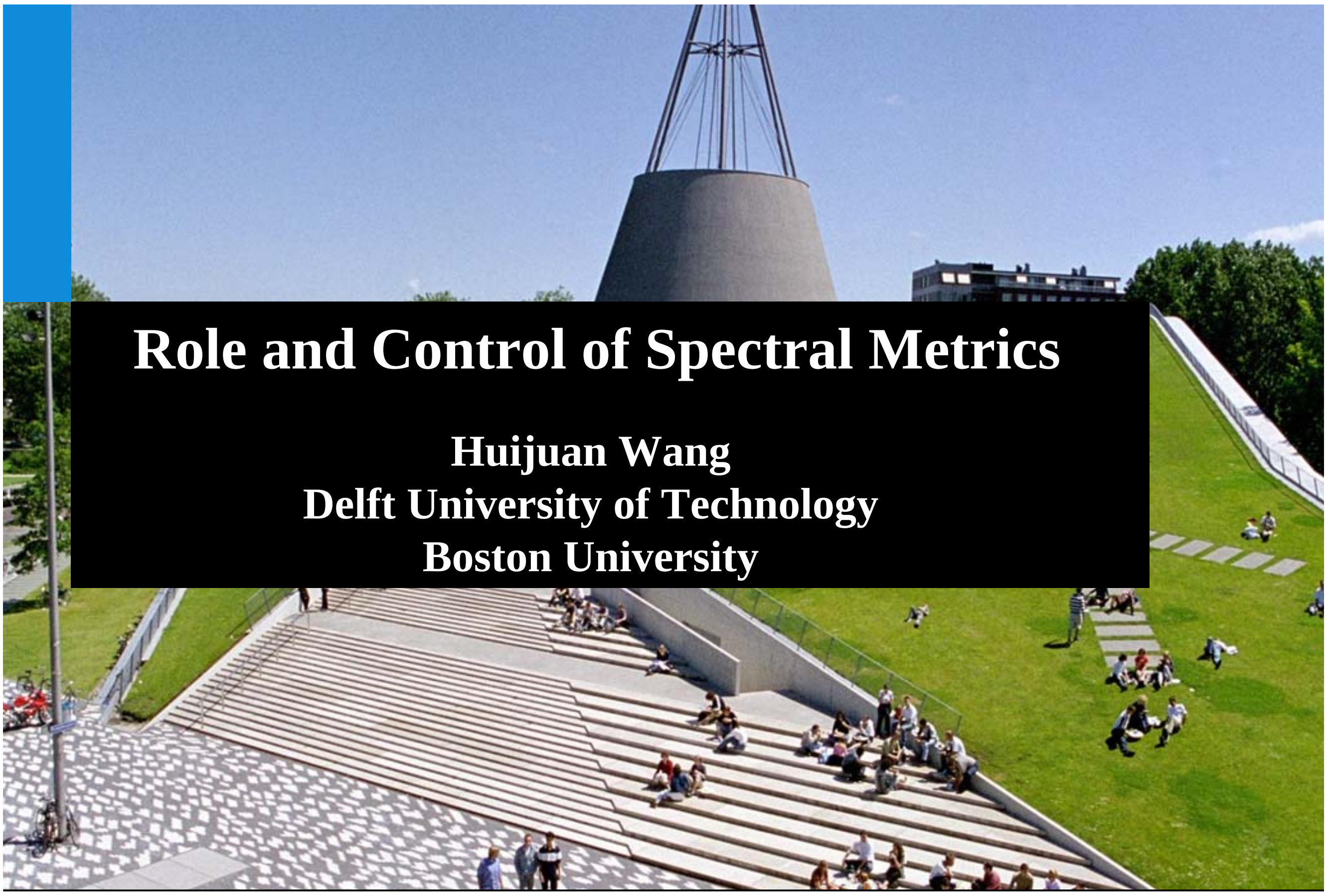


Role and Control of Spectral Metrics

Huijuan Wang
Delft University of Technology
Boston University

TU Delft — Delft University of Technology

ECT Trento Workshop

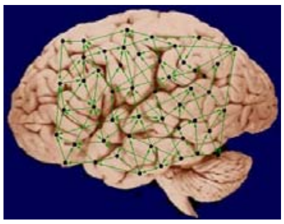
Challenge the future

"Metabolic network destruction: relating topology to robustness", Nano Communication Networks, 2011.

"Effect of tumor resection on the characteristics of functional brain networks", Physical Review E. 2010.

Biology

Brain



Economy

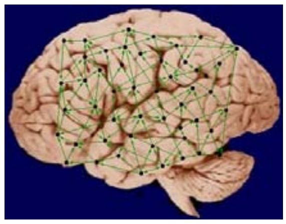
Network Science

"Multi-weighted Monetary Transaction Network", Advances in Complex Systems, 2011.

Man-made Infrastructures/Systems
Internet, power-grid

"The observable Part of a Network", IEEE/ACM Transaction on Networking, 2009.
"Betweenness Centrality in Weighted Networks", Physical Review E, 2008.

Effect of Network on Function



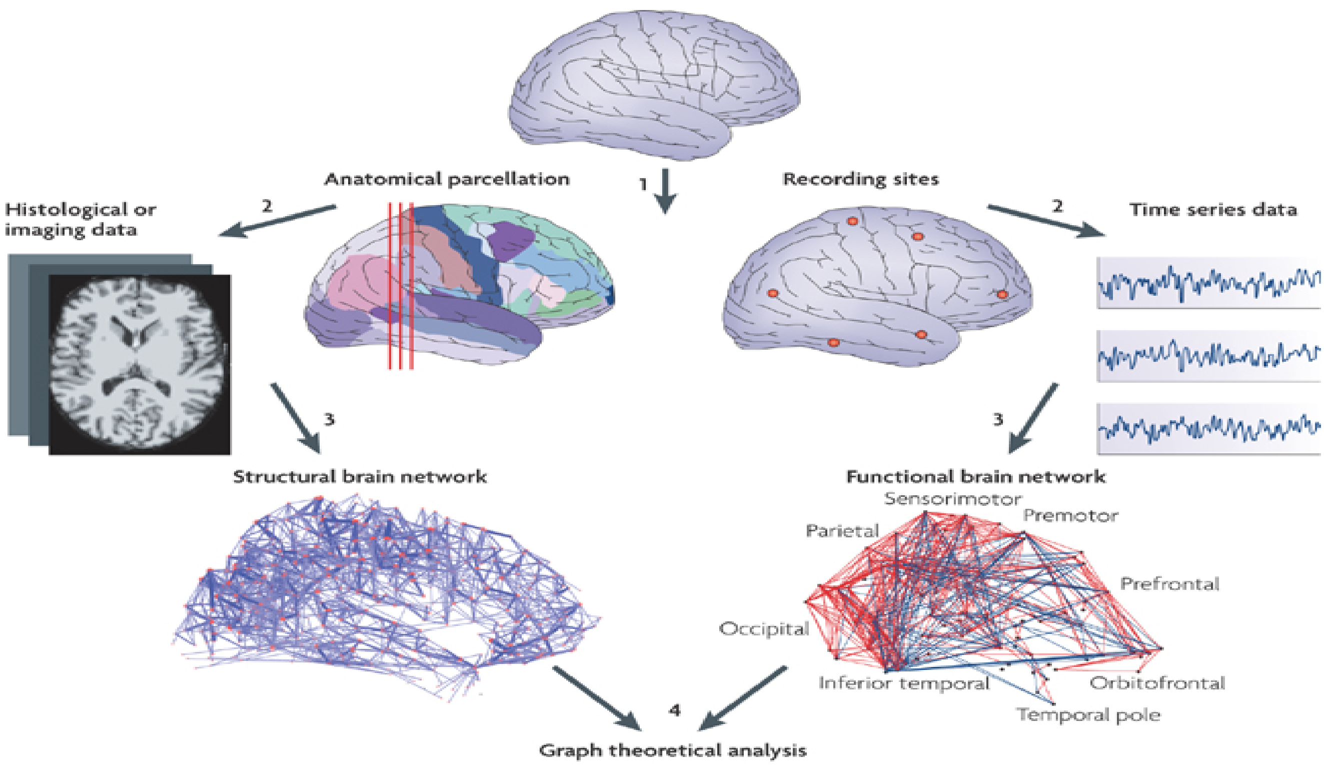
Biology

Brain

Economy

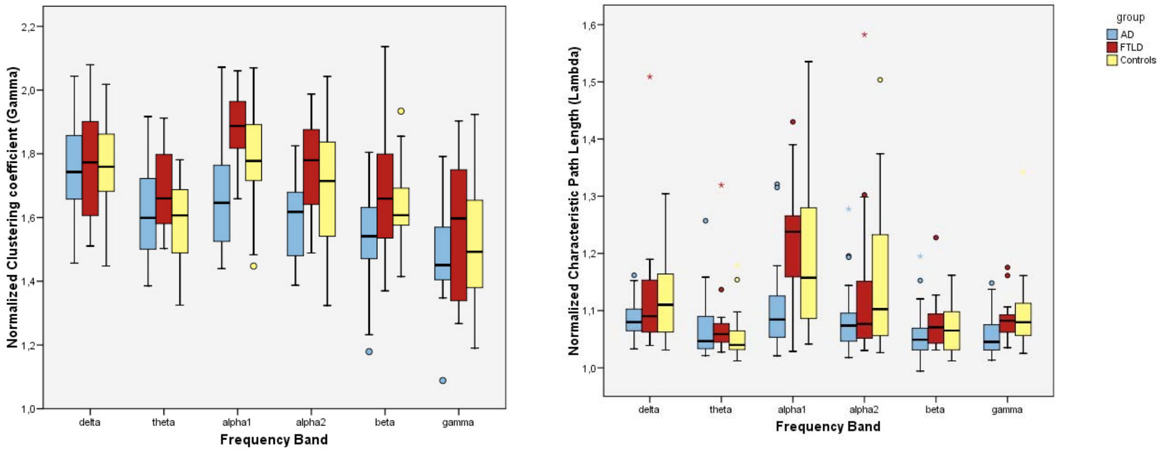
Network Science

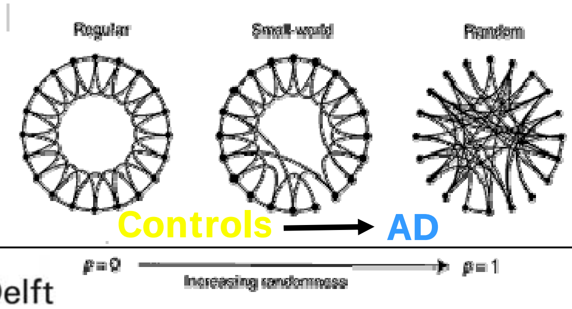
Man-made Infrastructures/Systems
Internet, power-grid



Nature Reviews | Neuroscience

Brain Networks vs. Alzheimer’s Diseases

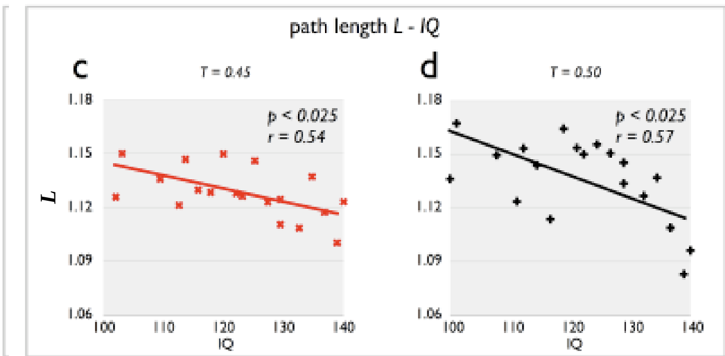




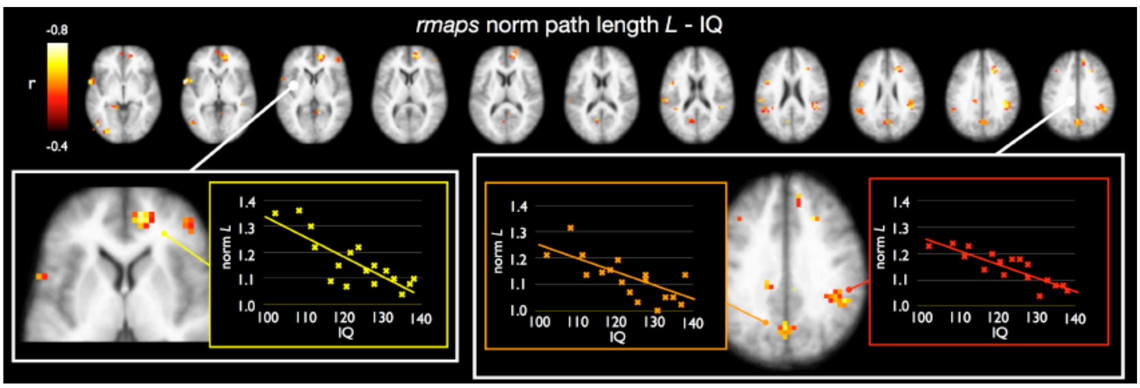
L (AD) < L (Controls)
C (AD) < C (Controls)

De Haan et al.
BMC Neuroscience 2009, 10: 101.

Brain Networks vs. IQ



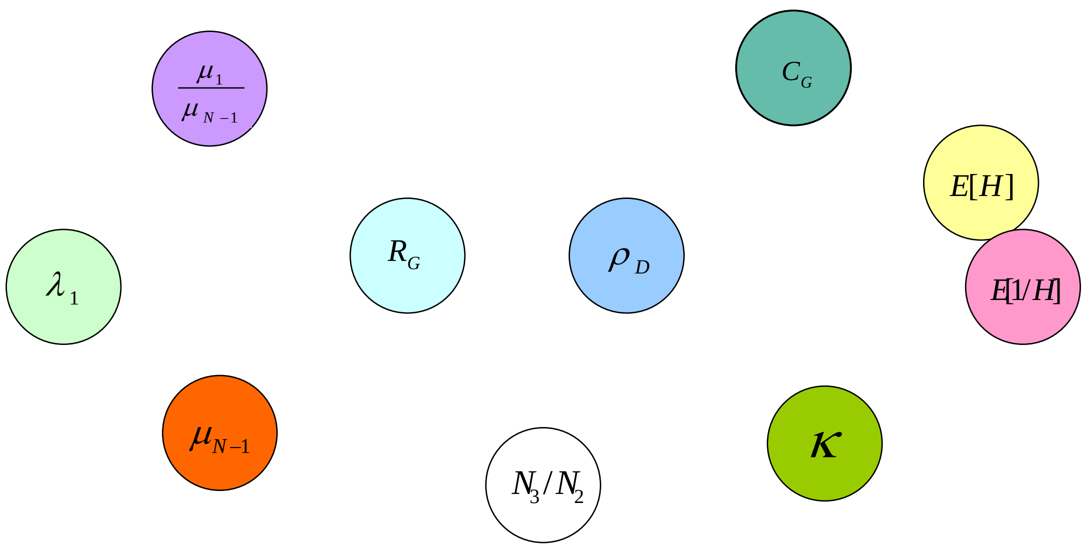
van den Heuvel et al.
J Neurosci. 2009, 29(23):7619-24.



Metric Correlations in Complex Networks

Cong Li, Huijuan Wang and Piet Van Mieghem
Journal of Statistical Mechanics: Theory and Experiment, P11018, 2011.

Network metrics



Metric Correlation is topology dependent

ER random graph: $E[H]$ is independent of N

D-lattice: $$E[H] \sim \frac{D}{3} N^{\frac{1}{D}}$$

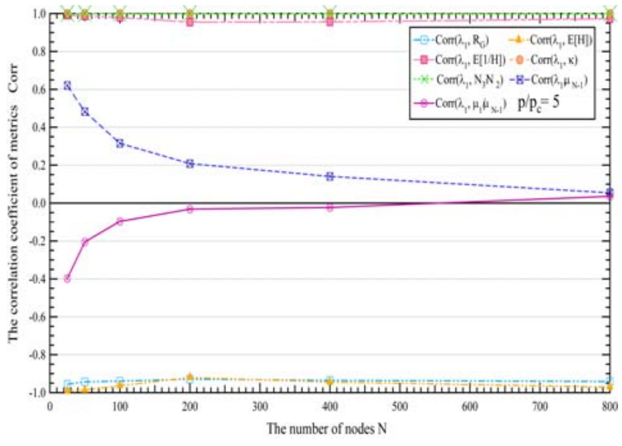
Consider: Erdös-Rényi random graphs
Bárabasi-Albert graphs

Application: Functional brain networks (MEG, health controls)

Metric correlation pattern

C_{ij}	R_G	μ_{N-1}	λ_1	$\frac{\mu_1}{\mu_{N-1}}$	$E[H]$
R_G	1.00	-0.77	-0.94	0.31	0.98
μ_{N-1}	-0.77	1.00	0.66	-0.73	-0.73
λ_1	-0.94	0.66	1.00	-0.11	-0.97
$\frac{\mu_1}{\mu_{N-1}}$	0.31	-0.73	-0.11	1.00	0.20
$E[H]$	0.98	-0.73	-0.97	0.20	1.00

Erdös-Rényi random graph



Metric correlation pattern

Erdös-Rényi random graph

Bárabasi-Albert graph

Analytic relations between network metrics

$$\frac{R_G}{(N-1)^2} \geq \frac{1}{\lambda_1}$$

$$\lambda_1^{\,3} \geq \frac{1}{N}\left(\rho\left(\sum_{i=1}^{N} d_i^3 - \frac{N_2^2}{N_1}\right) + \frac{N_2^2}{N_1}\right)$$

Van Mieghem, P., H. Wang, X. Ge, S. Tang and F. A. Kuipers, 2010, "Influence of Assortativity and Degree-preserving Rewiring on the Spectra of Networks", The European Physical Journal B, vol. 76, No. 4, pp. 643-652.

Verification in Brain Networks

C_{ij}	R_G	μ_{N-1}	λ_1	$\frac{\mu_1}{\mu_{N-1}}$	$E[H]$	$E[\frac{1}{H}]$	C_G	ρ_D	κ	N_3/N_2
R_G	1.00	-0.77	-0.94	0.31	0.98	-0.97	-0.91	-0.33	-0.94	-0.94
μ_{N-1}	-0.77	1.00	0.66	-0.73	-0.73	0.72	0.60	0.23	0.67	0.67
λ_1	-0.94	0.66	1.00	-0.11	-0.97	0.98	0.92	0.37	1.00	1.00
$\frac{\mu_1}{\mu_{N-1}}$	0.31	-0.73	-0.11	1.00	0.20	-0.19	-0.16	-0.16	-0.12	-0.11
$E[H]$	0.98	-0.73	-0.97	0.20	1.00	-1.00	-0.88	-0.26	-0.97	-0.97
$E[\frac{1}{H}]$	-0.97	0.72	0.98	-0.19	-1.00	1.00	0.90	0.27	0.99	0.99
C_G	-0.91	0.60	0.92	-0.16	-0.88	0.90	1.00	0.46	0.91	0.92
ρ_D	-0.33	0.23	0.37	-0.16	-0.26	0.27	0.46	1.00	0.31	0.34
κ	-0.94	0.67	1.00	-0.11	-0.97	0.99	0.91	0.31	1.00	1.00
N_3/N_2	-0.94	0.67	1.00	-0.11	-0.97	0.99	0.92	0.34	1.00	1.00

T=0.019

Verification in Brain Networks

Erdös-Rényi random graph

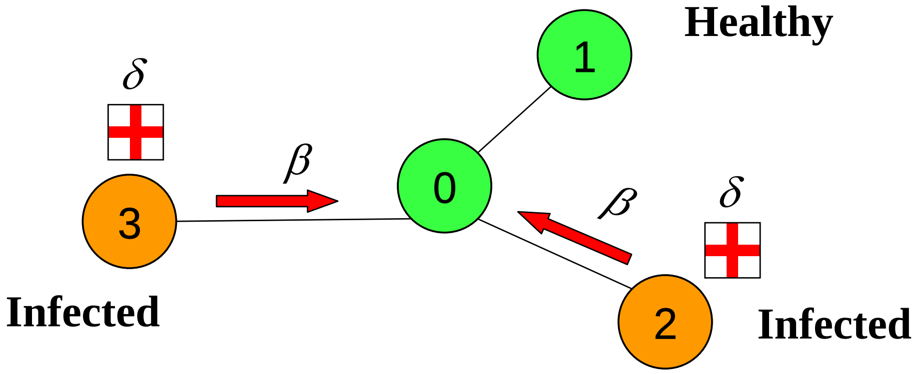
Functional Brain Networks

The Role of Spectral radius in Epidemics and Coupled Osillators

SIS model

- Homogeneous birth (infection) rate β on all edges between infected and susceptible nodes
- Homogeneous death (curing) rate δ for infected nodes

$\tau = \beta / \delta$: effective spreading rate

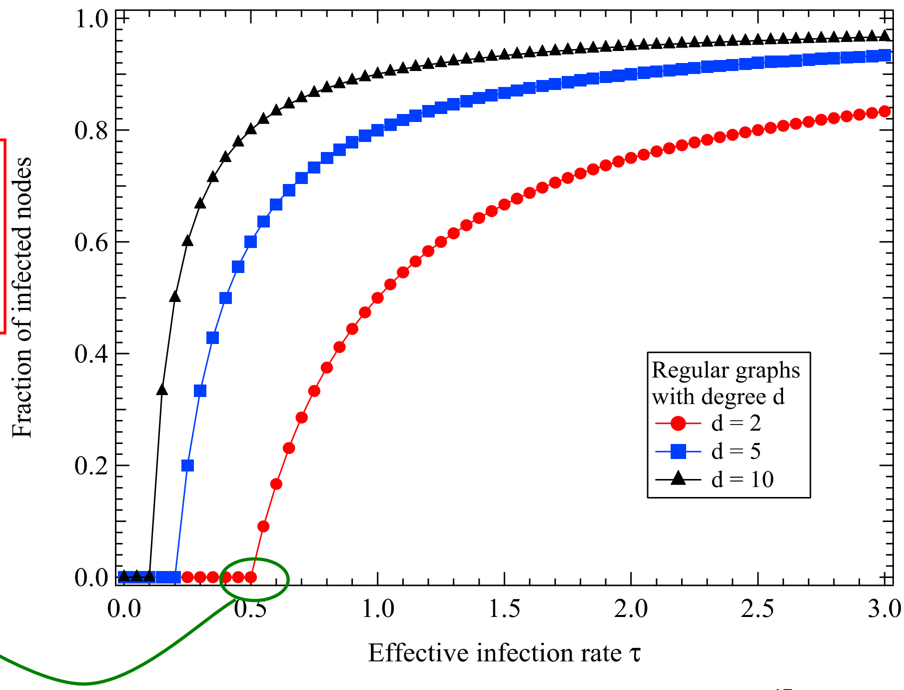


SIS model: epidemic threshold

β : infection rate per link

δ : curing rate per node

$\tau = \beta / \delta$: effective spreading rate

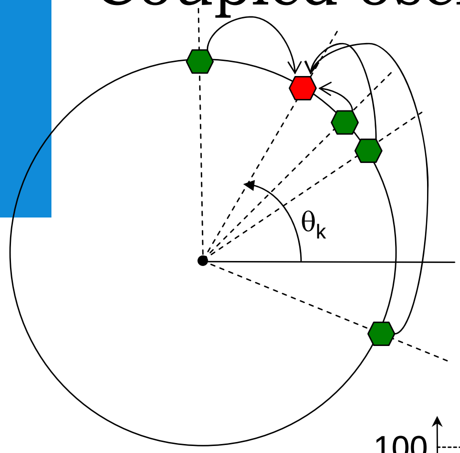


- **Epidemic threshold**

$$\tau_c = \frac{1}{\lambda_1(A)}$$

P. Van Mieghem, J. Omic, R. E. Kooij, "Virus Spread in Networks", IEEE/ACM Transaction on Networking, Vol. 17, No. 1, pp. 1-14, (2009).

Coupled oscillators (Kuramoto model)

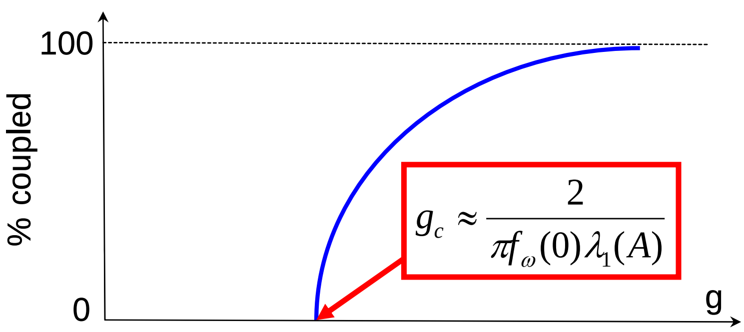


Interaction equals sums of sinus of phase difference of each neighbor:

$$\dot{\theta}_k = \omega_k + g\sum_{j=1}^{N} a_{kj} \sin\left(\theta_j - \theta_k\right)$$

coupling strength

natural frequency



$$g_c \approx \frac{2}{\pi f_\omega(0)\lambda_1(A)}$$

J. G. Restrepo, E. Ott, and B. R. Hunt. Onset of synchronization in large networks of coupled oscillators, Phys. Rev. E, vol. 71, 036151, 2005

Modify the spectral radius

- Degree-preserving rewiring

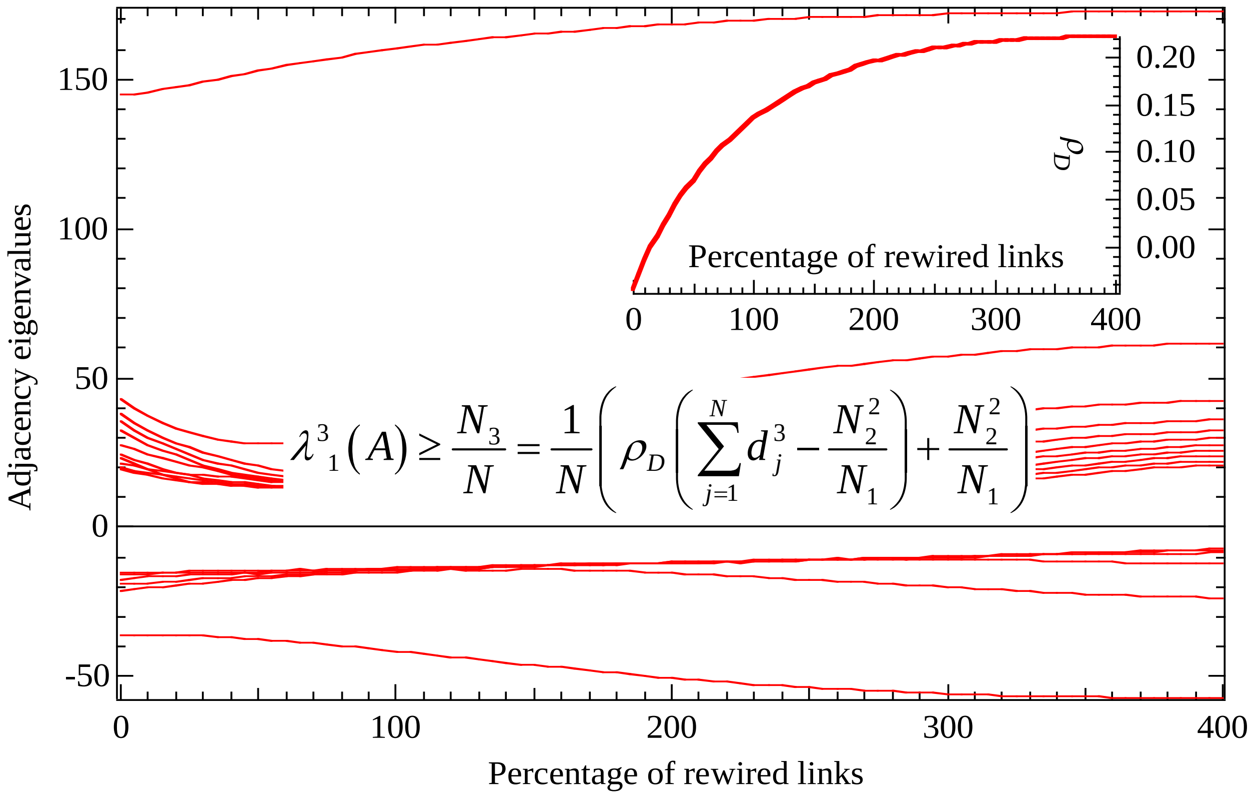
  "Influence of Assortativity and Degree-preserving Rewiring on the Spectra of Networks", The European Physical Journal B, vol. 76, No. 4, pp. 643-652, 2010.

- Removing links/nodes (optimal way is NP-complete)

  "Decreasing the spectral radius of a graph by link removals", Physical Review E, Vol. 84, 016101, 2011.

- Adding interacting links between two networks

Degree-preserving rewiring USA air transport network



Link / node removal

Removing m links/nodes to maximally decrease λ_1 is NP-hard.

$$\lambda_1(A) = x_1^T A x_1 = 2\sum_{l\in G}(x_1)_{l^+}(x_1)_{l^-}$$

$$\lambda_1(A_m) = w_1^T A_m w_1 = 2\sum_{l\in G\backslash M_m}(w_1)_{l^+}(w_1)_{l^-}, \text{where } l \text{ joins } l^+ \text{ and } l^-$$

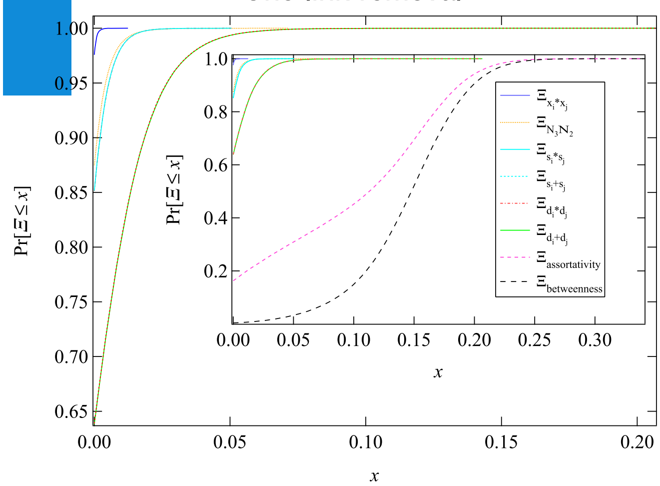
Lemma

$$2\sum_{l\in M_m}(w_1)_{l^+}(w_1)_{l^-} \le \lambda_1(A) - \lambda_1(A_m) \le 2\sum_{l\in M_m}(x_1)_{l^+}(x_1)_{l^-}$$

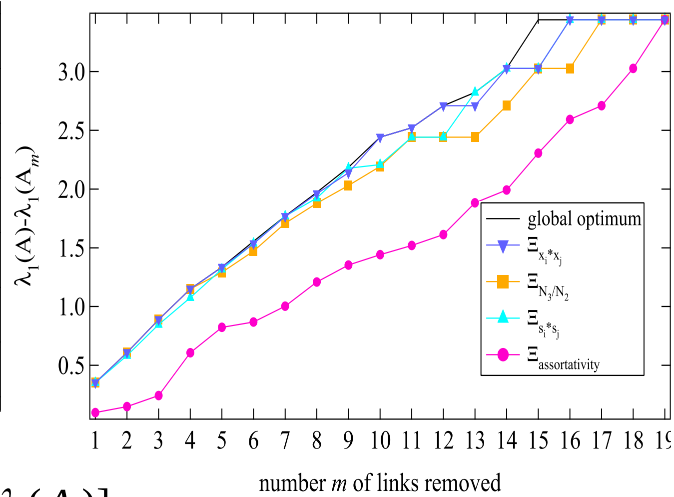
21

Link / node removal

One link removal



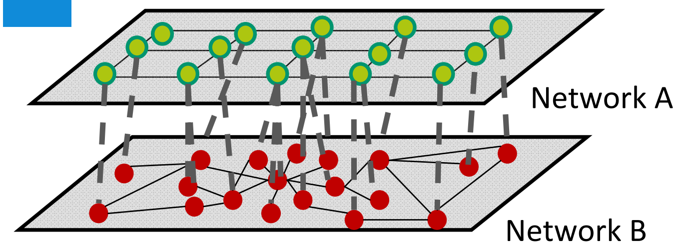
m link "greedy" removal



$$\Xi = [\lambda_1(A) - \lambda_1(A_1)]_{optimal} - [\lambda_1(A) - \lambda_1(A_1)]_{Strategy\ S}$$

Strategy $\Xi_{x_i * x_j}$: remove a link that maximizes $(x_1)_{l^+}(x_1)_{l^-}$

Interacting link addition



β : infection rate at connectivity link

δ : curing rate per node

$\alpha\ \beta$: infection rate at dependency link

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, B = \begin{bmatrix} 0 & B_{12} \\ B_{12}{}^T & 0 \end{bmatrix}$$

$$\lambda_1(A + \alpha B)$$

Two types of links: Connectivity & Dependency

S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley and S. Havlin, Nature 464, 1025 (2010)

Conclusion

- Metric correlations show that spectral metrics seem to be essential in network characterizations to discover the association between network and function.
- Epidemics and coupled oscillators: phase transition at $t_c = 1/\lambda_1$.
- Epidemic threshold engineering:
  - Degree-preserving *assortative* (disassortative) rewiring increases (decreases) λ_1.
  - Removing links/nodes to maximally decrease λ_1 is NP-hard, which motivates heuristic strategies.
  - How does adding interacting links increase λ_1?