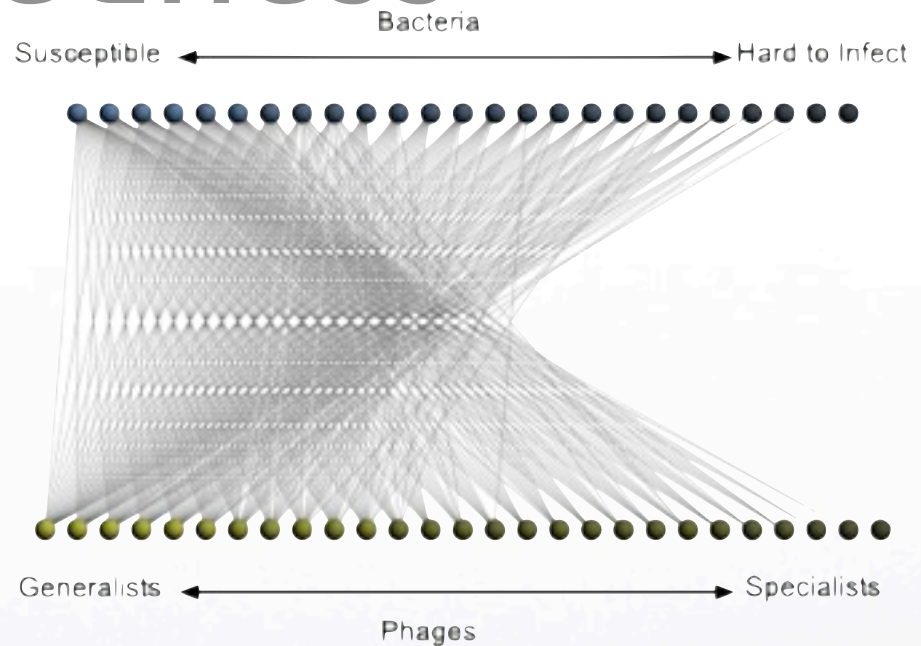
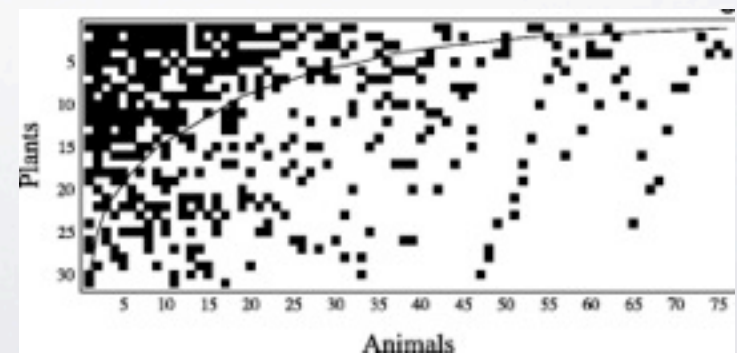
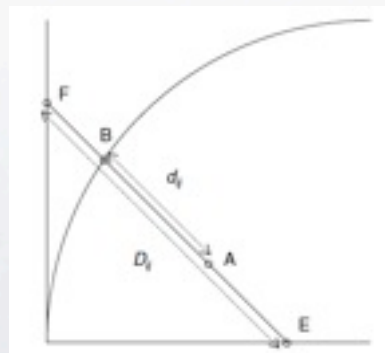
quantifying nestedness

BINMATNEST

M.A. Rodriguez-Girones and L. Santamaria,
Journal of Biogeography 33, 924 (2006)

isocline

$$f(x;p) = \frac{0.5}{n} + \frac{n-1}{n} \cdot \left(1 - \left(1 - \frac{m \cdot x - 0.5}{m-1} \right)^p \right)^{1/p}$$





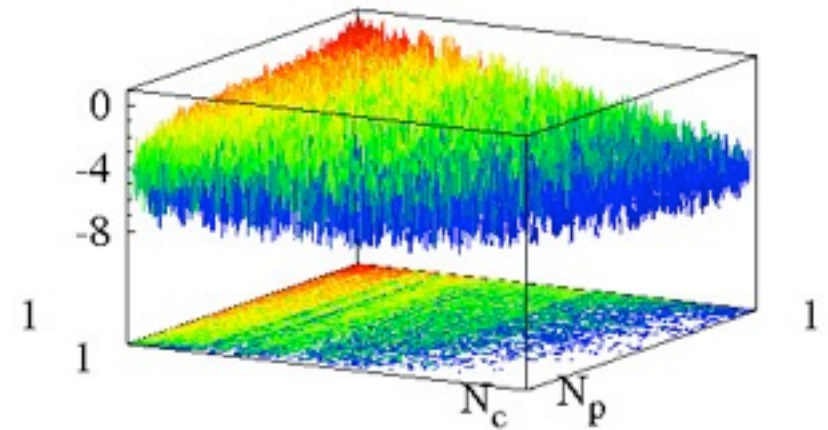
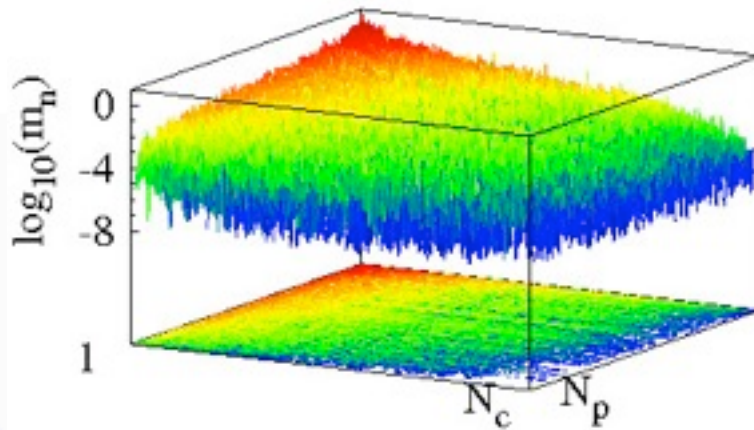
Mutualistic Networks (countries-products)

$$m^{(i,e)} = M^{(i,e)} / M_{\max}$$

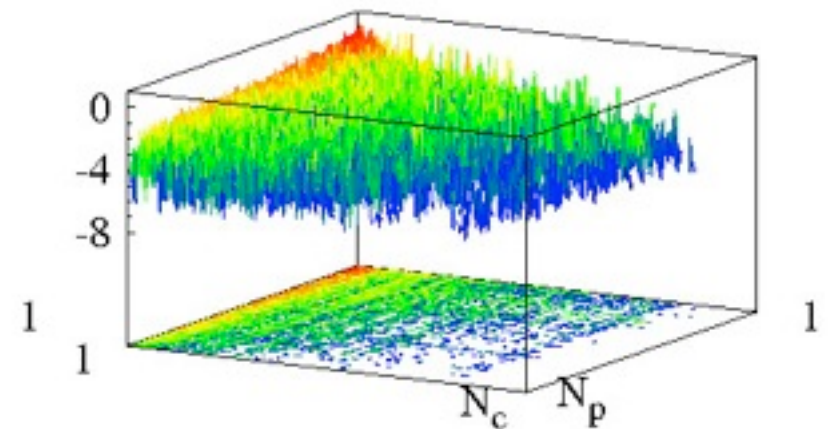
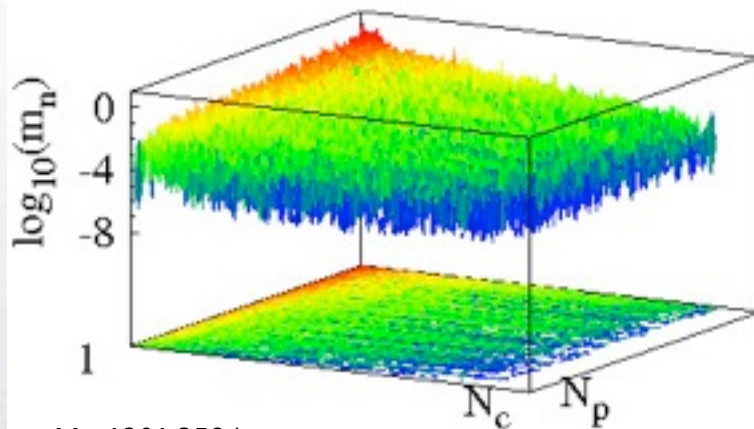
$$M_{p,c}^{(i)} = \sum_{c'=1}^{N_c} M_{c,c'}^p$$

$$M_{p,c}^{(e)} = \sum_{c'=1}^{N_c} M_{c',c}^p$$

2008



1968

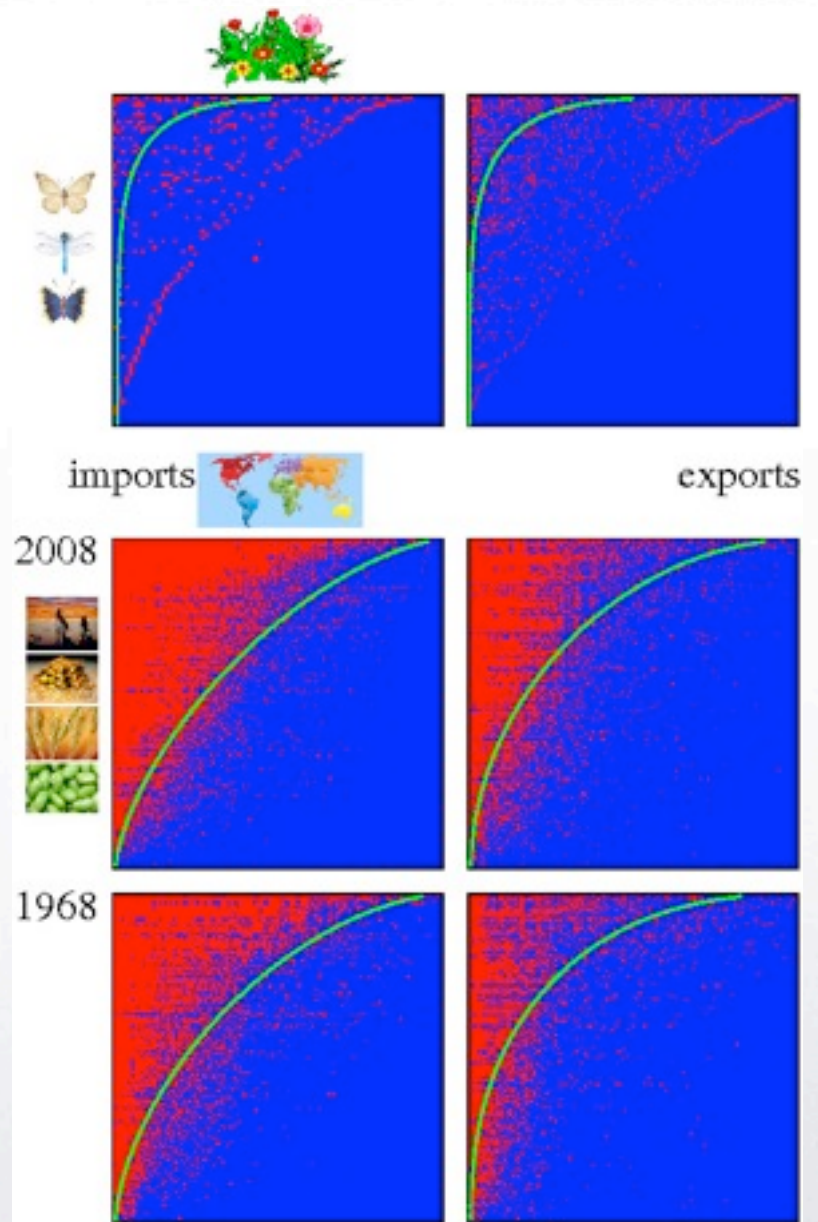


L. Ermann and D.L. Shepelyansky, arXiv:1201.3584



Binary mutualistic Networks

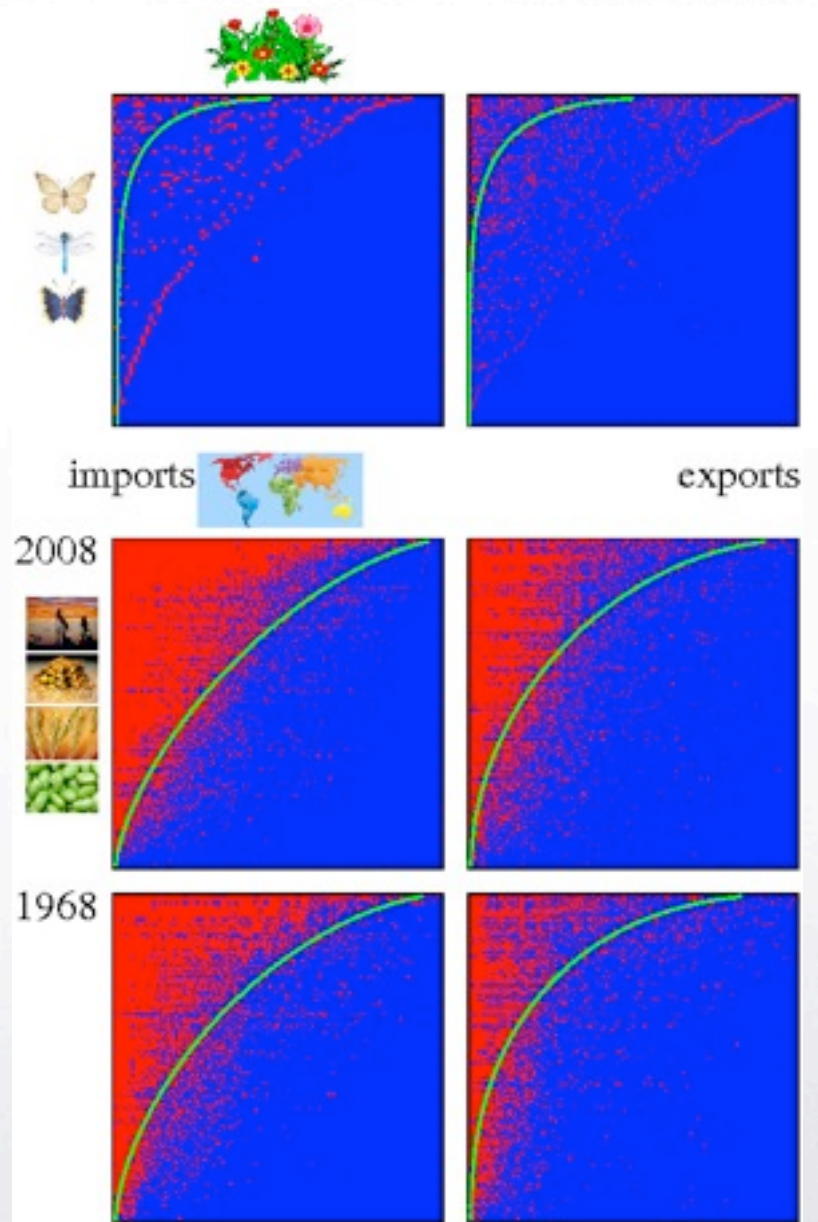
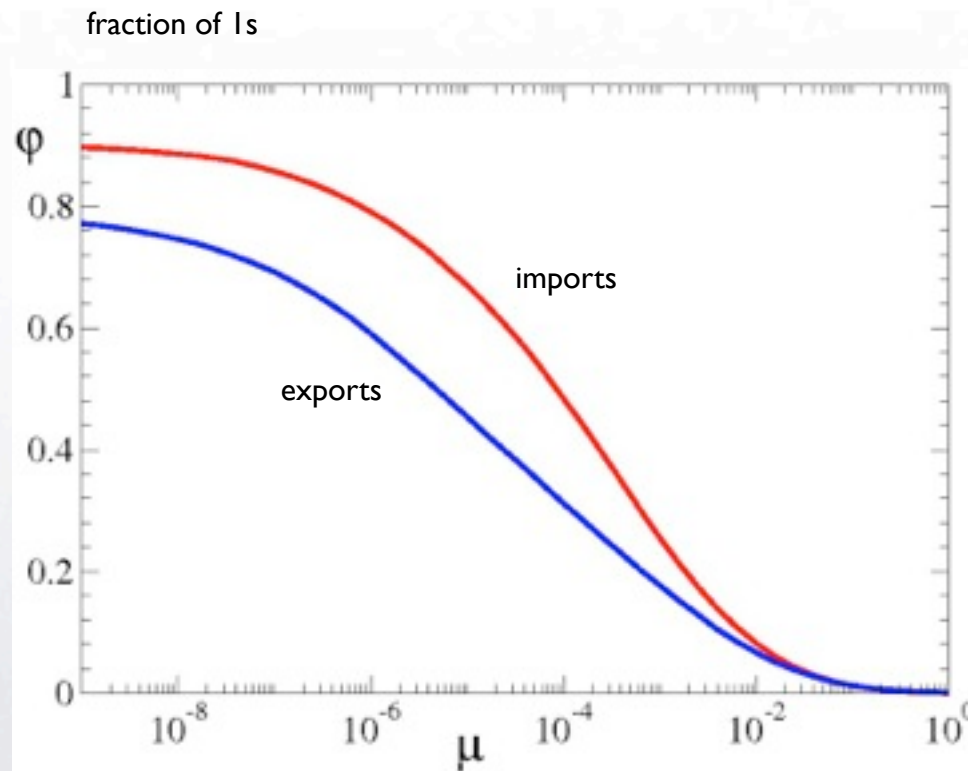
$$Q_{c,p}^{(i,e)} = \begin{cases} 1 & \text{if } m_{c,p}^{(i,e)} \geq \mu \\ 0 & \text{if } m_{c,p}^{(i,e)} < \mu \end{cases}$$

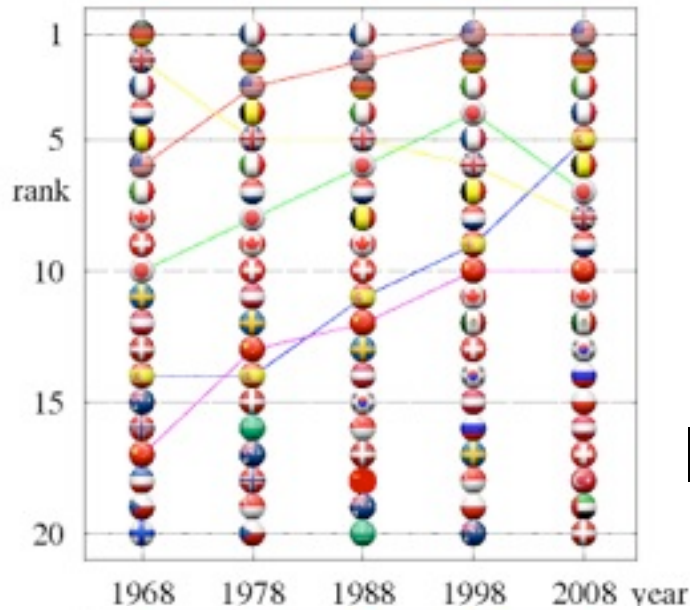




Binary mutualistic Networks

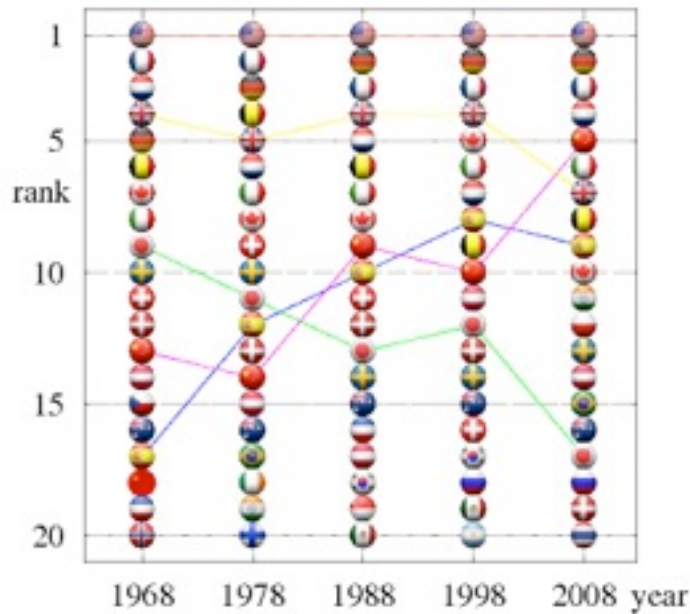
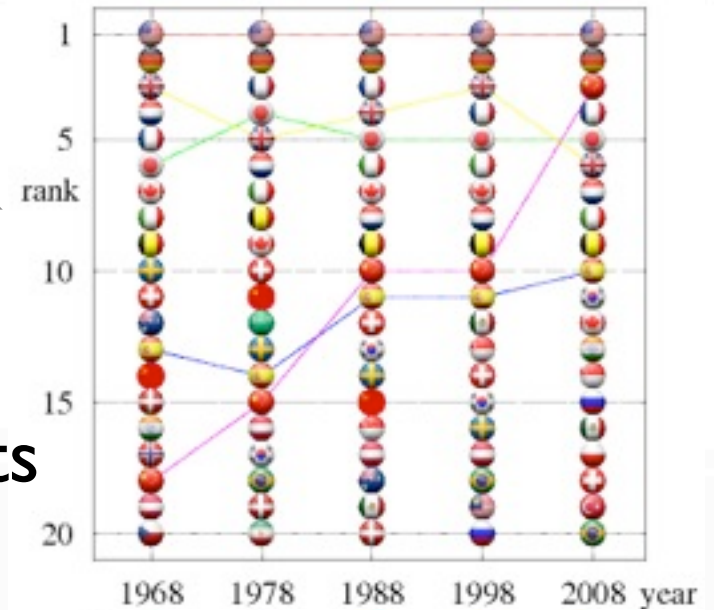
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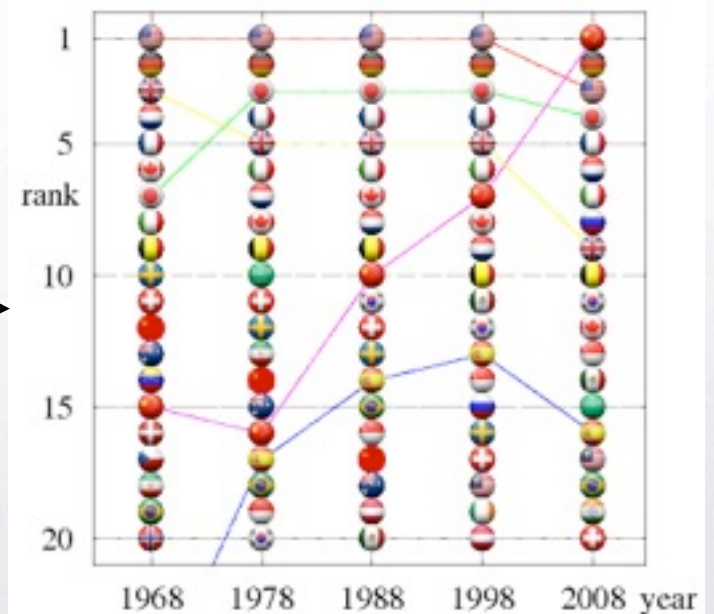


EcoloRank Countries

Imports Exports



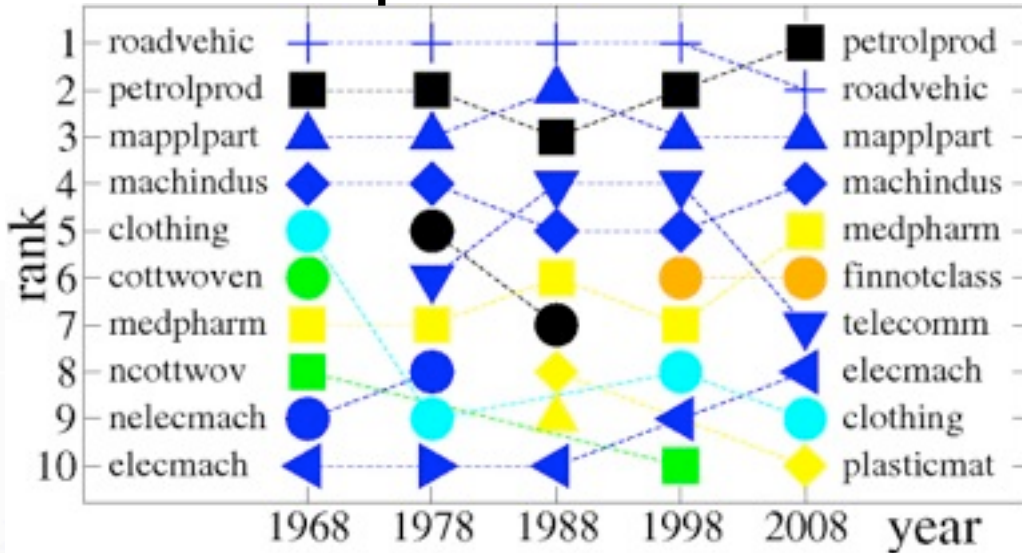
← money ranking →



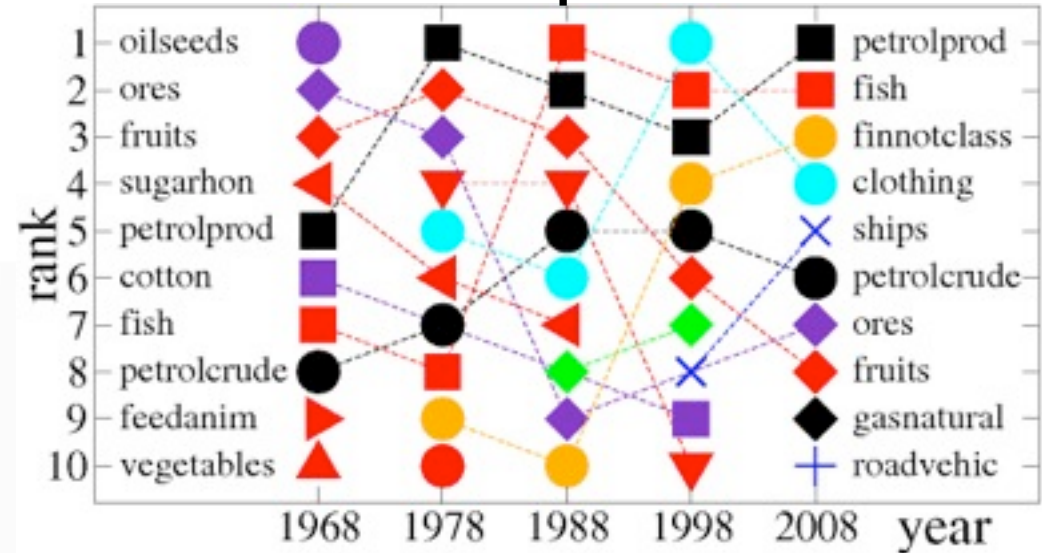


EoloRank products

imports



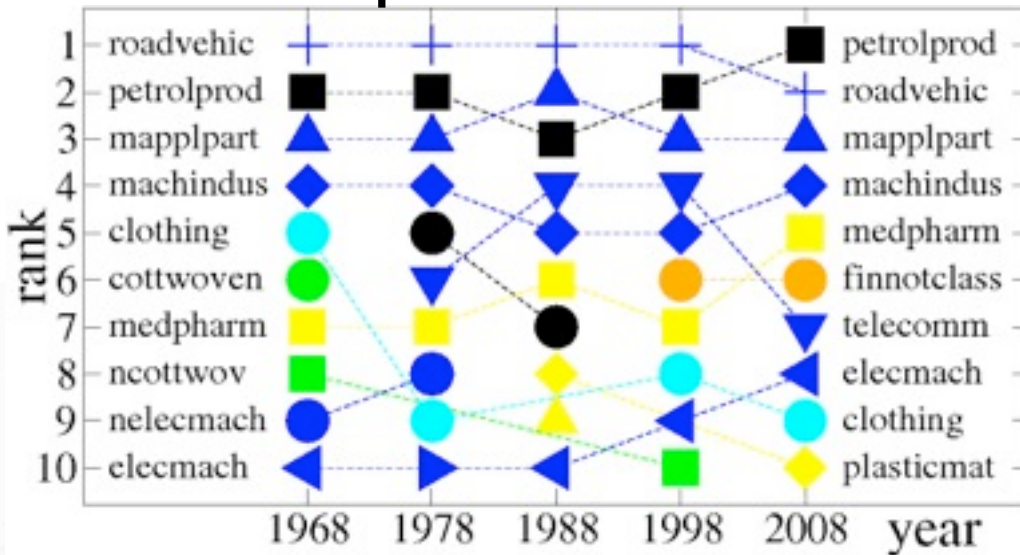
exports



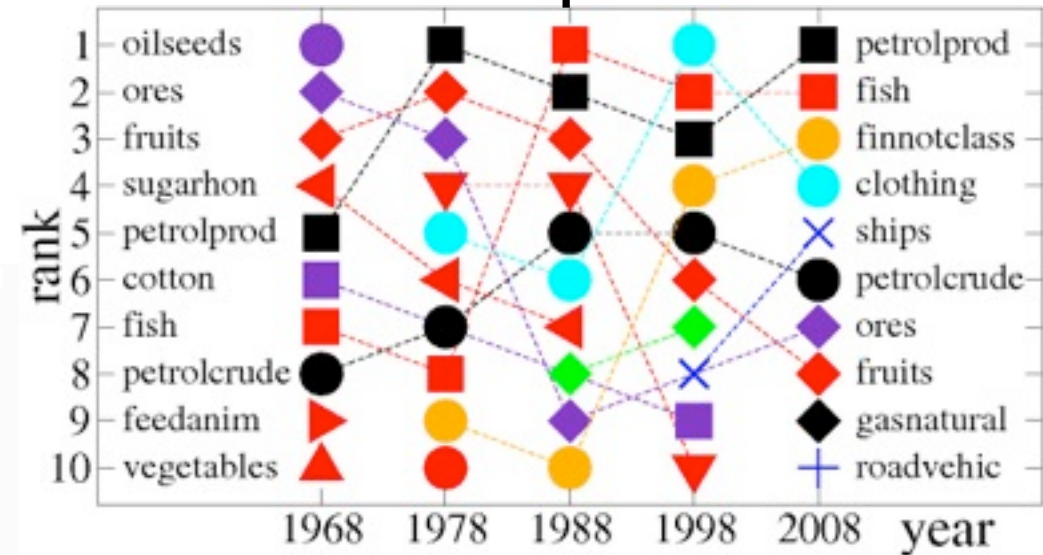


EoloRank products

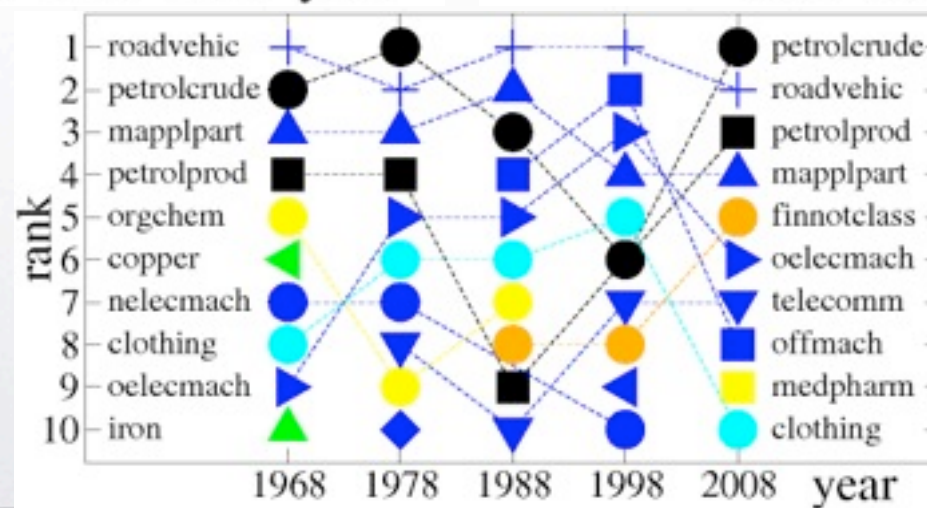
imports



exports



money rank



001	animals	Live animals
001	fish	Fish, fresh & simply preserved
001	fruits	Fruit, fresh, and nuts excl. Oil nuts
004	vegetables	Vegetables, roots & tubers, fresh or dried
001	sugarhon	Sugar and honey
071	coffee	Coffee
081	feedanim	Feed, stuff for animals excl. unshelled cereals
221	oilseeds	Oil seeds, oil nuts and oil kernels
263	cotton	Cotton
283	ores	Ores & concentrates of non ferrous base metals
331	petrolcrude	Petroleum, crude and partly refined
302	petrolprod	Petroleum products
341	gas	Gas, natural and manufactured
512	orgchem	Organic chemicals
541	medpharm	Medicinal & pharmaceutical products
581	plasticmat	Plastic materials, prepared, cellulose & resins
599	chemmat	Chemical materials and products, non
602	cottwoven	Cotton fabric, woven ex. narrow or spec.fabrica
603	ncottwov	Text fabrics woven ex narrow, spec, not cotton
607	pearlprec	Pearls and precious and semi precious stones
674	iron	Universals, plates and sheets of iron or steel
682	copper	Copper
711	selecmach	Power generating machinery, other than electric
714	offmach	Office machines
718	machindus	Machinery for special industries
719	mapppart	Machinery and appliances non electrical parts
722	elecmach	Electric power machinery and switchgear
724	telecomm	Telecommunications apparatus
729	oelecmach	Other electrical machinery and apparatus
731	ships	Ships and boats
841	clothing	Clothing except for clothing
901	finnotclass	Special transactions not classf. accord.to kind



III

Multi-products and Crisis (work in progress)



Multi-product WTN

(multiplexity in networks)

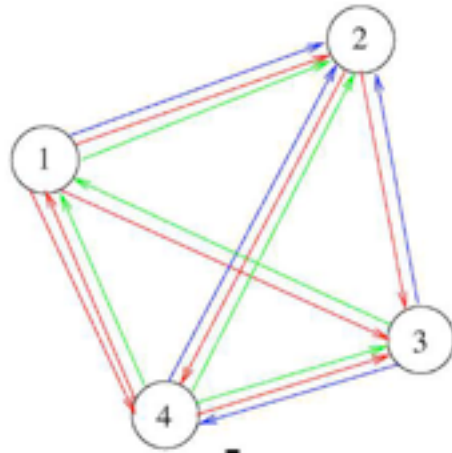
$N_c=227$ (2008)

(SITCI Rev.)

$N_p=10$ (1d)

$N_p=61$ (2d)

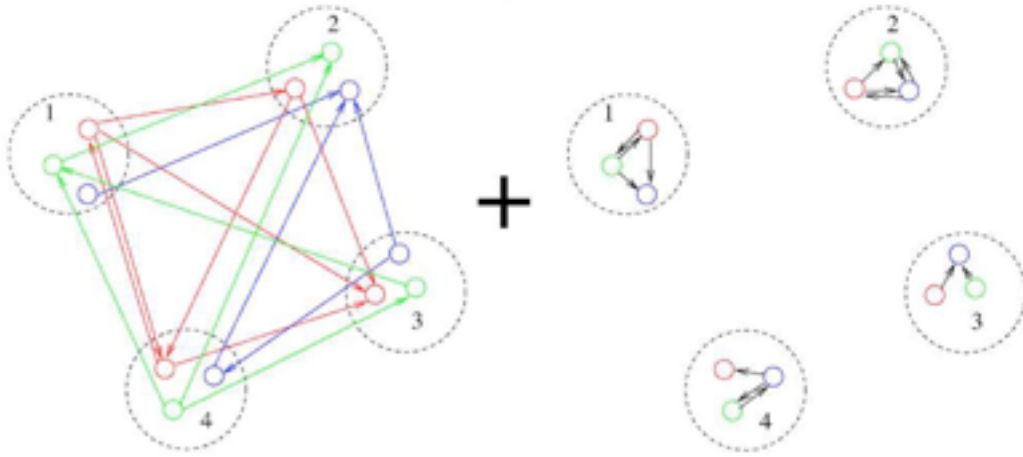
$N_p=182$ (3d)



S

C

+





Multi-product WTN

(multiplexity in networks)

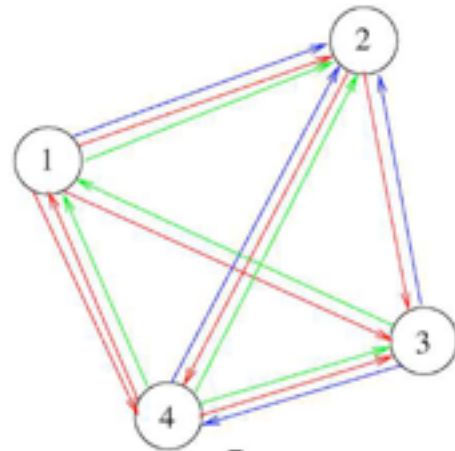
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$N_p=182$ (3d)

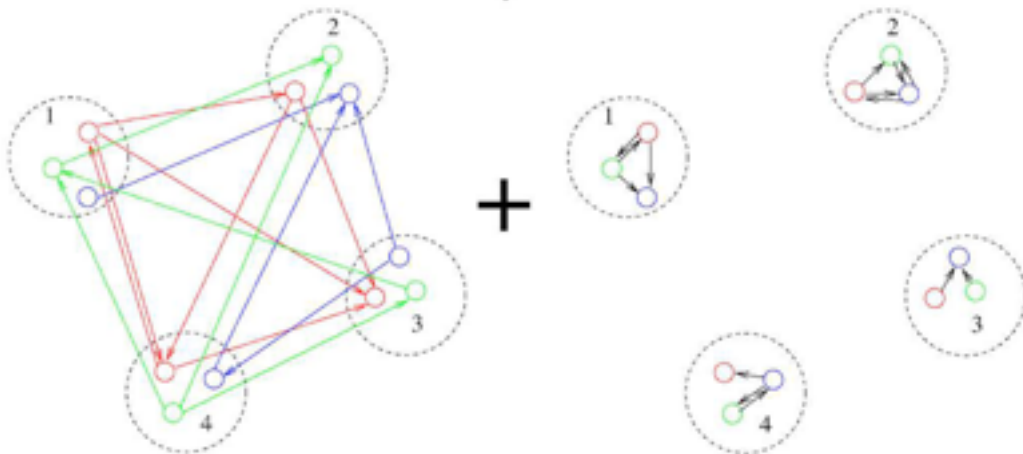


S

C



+



$$G = \alpha\beta S + \alpha(1 - \beta)C + \frac{(1 - \alpha)}{N}E$$

$$G^* = \alpha\beta S^* + \alpha(1 - \beta)C^* + \frac{(1 - \alpha)}{N}E$$

$$S_{i',i} = \begin{cases} \frac{M_{c',c}^{(p)}}{m_c^{(p)}} \delta_{p',p} & \text{if } m_c^{(p)} \neq 0 \\ \frac{1}{N} & \text{if } m_c^{(p)} = 0 \end{cases}$$

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$$C_{i',i} = \frac{m_c^{*(p')}}{\sum_{p''} m_c^{*(p'')}} \delta_{c',c}$$

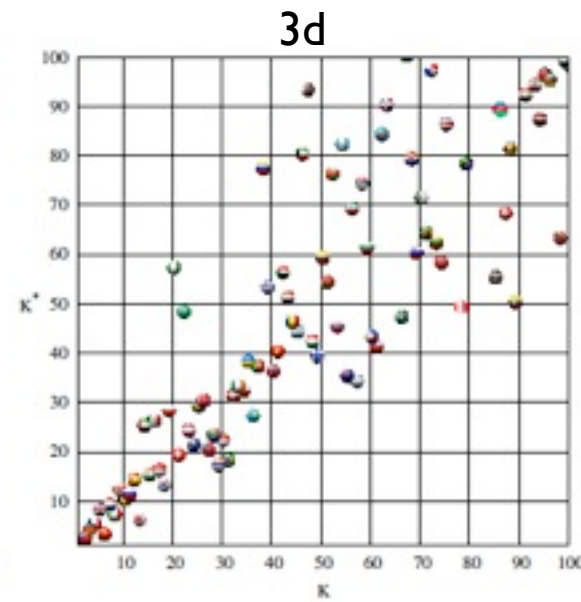
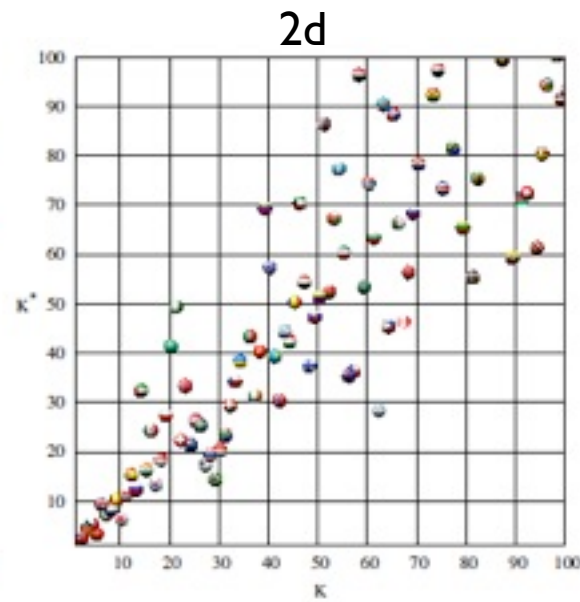
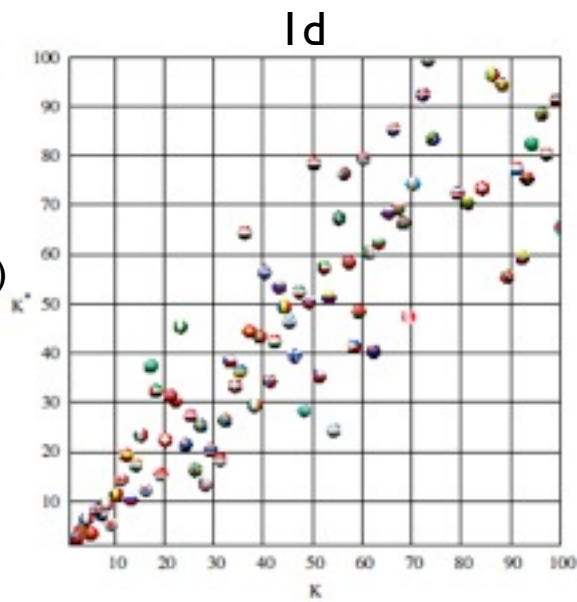
$$C_{i',i}^* = \frac{m_c^{(p')}}{\sum_{p''} m_c^{(p'')}} \delta_{c',c}$$

$$E_{i',i} = 1$$



Multi-product WTN

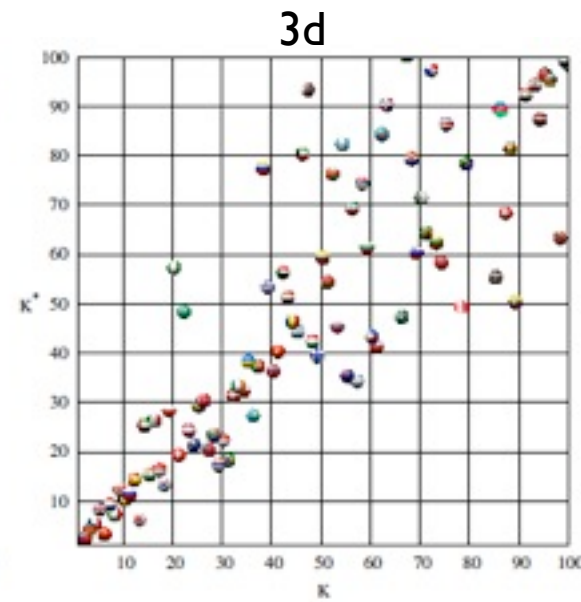
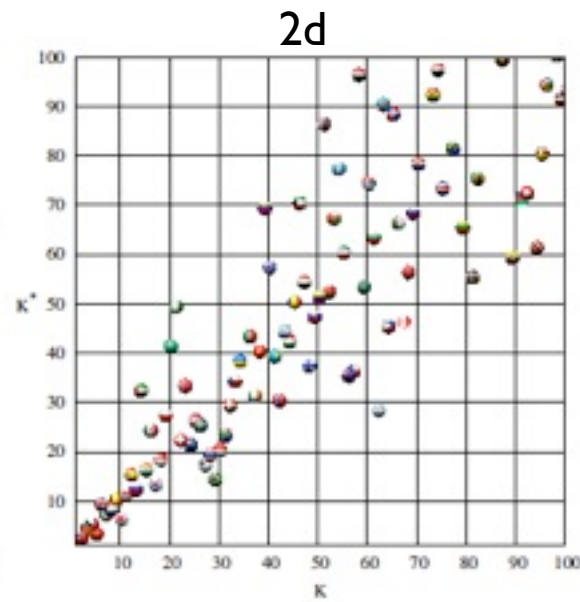
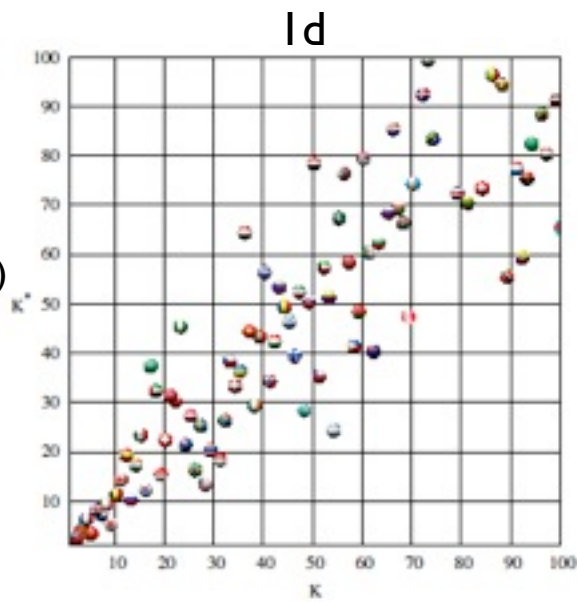
$K-K^*$
(tracing out products)



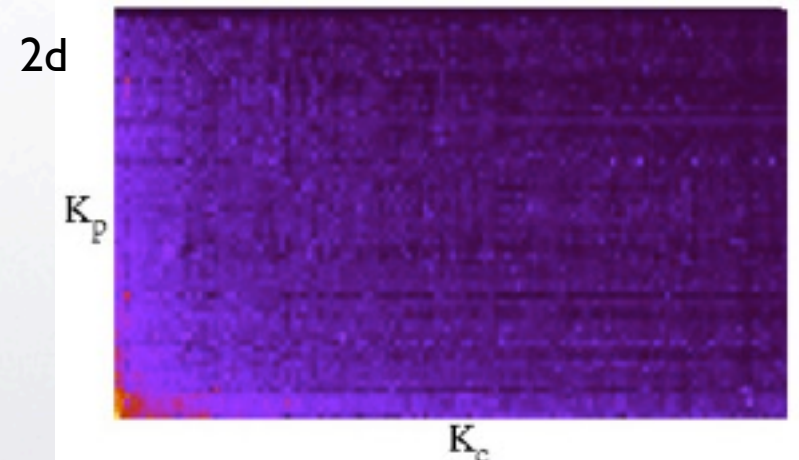
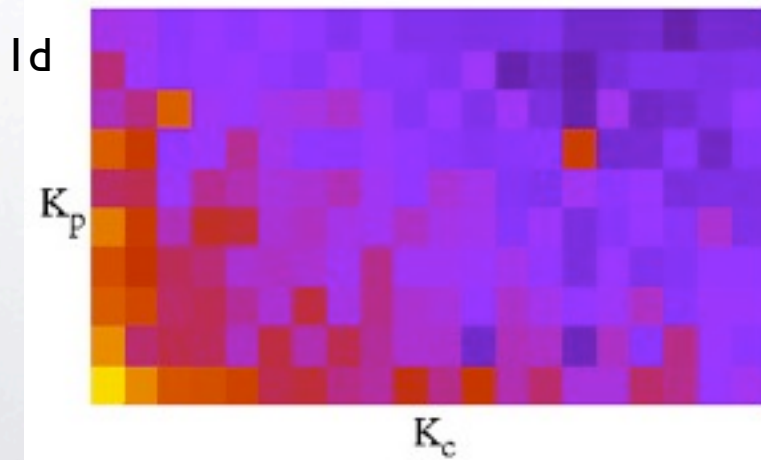


Multi-product WTN

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2-dimensional
PageRank





balance $b_i = \frac{P(i) - P^*(i)}{P(i) + P^*(i)}$

weight $w_i = P(i) + P^*(i)$

Crisis

2008; $w > 0.05$ (~20%)

Rank	country	b_i (positive)	global rank	country	b_i (positive)	global rank
1	Greece	0.5131	1	Iran	-0.2846	7
2	Spain	0.2505	15	Malaysia	-0.2676	8
3	Romania	0.2322	19	China	-0.2506	10
4	Portugal	0.2222	23	Saudi Arabia	-0.2470	12
5	Mexico	0.1743	37	Argentina	-0.2388	13
6	Canada	0.1633	40	Russian Fed.	-0.2340	15
7	USA	0.1457	45	Brazil	-0.1939	20
8	UK	0.1397	49	Singapore	-0.1814	22
9	Poland	0.1326	51	Rep. of Korea	-0.1788	23
10	France	0.1086	62	Australia	-0.1693	25

$w > 0.035$



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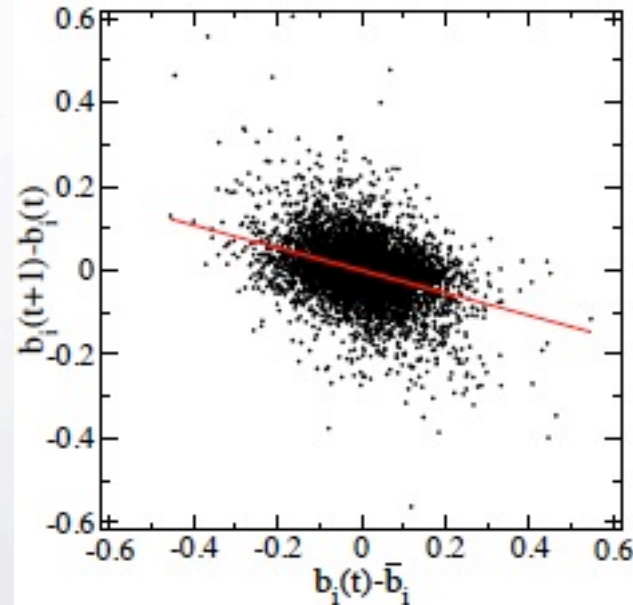
weight $w_i = P(i) + P^*(i)$

2008; $w > 0.05$ (~20%)

Rank	year	country	$b_i(t+1) - b_i(t)$
1	1973	Lebanon	-0.376
2	2001	Argentina	-0.269
3	1981	Mexico	-0.258
4	1983	Nigeria	-0.253
5	2002	Saudi Arabia	-0.250
6	1982	Venezuela	-0.247
7	1997	Indonesia	-0.244
8	1962	Venezuela	-0.236
9	1973	Nigeria	-0.230
10	1994	Mexico	-0.230
11	1997	Rep. of Korea	-0.219
12	1983	U. Arab Emir.	-0.213
13	2005	Iran	-0.210
14	1978	Iran	-0.210
15	1993	Turkey	-0.204
16	1975	India	-0.202
17	1998	Russian Fed.	-0.202
18	1976	Iraq	-0.200
19	1987	Argentina	-0.196
20	1989	Venezuela	-0.192

$w > 0.035$

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Crisis model

toy model:

- (1) if $b_i \geq \kappa \Rightarrow$ imports of i are closed
- (2) compute $G, G^* \Rightarrow b_i \Rightarrow (1)$

C_c (local)

N_c (global)

t

τ

κ

f_p



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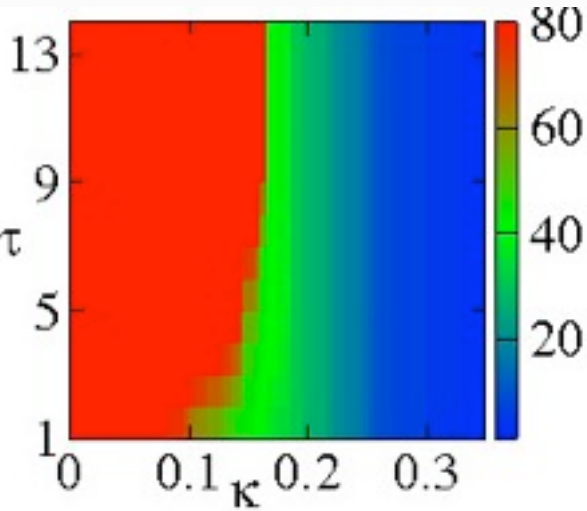
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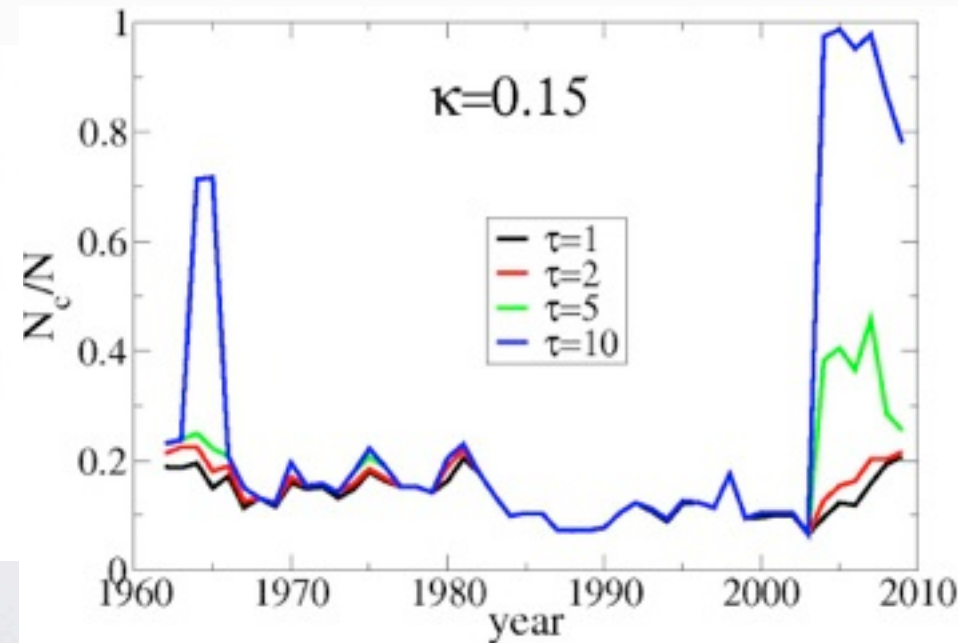
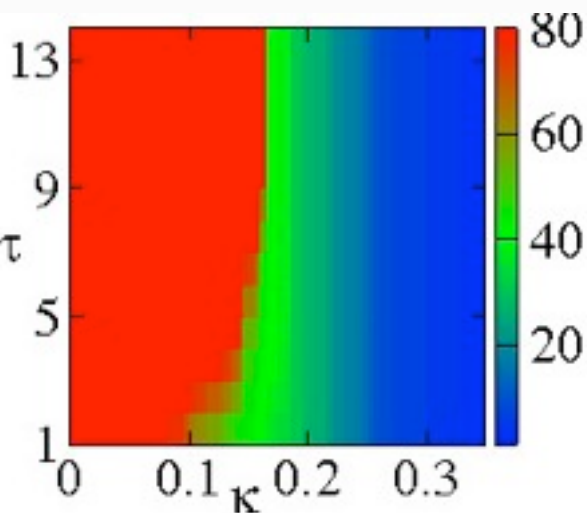
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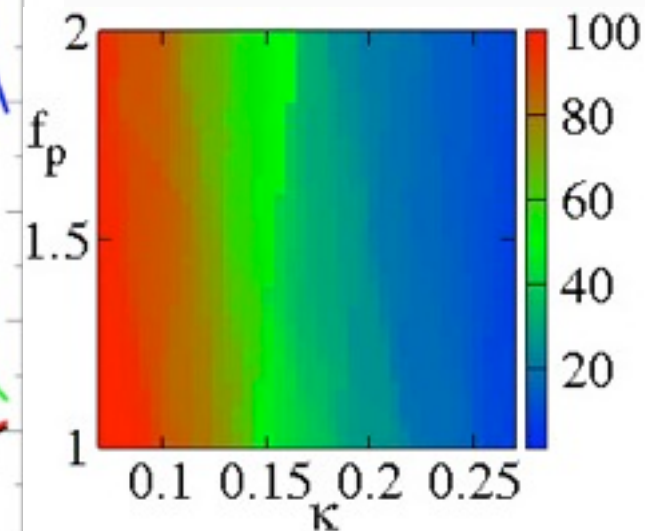
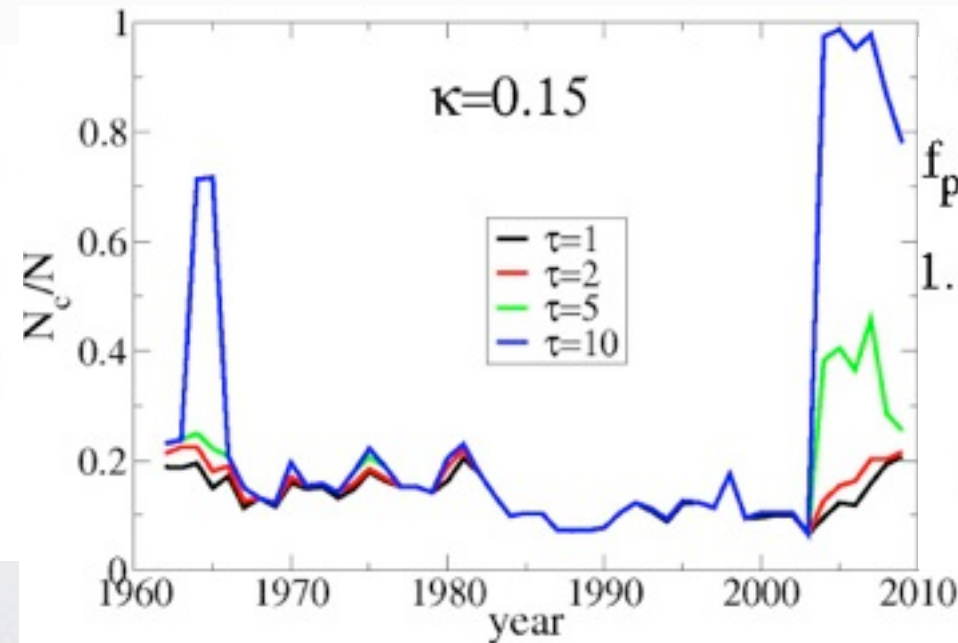
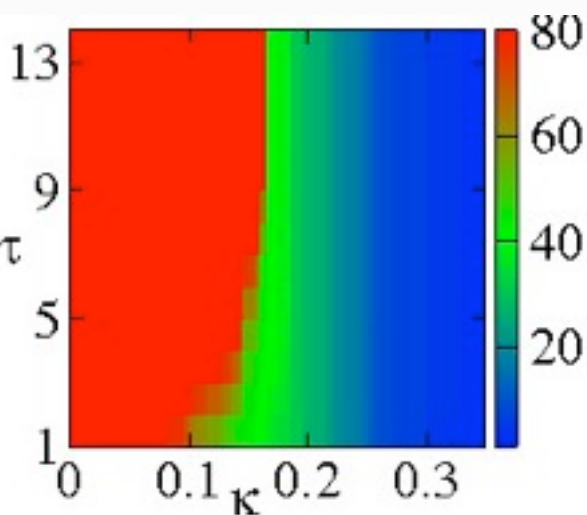
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(network properties, democratic, i-e symmetry)
- Comparison with Import-Export
- Model of M (directed by randomness, preserves Zipf law)



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- III (2-dimensional PageRank, β parameter controls internal market)
- **Crisis**(analyze b and $b(t+1)-b(t)$, toy model to study global crisis)



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Thank You