Fakultät für Physik

UNIVERSITÄT DUISBURG ESSEN

Credits and the Instability of the Financial System: a Physicist's Point of View

Thomas Guhr

Spectral Properties of Complex Networks

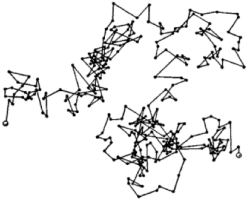
ECT* Trento, July 2012

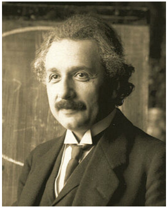
Outline

- Introduction — econophysics, credit risk
- Structural model and loss distribution
- Numerical simulations and random matrix approach
- Conclusions — general, present credit crisis

Introduction — Econophysics

Some History: Connection Physics–Economics





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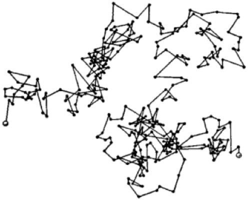

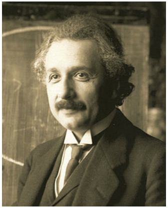

Einstein 1905

Bachelier 1900

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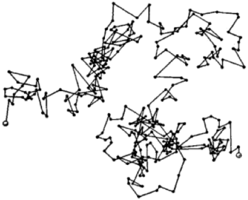

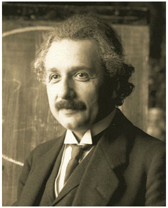

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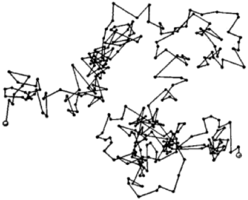

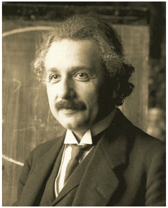

Einstein 1905

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Mandelbrot 60's

Growing Jobmarket for Physicists

Frankfurter Allgemeine

ZEITUNG FÜR DEUTSCHLAND

Frankfurter Allgemeine Zeitung — Finanzmärkte und Geldanlage — Freitag, 20. Mai 2005, Nr. 115 / Seite 27

Einsteins Erben in den Banken

Die Mathematik der „Brownschen Bewegung" ist Grundlage sowohl der Atomphysik als auch der modernen Finanzmathematik / Von Benedikt Fehr

Vor 100 Jahren, im Mai 1905, lieferte Albert Einstein mit seiner „Theorie der Brownschen Bewegung" einen Beweis für die moderne Vorstellung vom Atom. Unbeabsichtigt, indirekt und auf verschlungenen Wegen trug Einsteins Geniestreich ein dreiviertel Jahrhundert später dazu bei, eine weitere Revolution zu zünden – auf den Finanzmärkten. Denn auch Börsenkurse lassen sich als „Brownsche Bewegung" deuten. Physikern und Mathematikern sind deshalb die komplizierten Formeln geläufig, ohne die im modernen Bank- und Finanzgeschäft nichts mehr geht. Eine erste Anwendung fand die hochgezüchtete Mathematik in der Optionspreistheorie, die in den siebziger Jahren entwikkelt wurde. Inzwischen geht die Wirkung weit darüber hinaus. So schätzen Finanzmathematiker zum Beispiel auch die Ausfallwahrscheinlichkeiten von Krediten und die Risikoprämien für Kreditausfallversicherungen mit diesen hochabstrakten Modellen. Die neuen Instrumente tragen dazu bei, unternehmerische Risiken besser beherrschbar zu machen – was das rasche Wachstum der Märkte für diese Finanzinnovationen erklärt. Doch falsch angewendet, können sie selbst zu einem Risiko für die Stabilität des Finanzsystems werden.

Überraschende Karrieren: Goetz Giese hat 1996 über die „Dynamik granularer Teilchen" seinen Doktor in Physik gemacht, heute arbeitet er im Risiko-Controlling der Commerzbank. Roland

gilt er als der frühe Begründer der modernen Finanzmathematik.

Bacheliers später Siegeszug beginnt in den fünfziger Jahren. Inzwischen hatte John M. Keynes, der selbst ein leiden-

Deutschland einstellt, von der Ausbildung her Naturwissenschaftler. Im Derivate-Eigenhandel der Hypo-Vereinsbank sind fünf von sechs Mitarbeitern Mathematiker und Physiker.

Risikokontrolle ist deshalb zu einem wichtigen Wettbewerbsparameter, zu einer Art Produktionsfaktor geworden.

Die Optionspreismodelle wiederum erlauben es den Banken, die Derivate, die

rance", die Mitte der achtziger Jahre in Wall Street mit viel Marketinggetöse an Investoren verkauft wurde. Diese „synthetische Option" sollte Anleger gegen einen Verfall der Aktienkurse schützen. Das

ker Ernst Eberlein wiederum, Generalsekretär der „Bachelier Finance Society", tüftelt an hochgezüchteten „Lévy-Modellen", in denen die Brownsche Bewegung nur ein Spezialfall unter vielen ist.

"Every tenth academic hired by Deutsche Bank is a natural scientist."

A New Interdisciplinary Direction in Basic Research

Theoretical physics: construction and analysis of mathematical models based on experiments or empirical information

physics $\longrightarrow$ economics:

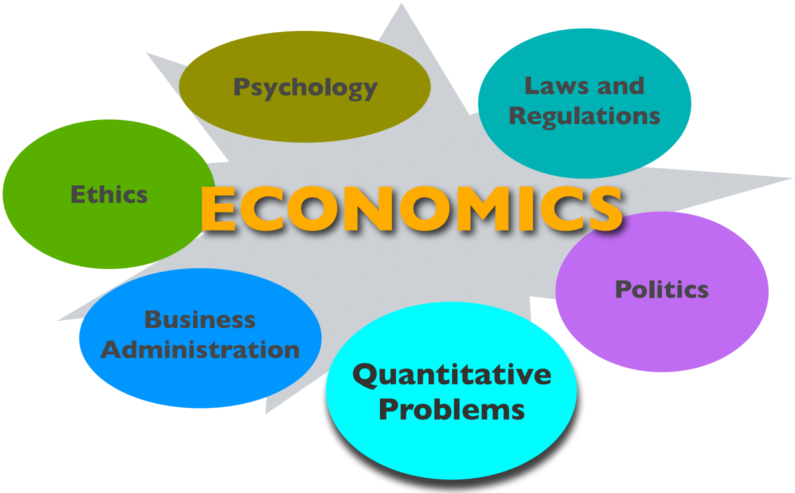
much better economic data now, growing interest in complex systems

Study economy as complex system in its own right

economics $\longrightarrow$ physics:

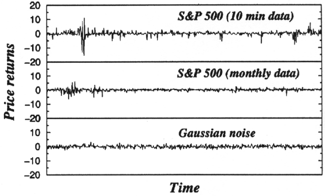
risk managment, expertise in model building based on empirical data

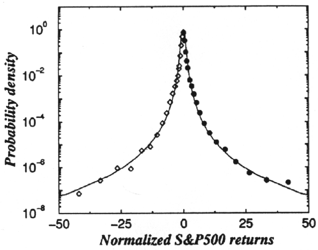
Economics — a Broad Range of Different Aspects



Example: Return Distributions

$$R_{\Delta t}(t) = \frac{S(t+\Delta t) - S(t)}{S(t)}$$





non–Gaussian, heavy tails! (Mantegna, Stanley, ..., 90's)

Introduction — Credit Risk

Credits and Stability of the Economy

- credit crisis shakes economy $\longrightarrow$ dramatic instability

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- **claim: risk reduction by diversification**

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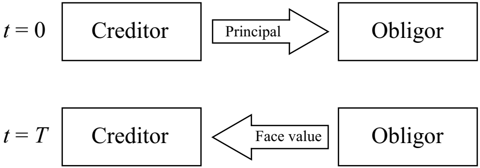
Credits and Stability of the Economy

- credit crisis shakes economy $\longrightarrow$ dramatic instability
- **claim: risk reduction by diversification**
- questioned with qualitative reasoning by several economists
- I now present our quantitative study and answer

Defaults and Losses

- default occurs if obligor fails to repay $\rightarrow$ loss
- possible losses have to be priced into credit contract
- correlations are important to evaluate risk of credit portfolio
- statistical model to estimate loss distribution

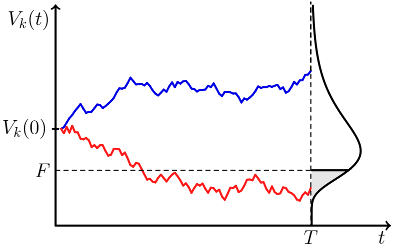
Zero–Coupon Bond



- principal: borrowed amount
- face value F:
  borrowed amount + interest + risk compensation
- credit contract with simplest cash-flow
- credit portfolio comprises many such contracts

Modeling Credit Risk

Structural Models of Merton Type



- microscopic approach for K companies
- economic state: risk elements $V_k(t),\ k = 1, \ldots, K$
- default occurs if $V_k(T)$ falls below face value F_k
- then the (normalized) loss is $L_k = \dfrac{F_k - V_k(T)}{F_k}$

Geometric Brownian Motion with Jumps

K companies, risk elements $V_k(t),\ k = 1, \ldots, K$ represent economic states, closely related to stock prices

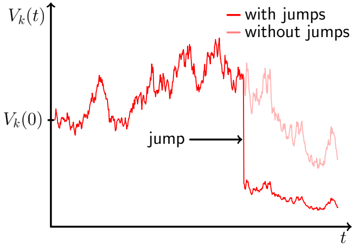
$$\frac{dV_k(t)}{V_k(t)} = \mu_k \, dt + \sigma_k \varepsilon_k(t) \sqrt{dt} + dJ_k(t)$$

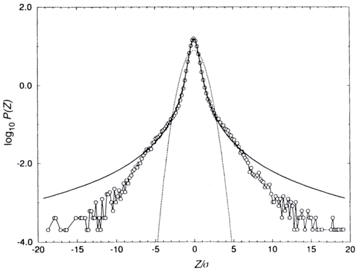
we include jumps !

- drift term (deterministic) $\mu_k \, dt$
- diffusion term (stochastic) $\sigma_k \varepsilon_k(t) \sqrt{dt}$
- jump term (stochastic) $dJ_k(t)$

parameters can be tuned to describe the empirical distributions

Jump Process and Price or Return Distributions



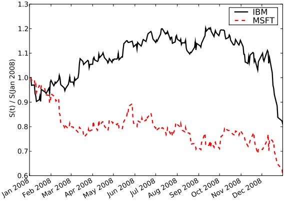


jumps reproduce empirically found heavy tails

Financial Correlations

asset values $V_k(t'),\ k = 1, \ldots, K$
measured at $t' = 1, \ldots, T'$

returns $R_k(t') = \dfrac{dV_k(t')}{V_k(t')}$



normalization $M_k(t') = \dfrac{R_k(t') - \langle R_k(t')\rangle}{\sqrt{\langle R_k^2(t')\rangle - \langle R_k(t')\rangle^2}}$

correlation $C_{kl} = \langle M_k(t') M_l(t')\rangle$, $\langle u(t')\rangle = \dfrac{1}{T'}\sum_{t'=1}^{T'} u(t')$

$K \times T'$ data matrix M such that $C = \dfrac{1}{T'} M M^\dagger$

Inclusion of Correlations in Risk Elements

- $\varepsilon_i(t), \; i = 1, \ldots, I$ set of random variables
- $K \times I$ structure matrix A
- correlated diffusion, uncorrelated drift, uncorrelated jumps

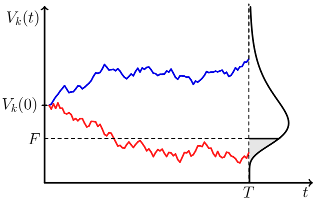
$$\frac{dV_k(t)}{V_k(t)} = \mu_k \, dt + \sigma_k \sum_{i=1}^{I} A_{ki} \varepsilon_i(t) \sqrt{dt} + dJ_k(t)$$

for $T \to \infty$ correlation matrix is $C = AA^\dagger$

covariance matrix is $\Sigma = \sigma C \sigma$ with $\sigma = \mathrm{diag}\,(\sigma_1, \ldots, \sigma_K)$

Loss Distribution

Individual Losses



normalized loss at maturity
$t = T$

$$L_k = \frac{F_k - V_k(T)}{F_k}\Theta(F_k - V_k(T))$$

if default occurs

Portfolio Loss Distribution

- homogeneous portfolio
- portfolio loss $L = \frac{1}{K} \sum_{k=1}^{K} L_k$
- stock prices at maturity $V = (V_1(T), \ldots, V_K(T))$
- distribution $p^{(\mathrm{mv})}(V, \Sigma)$ with $\Sigma = \sigma C \sigma$

want to calculate

$$p(L) = \int d[V] p^{(\mathrm{mv})}(V, \Sigma)\, \delta \left(L - \frac{1}{K} \sum_{k=1}^{K} L_k \right)$$

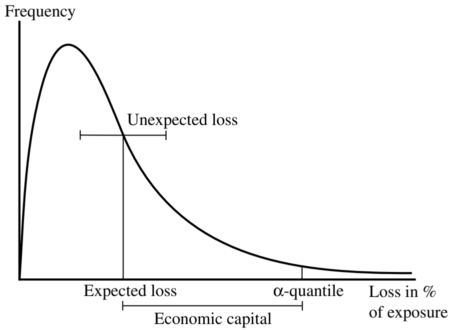
Large Portfolios

Real portfolios comprise several hundred or more individual contracts $\longrightarrow$ K is large.

Central Limit Theorem: For very large K, portfolio loss distribution $p(L)$ must become Gaussian.

Question: how large is "very large" ?

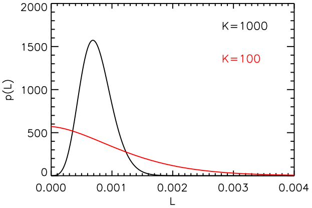
Typical Portfolio Loss Distributions

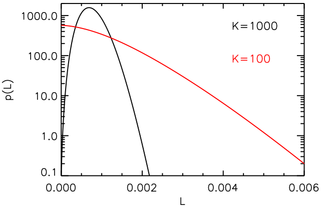


- highly asymetric, heavy tails, rare but drastic events
- mean of loss distribution is called expected loss (EL)
- standard deviation is called unexpected loss (UL)

Simplified Model — No Jumps, No Correlations

- analytical, good approximations
- slow convergence to Gaussian for large portfolio
- kurtosis excess of uncorrelated portfolios scales as $1/K$





Simplified Model — No Jumps, No Correlations

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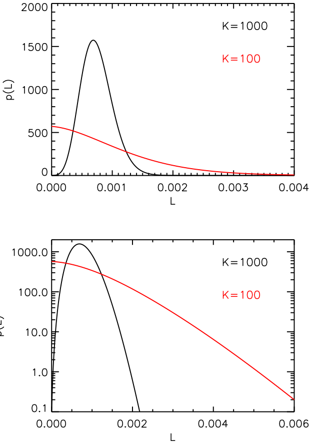
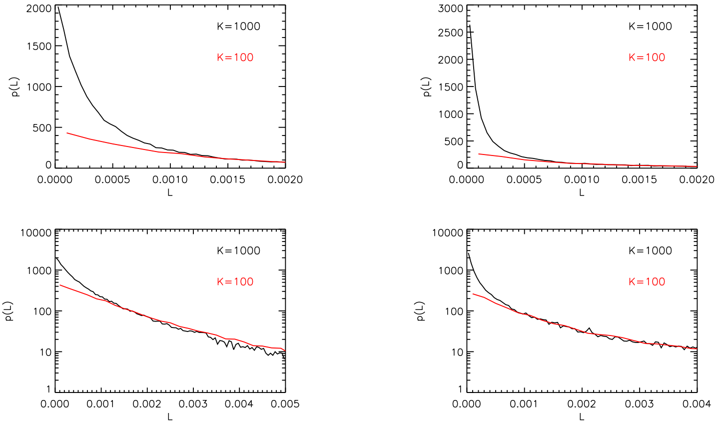
- kurtosis excess of uncorrelated portfolios scales as $1/K$
- diversification works slowly, but it works!



Numerical Simulations

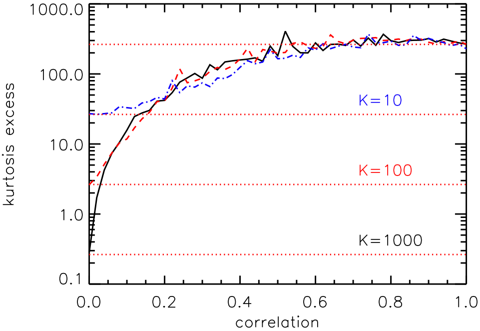
Numerical Simulations: Influence of Correlations, No Jumps

fixed correlation $C_{kl} = c, \; k \neq l$, and $C_{kk} = 1$



$c = 0.2$ $\qquad$ $c = 0.5$

Kurtosis Excess versus Fixed Correlation

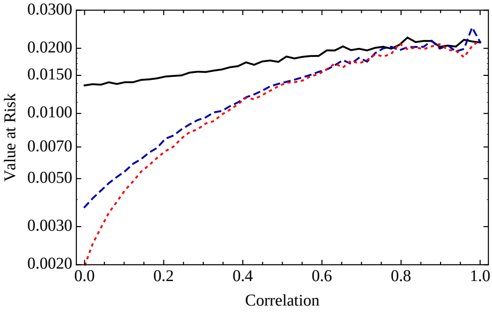


$$\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3$$

limiting tail behavior quickly reached

$\longrightarrow$ diversification does not work

Value at Risk versus Fixed Correlation



$$\int_0^{\mathrm{VaR}} p(L)dL = \alpha$$

here $\alpha = 0.99$

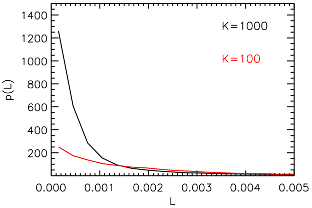
$K = 10, 100, 1000$

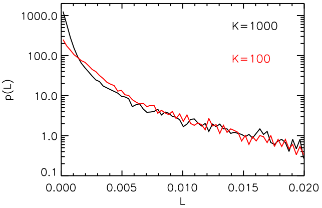
99% quantile, portfolio losses are with probability 0.99 smaller than VaR, and with probability 0.01 larger than VaR

diversification does not work, it does not reduce risk !

Numerical Simulations: Correlations and Jumps

- correlated jump–diffusion
- fixed correlation $c = 0.5$
- jumps change picture only slightly
- tail behavior stays similar with increasing K





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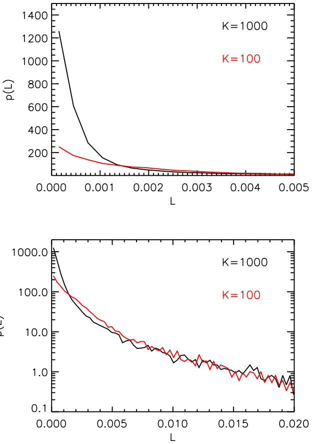
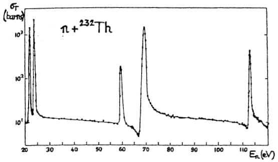


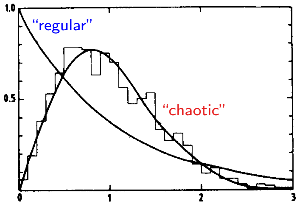
Random Matrix Approach

Quantum Chaos

result in statistical nuclear physics (Bohigas, Haq, Pandey, 80's)



resonances



spacing distribution

universal in a huge variety of systems: nuclei, atoms, molecules, disordered systems, lattice gauge quantum chromodynamics, elasticity, electrodynamics

$\longrightarrow$ quantum chaos $\longrightarrow$ random matrix theory

Search for Generic Features of Loss Distribution

- large portfolio $\rightarrow$ large K
- correlation matrix C is $K \times K$
- "second ergodicity": spectral average = ensemble average
- set $C = WW^\dagger$ and choose W as random matrix

Search for Generic Features of Loss Distribution

- large portfolio $\rightarrow$ large K
- correlation matrix C is $K \times K$
- "second ergodicity": spectral average = ensemble average
- set $C = WW^\dagger$ and choose W as random matrix
- additional motivation: correlations vary over time

Price Distribution at Maturity

Brownian motion, $V = (V_1(T), \ldots, V_K(T))$, price distribution

$$p^{(\mathrm{mv})}(V, \Sigma) = \frac{1}{\sqrt{2\pi T}^K} \frac{1}{\sqrt{\det \Sigma}} \exp\left(-\frac{1}{2T}(V - \mu T)^\dagger \Sigma^{-1} (V - \mu T)\right)$$

$C = WW^\dagger$ with W rectangular real $K \times N$,
N free parameter, such that $\Sigma = \sigma WW^\dagger \sigma$

assume Gaussian distribution for W with variance $1/N$

$$p^{(\mathrm{corr})}(W) = \sqrt{\frac{N}{2\pi}}^{KN} \exp\left(-\frac{N}{2} \mathrm{tr}\, W^\dagger W\right)$$

average correlation is zero, that is $\langle WW^\dagger \rangle = 1_K$

Average Price Distribution

$$\langle p^{(\mathrm{mv})}(\rho)\rangle = \int d[W] p^{(\mathrm{corr})}(W) p^{(\mathrm{mv})}(V, \sigma W W^\dagger \sigma)$$

$$= \sqrt{\frac{N}{2\pi T}}^K \frac{2^{1-\frac{N}{2}}}{\Gamma(N/2)} \rho^{\frac{N+K-1}{2}} \sqrt{\frac{N}{T}}^{\frac{N-K}{2}} \mathcal{K}_{\frac{N-K}{2}}\left(\rho\sqrt{\frac{N}{T}}\right)$$

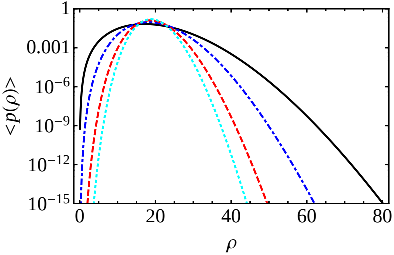
with hyperradius $\rho = \sqrt{\sum_{k=1}^{K} \frac{V_k^2(T)}{\sigma_k^2}}$

similar to statistics of extreme events

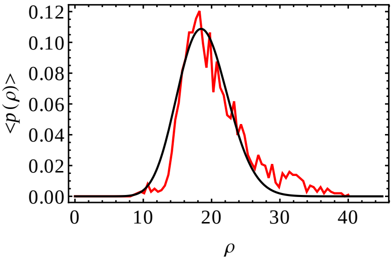
easily transferred to geometric Brownian motion

Heavy Tailed Average Return Distribution

about $K = 400$ stocks with complete time series from S&P500



$N = 5, 10, 20, 30$ (theory)

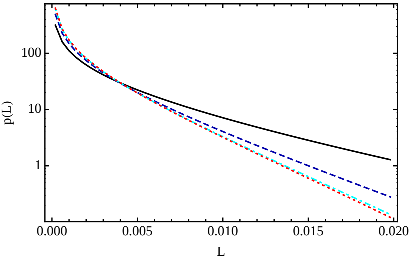


$N = 14$ (fit to data)

N smaller $\longrightarrow$ stronger correlated $\longrightarrow$ heavier tails

Average Loss Distribution

$$\langle p(L) \rangle = \int d[V] \langle p^{(\mathrm{mv})}(\rho) \rangle \, \delta \left(L - \frac{1}{K} \sum_{k=1}^{K} L_k \right)$$



$\langle C_{kl} \rangle = 0 \; , \quad k \neq l$

$N = 5 \;\rightarrow\; \mathrm{std}\left(C_{kl}\right) = 0.45$

$K = 10, 100, 1000, 10000$

best case scenario, but heavy tails remain

$\longrightarrow$ **little diversification benefit**

General Conclusions

- uncorrelated portfolios: diversification works (slowly)
- unexpectedly strong impact of correlations due to peculiar shape of loss distribution
- correlations lead to extremely fat–tailed distribution
- fixed correlations: diversification does not work
- ensemble average reveals generic features of loss distributions
- average correlation zero (best case scenario), but still: heavy tails remain, little diversification benefit
- non–zero average correlation: work in progress

Conclusions in View of the Present Credit Crisis

- contracts with high default probability
- rating agencies rated way too high
- credit institutes resold the risk of credit portfolios, grouped by credit rating
- lower ratings → higher risk and higher potential return
- **effect of correlations underestimated**
- **benefit of diversification vastly overestimated**

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Conclusions in View of the Present Credit Crisis


SPECIAL EDITION Marketplace | BBC WORLD SERVICE BUSINESS DAILY SPECIAL EDITION

EURO CRISIS CONTAGION

LONDON
Debt woes cripple Europe, dent global growth

LOS ANGELES
For US markets, Euro crisis is a state of mind

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R. Schäfer, M. Sjölin, A. Sundin, M. Wolanski and T. Guhr,
Credit Risk - A Structural Model with Jumps and Correlations,
Physica **A383** (2007) 533

M.C. Münnix, R. Schäfer and T. Guhr,
A Random Matrix Approach to Credit Risk,
arXiv:1102.3900

both ranked for several months among the top–ten
new credit risk papers on www.defaultrisk.com

R. Schäfer, A. Koivusalo and T. Guhr,
Credit Portfolio Risk and Diversification, invited contribution to
"Credit Portfolio Securitizations and Derivatives",
D. Rösch and H. Scheule (eds.), Wiley, 2012