Nonlinearity, interactions and Anderson localization

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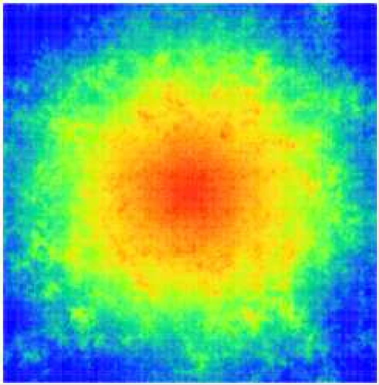
Overview of results 1993 - 2008



I.Garcia-Mata, DLS arXiv:0805.0539 (2008)

- Nonlinearity induced delocalization in the Anderson model
- Many-body Interactions: Two Interacting Particles effect
- Slow Metal
- Attractive Interactions and Superconductor-Insulator Transition
- Wigner crystal in a periodic potential
- Dynamical or Chirikov localization

Nonlinearity and Anderson localization: estimates

$$i\hbar\frac{\partial\psi_n}{\partial t} = E_n\psi_n + \beta|\psi_n|^2\psi_n + V(\psi_{n+1}+\psi_{n-1})\;;[-W/2 < E_n < W/2]$$

localization length $l \approx 96(V/W)^2$ (1D); $\ln l \sim (V/W)^2$ (2D) Amplitudes C in the linear eigenbasis are described by the equation

$$i\frac{\partial C_m}{\partial t} = \epsilon_m C_m + \beta\sum_{m_1m_2m_3} U_{mm_1m_2m_3}C_{m_1}C^*_{m_2}C_{m_3}$$

the transition matrix elements are $U_{mm_1m_2m_3} = \sum_n Q^{-1}_{nm}Q_{nm_1}Q^*_{nm_2}Q_{nm_3} \sim 1/l^{3d/2}$. There are about l^{3d} random terms in the sum with $U \sim l^{-3d/2}$ so that we have $idC/dt \sim \beta C^3$. We assume that the probability is distributed over $\Delta n > l^d$ states of the lattice basis. Then from the normalization condition we have $C_m \sim 1/(\Delta n)^{1/2}$ and the transition rate to new non-populated states in the basis $\boldsymbol{m}$ is $\Gamma \sim \beta^2|C|^6 \sim \beta^2/(\Delta n)^3$. Due to localization these transitions take place on a size l and hence the diffusion rate in the distance $\Delta R \sim (\Delta n)^{1/d}$ of $d-$ dimensional $\boldsymbol{m}-$ space is $d(\Delta R)^2/dt \sim l^2\Gamma \sim \beta^2 l^2/(\Delta n)^3 \sim \beta^2 l^2/(\Delta R)^{3d}$. At large time scales $\Delta R \sim R$ and we obtain

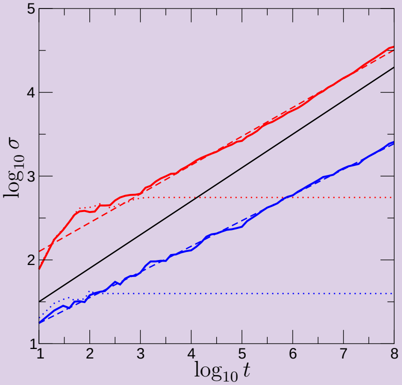
$$\Delta n \sim R^d \sim (\beta l)^{2d/(3d+2)} t^{d/(3d+2)}$$

Chaos criterion:

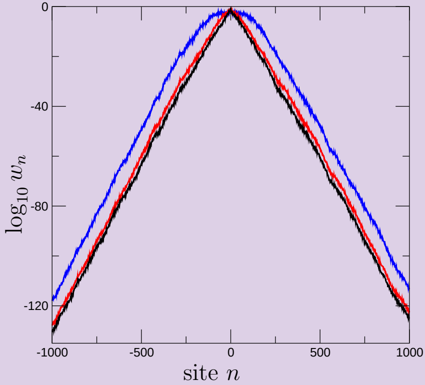
$$S = \delta\omega/\Delta\omega \sim \beta > \beta_c \sim 1$$

there $\delta\omega \sim \beta|\psi_n|^2 \sim \beta/\Delta n$ is nonlinear frequency shift
and $\Delta\omega \sim 1/\Delta n$ is spacing between exites eigenmodes
DLS PRL **70**, 1787 (1993) ($d = 1$); I.Garcia-Mata, DLS arXiv:0805.0539 (2008) ($d \geq 1$)

Nonlinearity and Anderson localization (1D)



$W/V = 2, 4$, $\beta = 0, 1$; $\sigma = (\Delta n)^2 \propto t^{2/5}$

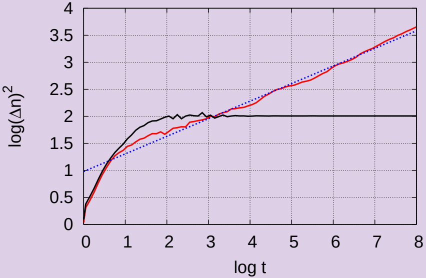


$W/V = 4$, $\beta = 1$, $t = 10^8$, $\beta = 0$

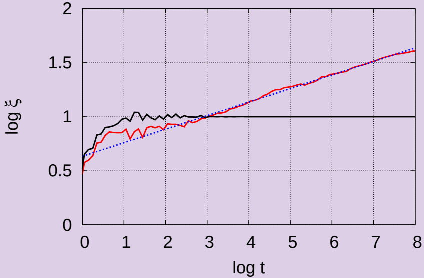
$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\psi_n|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1})\ ; [-W/2 < E_n < W/2]$$

A.S.Pikovsky, DLS PRL **100**, 094101 (2008)

Nonlinearity and Anderson localization (1D)



$W/V = 4$, $\beta = 1$, $\beta = 0$; $\alpha_1 = 0.325 \pm 0.003$ (theory 0.4)

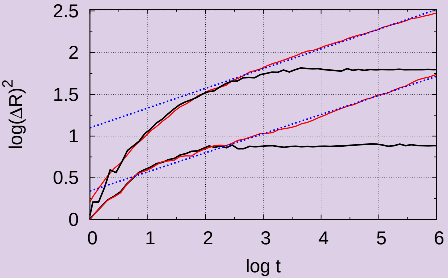


$\nu = 0.125 \pm 0.001$ (theory 0.2); ξ is participation ratio

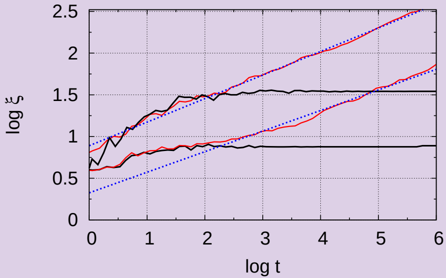
$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\psi_n|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1})$$

I.Garcia-Mata, DLS arXiv:0805.0539 (2008)

Nonlinearity and Anderson localization (2D)



$W/V = 10, 15$, $\beta = 0, 1$; $\alpha_2 = 0.236, 0.229 \pm 0.003$ (theory 0.25)

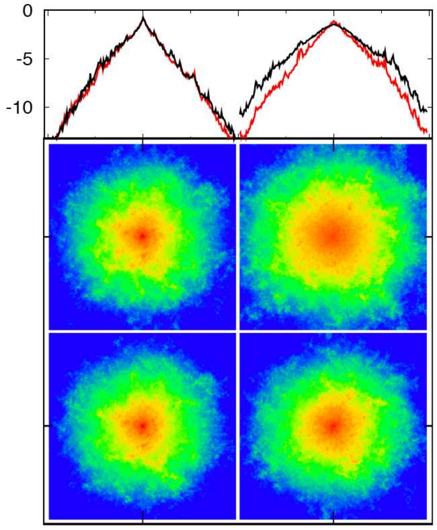


$\nu = 0.282, 0.247 \pm 0.005$ (theory 0.25); ξ is participation ratio

$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\psi_n|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1})$$

I.Garcia-Mata, DLS arXiv:0805.0539 (2008)

Nonlinearity and Anderson localization (2D)



$W = 10; \beta = 0$(left), 1(right);
$t = 10^4$ (bottom), 10^6 (middle),
projecton on $x-$axis (top);
256×256 lattice

[also: kicked nonlinear rotator model (1d)]

I.Garcia-Mata, DLS arXiv:0805.0539 (2008)

Possible experimental tests & applications

- BEC in disordered potential (Aspect, Inguscio)
- BEC time reversal (Phillips)
- nonlinear wave propagation in disordered media (Segev, Silberberg)
- lasing in random media (Cao)
- energy propagation in complex molecular chains (proteins, Fermi-Pasta-Ulam problem)
- NONLINEAR SPIN-GLASS ?
- OTHER GROUPS:
  S.Aubry *et al.* PRL **100**, 084103 (2008)
  A.Dhar *et al.* PRL **100**, 134301 (2008)
  S.Fishman *et al.* J. Stat. Phys. bf 131, 843 (2008)
  S.Flach *et al.* arXiv:0805.4693[cond-mat] (2008)
  W.-M.Wang *et al.* arXiv:0805.4632[math.DS] (2008)
  see also the participant list of the NLSE Workshop
  at the Lewiner Institute, Technion, June 2008

Two Interacting Particles (TIP) effect

Anderson model in d-space + onsite Hubbard interaction U, $V \sim E_F$ is one-particle hopping; exited states $\psi_n \sim \exp(-|n-m|/l)/\sqrt{l}$; $l \gg 1$.
Equation in the basis of noninteracting eigenstates $\chi_{m_1 m_2}$:

$$i\partial\chi_{m_1m_2}/\partial t = \epsilon_{m_1m_2}\chi_{m_1m_2} + \sum_{m'_1 m'_2} U_{m_1m_2m'_1m'_2}\chi_{m'_1m'_2}$$

Sum runs over $M \sim l^d$ coupled states; interaction induced matrix elements $U_s \sim U_{m_1m_2m'_1m'_2} \sim (U/(l^{2d}) \times \sqrt{M}$, density of coupled states is $\rho_2 \sim l^{2d}/V$, TIP transition rate $\Gamma_s \sim {U_s}^2\rho_2 \sim U^2/(l^d V)$. Enhancement factor

$$\kappa = \Gamma_s\rho_2 \sim (U/V)^2 l^d > 1$$

TIP localization:
$l_2/l \sim (U/V)^2 l$ (1d);
$\ln(l_2/l) \sim (U/V)^2 l^2$ (2d);
delocalization for $\kappa \sim (U/V)^2 l^3 > 1$ (3d)

DLS PRL **73**, 2607 (1994); Y.Imry EPL **30**, 405 (1995)

Slow Metal (2D)

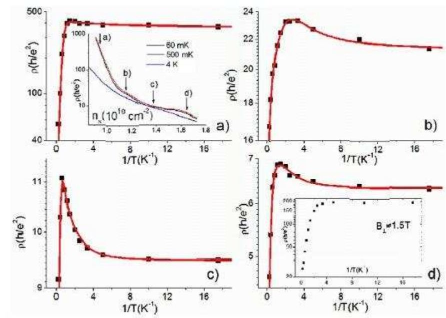


FIG. 2 (color online). Resistivity as a function of inverse temperature $1/T$ at $B = 0$ T (symbols). At all densities, the strongly insulating T dependence at higher temperatures is followed by a decrease in resistance at low T. Device dimensions are $W \times L = 8\ \mu\text{m} \times 0.5\ \mu\text{m}$, spacer $\delta = 40$ nm. Electron densities are indicated by arrows in the inset to (a). Solid lines represent a fit of Eq. (1) to the data. Inset to (a): Resistivity as a function of electron density at $T = 60$ mK, 500 mK, 4 K. Inset to (d): ρ as function of $1/T$ at the same density as (d) but at $B_\perp = 1.5$ T.

TIP diffusion

$D \sim \Gamma_s l^2 \sim U^2/V$ at $(Ul/V)^2 > 1$

vs. usual diffusion $D_0 \sim v_F \ell \sim V$

Thus it is possible to have diffusion with conductance g and resistivity per square ρ_0 (in natural units):

$g \sim 1/\rho_0 \sim D/D_0 \sim (U/V)^2 \ll 1$

With up to $(Ul/V)^2 \sim 1$ and

$g \sim 1/l^2 \ll 1$

Problems: finite particle density, small density of states near the ground state

Experiment suggestion: to measure a charge of quasi-particles from noise fluctuations

M.Baenninger, A.Ghosh, M.Pepper, H.E.Beere, I.Farrer, D.A.Ritchie PRL **100**, 016805 (2008) vs. S.Kravchenko *et al.* RMP **73**,251 (2001)

TIP near the Fermi level

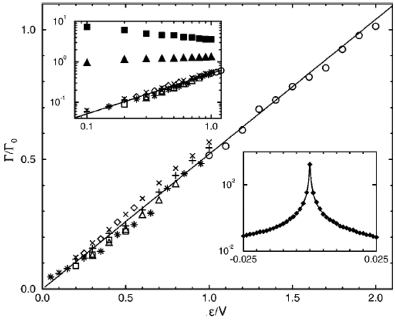


FIG. 1. Energy dependence of the rescaled Breit-Wigner width Γ/Γ_0 in 2D. Direct diagonalization (DD) data at $W/V = 2$: $U/V = 0.6$ with $L = 8$(○), $L = 15$ (△), $L = 20$ (□); $U/V = 1.5$ and $L = 20$ (◇). Fermi golden rule (FGR) data: $W/V = 2$ with $L = 20$ (+), $L = 25$ (×); $W/V = 1$ with $L = 15$ (∗). The straight line $\Gamma(\epsilon)/\Gamma_0 = C\epsilon/V$ with $C = 0.52$ shows the Imry estimate. Upper inset: the same on a log-log scale with FGR data at higher disorders [$W/V = 6$ (▲) and $W/V = 10$ (■) ($L = 30$)]. Lower inset: ρ_W vs E for $L = 20, W/2 = V = 1, U = 0.6, \epsilon = 0.4$ fitted by ρ_{BW} with $\Gamma = 0.18\Gamma_0$ (solid curve).

Small ϵ energy excitations above the Fermi level:

a)box size $L \ll l$

$\rho_2 \sim L^{2d}\epsilon/V$, $\Gamma = C\Gamma_0\epsilon/V$,

$\Gamma_0 = U^2/(VL^d)$, $C = const$

b)box size $L \sim l$

$U_s^2 \sim \Delta^2(U/V)^2(1+\epsilon/E_c)^{d/2-2}/g^2$,

with $g = E_c/\Delta > 1$ and for

$\epsilon > E_c \sim V/L^2 > \Delta \sim V/L^d$

$\kappa = \Gamma\rho_2 \sim (U/V)^2(\epsilon/\Delta)^{d/2-1}$

for $L \sim l$, $d = 2$ we have κ independent of ϵ for $\epsilon \sim \Delta$.

Problems:

there is no enhancement at E_F, $\kappa \sim 1$

P.Jacquod, DLS PRL **78**, 4986 (1997)

Many electrons near the Fermi level (Coulomb interaction, no spin)

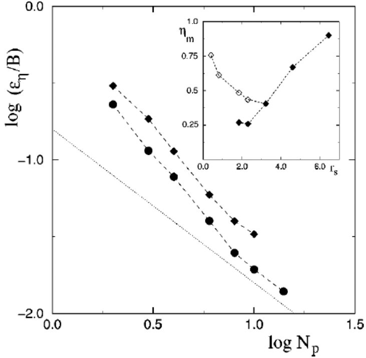


FIG. 4. Dependence of ϵ_η/B on the number of particles N_p, obtained from Fig. 2: $W/V=10$ with $\eta(E_\eta)=0.4$ (full diamond) and $W/V=7$ with $\eta(E_\eta)=0.2$ (●), where $\epsilon_\eta=E_\eta/N_p$. The straight line shows the slope when $E_\eta=$const. The inset gives the dependence of maximal η on r_s for $W/V=7$ and $N_p=6$: $U/V=2$, $8\leqslant L\leqslant 28$ (full diamond), and $L=14$, $0.25\leqslant U/V\leqslant 2$ (◇).

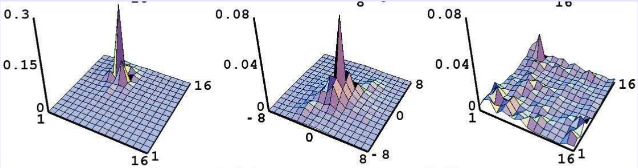
Level-spacing statistics $P(s)$:
$\eta = 1$ Poisson distribution,
$\eta = 0$ Wigner-Dyson distribution
ϵ_η - exitation energy per particle
at a given $\eta = const$ $(B = 4V)$
$r_s = U/(2V\sqrt{\pi\nu})$, $\nu = N_p/L^2 \approx 1/32$
usually $U = 2V$, $r_s \approx 3.2$,
$2 \leq N_p \leq 20$, $8 \leq L \leq 25$

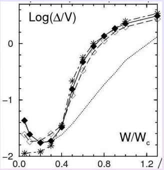
Result: chaotic, ergodic states at temperature going to zero

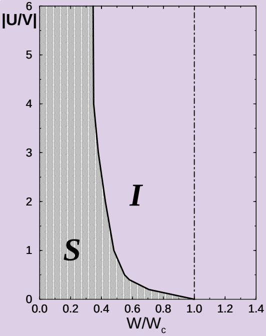
Problems: transport properties ?

DLS PRB **61**, 4588 (200); P.H.Song, DLS PRB **61**, 15546 (2000)

Cooper problem in the vicinity of the Anderson transition







Top (left): $3d$, $W/W_c = 0.5$, $W_c/V = 16.5$, $U/V = -4$ (left/middle); $U/V = 0$ (right); left/right: particle density projected on (x, y) plane; middle: interparticle distance probability
Top (right): large coupling gap Δ, not reproduced by mean field (dashed curve $L = 12$); $U/V = -4$, $L = 10, 12, 14$ (symbols)
Left: Diagram of bi-particle localized (BLS) phase
Result: localized pairs inside noninteracting metallic phase with $g \gg 1$; mean field does not give this BLS phase

J.Lages, DLS PRB **62**, 8665 (2000)

3d Hubbard model of spin-1/2 fermions (projector quantum Monte Carlo)

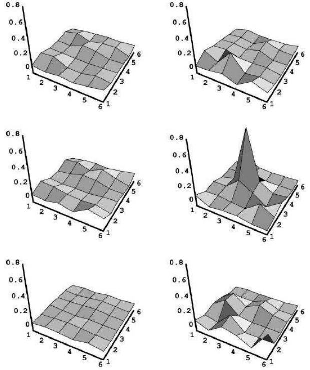


FIG. 1. Distribution of charge density difference for an added pair, $\delta\rho_p$, projected on the (x,y) plane for a $6\times6\times6$ lattice for the same single disorder realization, with $W/t=2$ (left) and $W/t=7$ (right), $N=108$. Top: exact computation for $U=0$, $\xi=70;55$ (left; right). Middle: PQMC calculation for $U/t=-4$, $\xi=48;6.5$ (left; right). Bottom: BdG mean-field calculation for $U/t=-4$, $\xi=132;25$ (left; right). All quantities presented in all figures are in dimensionless units (see text).

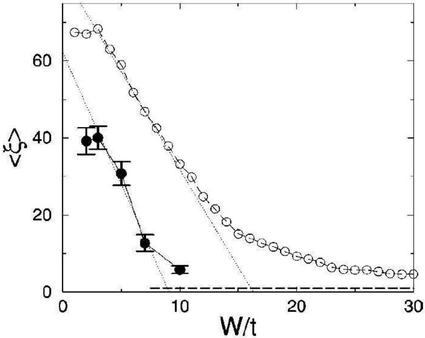


FIG. 2. Inverse participation ratio $\langle\xi\rangle$ averaged over disorder realizations, as a function of disorder strength W for a $6\times6\times6$ lattice, at $U=0$ (open circles) and $U/t=-4$ (solid circles). Dotted lines show linear fits to the data, the dashed line represents $\xi=1$ (see text), and error bars indicate statistical errors.

B.Srinivasan, G.Benenti, DLS PRB **66**, 172506 (2002) (up to $N=110$ fermions; $t=V$)

Superconductor-Insulator Transition: experiment

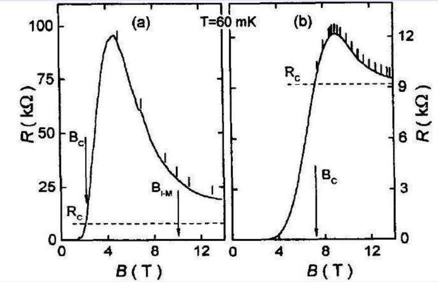


Fig.1. Magnetoresistance of the film in state 1 (a) and in state 2 (b). The critical R_c and B_c values at $T = 0$ are indicated. Also shown is the position of metal-insulator transition, B_{I-M}, determined from Fig.2. The temperature dependences of the resistance are analyzed at fields marked by vertical bars

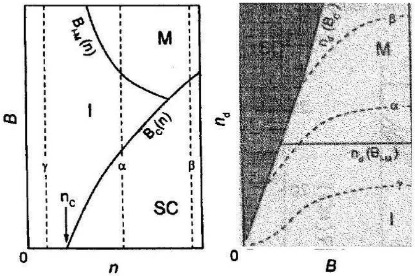
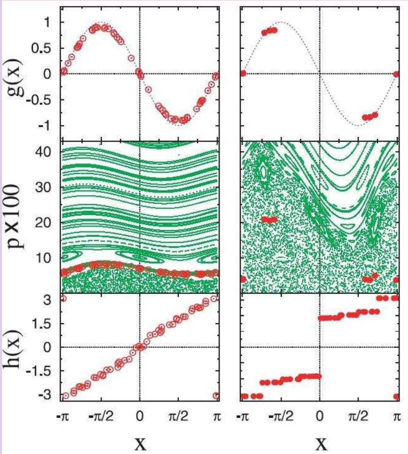
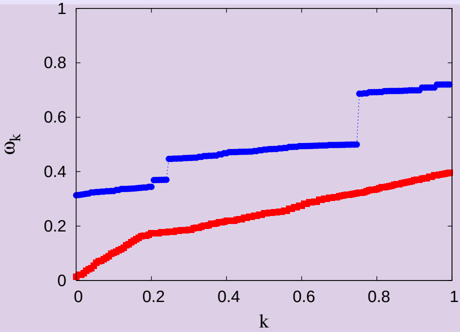


Fig.4. Schematic phase diagram of the observed transitions in the (n, B) and (B, n_d) planes. The evolution of states α, β, γ with magnetic field is shown by dashed lines. In shaded area the value n_d is not defined

V.F.Gantmakher *et al.* Pis'ma ZhETF **68**, 337 (1998)

Wigner crystal in a periodic potential (classical)





Hamiltonian:
$H = \sum_{i=1}^{N} \left(\frac{P_i^2}{2} + \frac{\omega^2}{2} x_i^2 - K \cos x_i \right) + \sum_{i>j} \frac{1}{|x_i - x_j|}$
$N = 150$ ions; $K = 0.03 < K_c \approx 0.05$ (open circle/red), $K = 0.2 > K_c$ (full circles/blue)

I.Garcia-Mata, O.V.Zhirov, DLS EPJD **41**, 325 (2007)

Wigner crystal in a periodic potential (quantum)

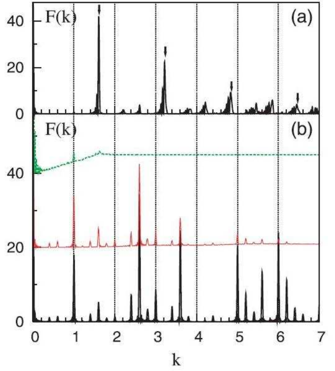


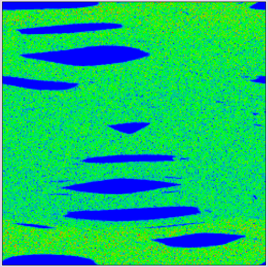
Fig. 11. (Color online) Formfactor $F(k)$ (see text) of the chain with $N = 150$ ions and $\omega = 0.00528$. (a) The classical incommensurate phase at $K = 0.03$, $\hbar = 0$, arrows mark the peaks at integer multiples of golden mean density ν_g. (b) The pinned phase at $K = 0.2$ for $\hbar = 0$ (bottom black curve), $\hbar = 0.1$ (middle red curve shifted 20 units upward), $\hbar = 2$ (top green curve shifted 40 units upward, for clarity $F(k)$ is multiplied by factor 5). The temperature of the quantum chain is $T = \hbar/\tau_0$ with $\tau_0 = 400$ so that $T \ll K$ and $T \ll \hbar\omega_0(K)$. (Compare

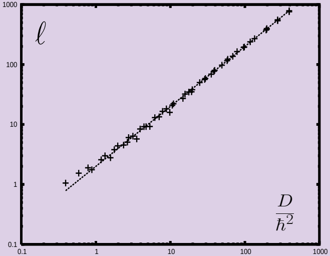
Quantum melting of the ground state:
transition from pinned instanton glass
to sliding phonons gas;
links to classical/quantum
Frenkel-Kontorova model
and Chirikov standard map

Conjecture:
similar mechanism for
delocalization of electrons in a
disordered potential

I.Garcia-Mata, O.V.Zhirov, DLS EPJD **41**, 325 (2007)

Chirikov localization: Chirikov typical map (1969)





- Standard map with random, periodically repeated phases ϕ_m:
  $\bar{p} = p + K \sin(x + \phi_m)$, $\bar{x} = x + \bar{p}$,
  $\phi_{m+T} = \phi_m$
  chaos border: $T^{-3/2} < K \ll 1$
  Kolmogorov-Sinai entropy: $h \sim K^{2/3} \ll 1$,
  diffusion rate per period T: $D = K^2 T/2$,
  => continuous time flow
  (Fig: Husimi function at $K = 0.1$, $T = 10$, $t = 2 \times 10^4$,
  $\hbar = 2\pi/N$, $N = 2^{16}$, initial coherent state at $p = 0$, $x = \pi$)

- $\ell \approx 2D/\hbar^2$: dynamical localization
  (Fig: $0.1 \leq K \leq 1$, $10 \leq T \leq 100$, $\hbar = 2\pi/17.618$)

K.Frahm, DLS, in preparation (2008)