Delocalization by nonlinearity and interactions in systems with disorder

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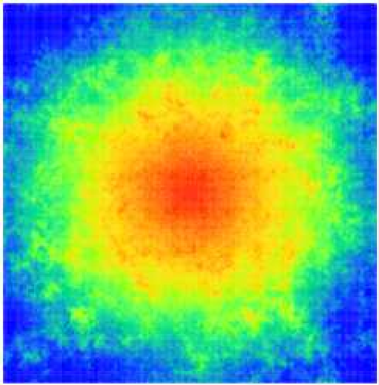


I.García-Mata, DLS arXiv:0805.0539 (2008)

- Anderson localization 1958 - 2008: Introduction, 50 years after
- Discrete Anderson nonlinear Schrödinger equation (DANSE) ($d = 1, 2$)
- Dynamical thermalization of nonlinear disordered lattices: anti-FPU
- Two interacting particles effect
- Åberg criterion for dynamical thermalization in many-body quantum systems

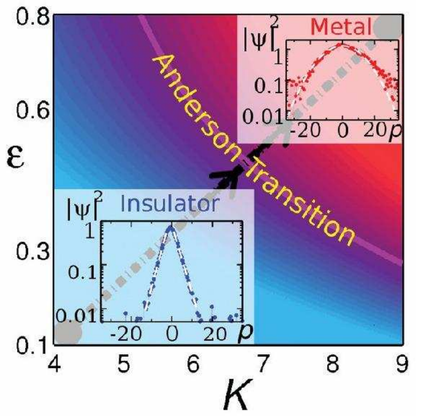
Anderson localization: introduction & perspectives

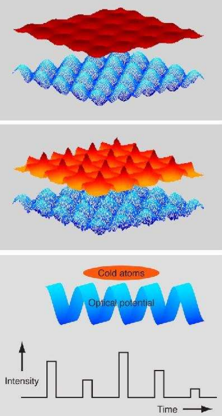
from the talk of P.W.Anderson at Newton Institute, July 21, 2008
see http://www.newton.ac.uk/programmes/MPA/seminars/072117001.html

"Well, In my country," said alice, still panting a little, "you would generally get to somehere else, if you ran very fast for a long time, as we've been doing". "A slow sort of country!", said the queen. "Now *here*, it takes all the running *you* can do, to stay in the same place."

Perspectives: a)localization in new type of systems; b)effects of interactions

3d-Dynamical de-localization of atomic waves





quantum chaos in kicked rotator => Chirikov localization in momentum space
=> dynamical analog of 3d Anderson transition

$$H = p^2/2 + K\cos x\,[1 + \epsilon\cos(\omega_2 t)\cos(\omega_3 t)]\sum_m \delta(t-m), \quad \hbar_{eff} = 2.89$$

J.C.Garreau *et al.* PRL **101**, 255702 (2008); theory prediction at PRL **62**, 345 (1989)

Nonlinearity and Anderson localization: estimates

$$i\hbar\frac{\partial\psi_n}{\partial t} = E_n\psi_n + \beta|\psi_n|^2\psi_n + V(\psi_{n+1}+\psi_{n-1})\;;[-W/2 < E_n < W/2]$$

localization length $l \approx 96(V/W)^2$ (1D); $\ln l \sim (V/W)^2$ (2D) Amplitudes C in the linear eigenbasis are described by the equation

$$i\frac{\partial C_m}{\partial t} = \epsilon_m C_m + \beta\sum_{m_1 m_2 m_3} U_{mm_1m_2m_3} C_{m_1} C^*_{m_2} C_{m_3}$$

the transition matrix elements are $U_{mm_1m_2m_3} = \sum_n Q^{-1}_{nm} Q_{nm_1} Q^*_{nm_2} Q_{nm_3} \sim 1/l^{3d/2}$. There are about l^{3d} random terms in the sum with $U \sim l^{-3d/2}$ so that we have $idC/dt \sim \beta C^3$. We assume that the probability is distributed over $\Delta n > l^d$ states of the lattice basis. Then from the normalization condition we have $C_m \sim 1/(\Delta n)^{1/2}$ and the transition rate to new non-populated states in the basis $\boldsymbol{m}$ is $\Gamma \sim \beta^2|C|^6 \sim \beta^2/(\Delta n)^3$. Due to localization these transitions take place on a size l and hence the diffusion rate in the distance $\Delta R \sim (\Delta n)^{1/d}$ of $d-$ dimensional $\boldsymbol{m}-$ space is $d(\Delta R)^2/dt \sim l^2\Gamma \sim \beta^2 l^2/(\Delta n)^3 \sim \beta^2 l^2/(\Delta R)^{3d}$. At large time scales $\Delta R \sim R$ and we obtain

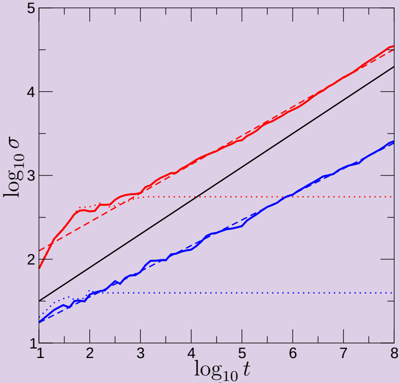
$$\Delta n \sim R^d \sim (\beta l)^{2d/(3d+2)} t^{d/(3d+2)};\; (\Delta n)^2 \propto t^\alpha;\; \alpha = 2/(3d+2)$$

Chaos criterion:

$$S = \delta\omega/\Delta\omega \sim \beta > \beta_c \sim 1$$

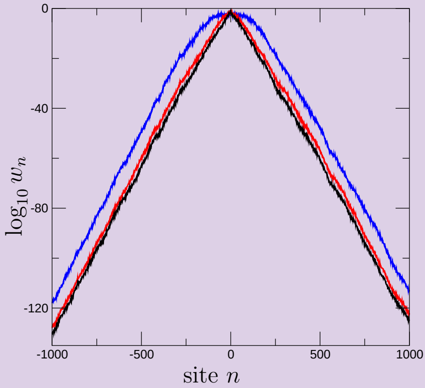
there $\delta\omega \sim \beta|\psi_n|^2 \sim \beta/\Delta n$ is nonlinear frequency shift
and $\Delta\omega \sim 1/\Delta n$ is spacing between exites eigenmodes
DLS PRL **70**, 1787 (1993) ($d = 1$); I.García-Mata, DLS arXiv:0805.0539 (2008) ($d \geq 1$)

Nonlinearity and Anderson localization (1D)



$W/V = 2, 4$, $\beta = 0, 1$; $\sigma = (\Delta n)^2 \propto t^\alpha$;

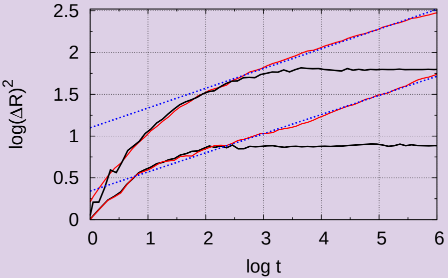
$\alpha = 2/5$ (theory) 0.34, 0.31 numerics



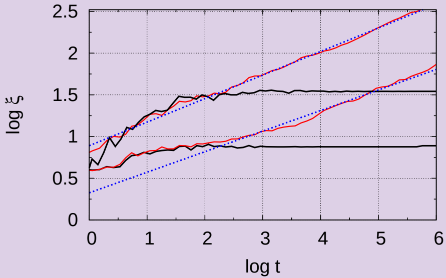
$W/V = 4$, $\beta = 1$, $t = 10^8$, $\beta = 0$

$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\psi_n|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1})\,;\,[-W/2 < E_n < W/2]$$

A.S.Pikovsky, DLS PRL **100**, 094101 (2008)



$W/V = 10, 15$, $\beta = 0, 1$; $\alpha_2 = 0.236, 0.229 \pm 0.003$ (theory 0.25)

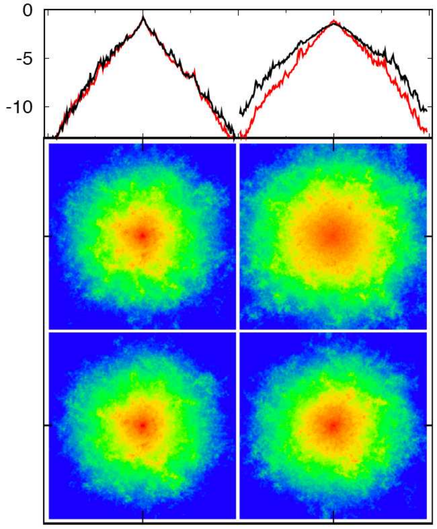


$\nu = 0.282, 0.247 \pm 0.005$ (theory 0.25); ξ is participation ratio

$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\psi_n|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1})$$

I.García-Mata, DLS arXiv:0805.0539 (2008)

Nonlinearity and Anderson localization (2D)



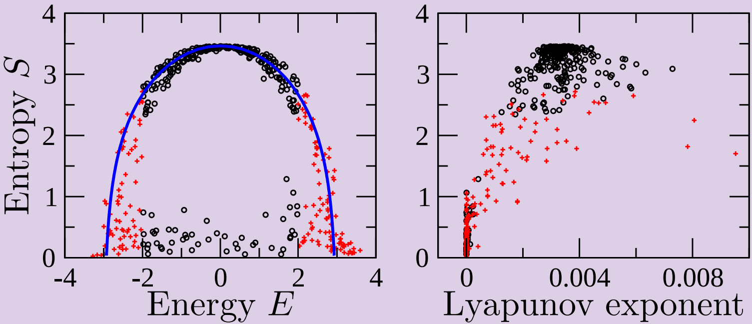
$W = 10; \beta = 0$(left), 1(right);
$t = 10^4$ (bottom), 10^6 (middle),
projecton on $x-$axis (top);
256×256 lattice

[also: kicked nonlinear rotator model (1d)]

I.García-Mata, DLS arXiv:0805.0539 (2008)

Dynamical thermalization in DANSE (1D)

starting from Fermi-Pasta-Ulam problem (1955):
regular lattice, delocalized linear modes $\rightarrow$ disorder localized modes



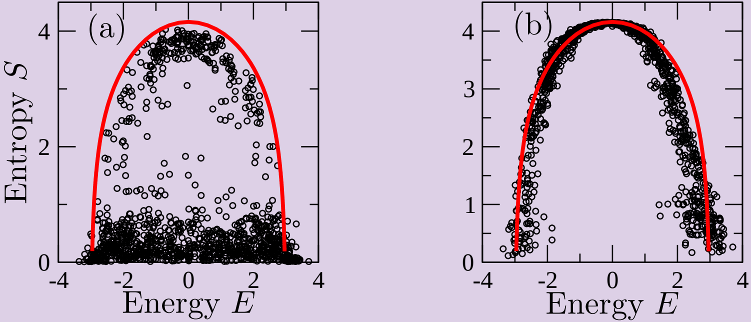
$N = 32$, $W = 4$, $\beta = 1$, $t = 10^7 + 10^6$, initial state: one linear eigenmode

Gibbs distribution with temperature T for localized linear modes, $\rho_m = |C_m|^2$:
entropy $S = -\sum_m \rho_m \ln \rho_m$, $\rho_m = Z^{-1} \exp(-\epsilon_m/T)$, $Z = \sum_m \exp(-\epsilon_m/T)$,
$E = T^2 \partial \ln Z / \partial T$, $S = E/T + \ln Z$. $\langle \ln Z \rangle \approx \ln N + \ln \sinh(\Delta/T) - \ln(\Delta/T)$, $\Delta \approx 3$

M.Mulansky, K.Ahnert, A.Pikovsky, DLS arxiv:0903.2191 (2009)

Dynamical thermalization in DANSE (1D)

weaker and stronger nonlinearity β

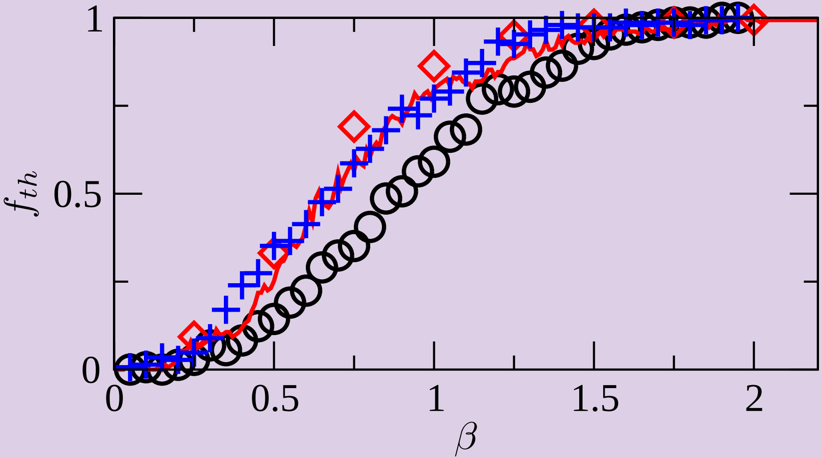


$N = 64$, $W = 4$, $\beta = 0.5(left), 2(right)$, $t = 10^6$, initial state: one linear eigenmode

Gibbs distribution with temperature T for localized linear modes, $\rho_m = |C_m|^2$:
entropy $S = -\sum_m \rho_m \ln \rho_m$, $\rho_m = Z^{-1} \exp(-\epsilon_m/T)$, $Z = \sum_m \exp(-\epsilon_m/T)$,
$E = T^2 \partial \ln Z / \partial T$, $S = E/T + \ln Z$. $\langle \ln Z \rangle \approx \ln N + \ln \sinh(\Delta/T) - \ln(\Delta/T)$, $\Delta \approx 3$

M.Mulansky, K.Ahnert, A.Pikovsky, DLS arxiv:0903.2191 (2009)

Dynamical thermalization in DANSE (1D)



Fraction of thermalized states: $N = 16$ (circles), 32 (curve), 64(+) ; $W = 4, t = 10^6$, (diamonds $N = 32, t = 10^7$)

M.Mulansky, K.Ahnert, A.Pikovsky, DLS arxiv:0903.2191 (2009)

Possible experimental tests & applications

- BEC in disordered potential (Aspect, Inguscio)
- kicked rotator with BEC (Phillips)
- nonlinear wave propagation in disordered media (Segev, Silberberg)
- lasing in random media (Cao)
- energy propagation in complex molecular chains (proteins, Fermi-Pasta-Ulam problem)
- NONLINEAR SPIN-GLASS ?

- OTHER GROUPS:
  S.Aubry *et al.* PRL **100**, 084103 (2008)
  A.Dhar *et al.* PRL **100**, 134301 (2008)
  S.Fishman *et al.* J. Stat. Phys. bf 131, 843 (2008)
  S.Flach *et al.* arXiv:0805.4693[cond-mat] (2008)
  W.-M.Wang *et al.* arXiv:0805.4632[math.DS] (2008)
  see also the participant list of the NLSE Workshop
  at the Lewiner Institute, Technion, June 2008

Quntum systems:
Two Interacting Particles (TIP) effect

Anderson model in d-space + onsite Hubbard interaction U, $V \sim E_F$ is one-particle hopping; exited states $\psi_n \sim \exp(-|n-m|/l)/\sqrt{l}$; $l \gg 1$.
Equation in the basis of noninteracting eigenstates $\chi_{m_1 m_2}$:

$$i\partial\chi_{m_1m_2}/\partial t = \epsilon_{m_1m_2}\chi_{m_1m_2} + \sum_{m'_1 m'_2} U_{m_1m_2m'_1m'_2}\chi_{m'_1m'_2}$$

Sum runs over $M \sim l^d$ coupled states; interaction induced matrix elements $U_s \sim U_{m_1m_2m'_1m'_2} \sim (U/(l^{2d}) \times \sqrt{M}$, density of coupled states is $\rho_2 \sim l^{2d}/V$, TIP transition rate $\Gamma_s \sim {U_s}^2\rho_2 \sim U^2/(l^d V)$. Enhancement factor

$$\kappa = \Gamma_s\rho_2 \sim (U/V)^2 l^d > 1$$

TIP localization:
$l_2/l \sim (U/V)^2 l$ (1d);
$\ln(l_2/l) \sim (U/V)^2 l^2$ (2d);
delocalization for $\kappa \sim (U/V)^2 l^3 > 1$ (3d)

DLS PRL **73**, 2607 (1994); Y.Imry EPL **30**, 405 (1995)

Slow Metal (2D)

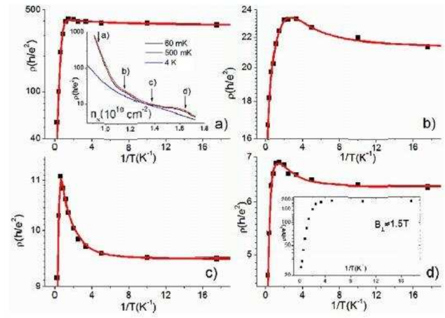


FIG. 2 (color online). Resistivity as a function of inverse temperature $1/T$ at $B = 0$ T (symbols). At all densities, the strongly insulating T dependence at higher temperatures is followed by a decrease in resistance at low T. Device dimensions are $W \times L = 8\ \mu\text{m} \times 0.5\ \mu\text{m}$, spacer $\delta = 40$ nm. Electron densities are indicated by arrows in the inset to (a). Solid lines represent a fit of Eq. (1) to the data. Inset to (a): Resistivity as a function of electron density at $T = 60$ mK, 500 mK, 4 K. Inset to (d): ρ as function of $1/T$ at the same density as (d) but at $B_\perp = 1.5$ T.

TIP diffusion
$D \sim \Gamma_s l^2 \sim U^2/V$ at $(Ul/V)^2 > 1$
vs. usual diffusion $D_0 \sim v_F \ell \sim V$
Thus it is possible to have diffusion with conductance g and resistivity per square ρ_0 (in natural units):
$g \sim 1/\rho_0 \sim D/D_0 \sim (U/V)^2 \ll 1$
With up to $(Ul/V)^2 \sim 1$ and
$g \sim 1/l^2 \ll 1$
Problems: finite particle density, small density of states near the ground state
Experiment suggestion: to measure a charge of quasi-particles from noise fluctuations

M.Baenninger, A.Ghosh, M.Pepper, H.E.Beere, I.Farrer, D.A.Ritchie
PRL **100**, 016805 (2008) vs. S.Kravchenko *et al.* RMP **73**,251 (2001)

Dyn-thermalization in many-body Q-systems

Åberg criterion $J > J_c \approx \Delta_c \gg \Delta_n \propto \exp(-n)$: two-body matrix element J should be larger than energy spacing between directly coupled states Δ_c

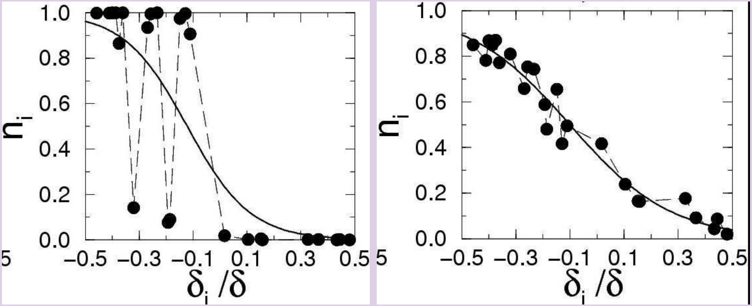
EXAMPLES

- Weakly interacting fermions:
  one-particle level spacing Δ,
  two-body interaction matrix element $J = U \sim \Delta/g$ with $g \gg 1$
  (two-body random interaction model - TBRIM).
  Quantum chaos border for dynamical thermalization:
  $\delta E \sim Tn_{eff} \sim T^2/\Delta > g^{2/3}\Delta \gg \Delta \gg \Delta_n \sim \Delta/\exp(n_{eff})$
- Quantum computer with static imperfections:
  QC Hamiltonian:
  $H = \sum_i \Gamma_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x$;
  $\Gamma_i = \Delta_0 + \delta_i$, $-\delta < 2\delta_i < \delta$, middle band with $S_z = \sum_{n_q} \sigma_z = 0$ $-J < J_{ij} < J$;
  Quantum chaos border for dynamical thermalization:
  $\rightarrow J_c \approx 4\delta/n_q \gg \Delta_n \sim \delta/2^{n_q}$
- Above the border: Gibbs description of quantum ergodicity;
  Wigner-Dyson level spacing statistics

S.Åberg PRL **64**, 3119 (1990); DLS, O.Sushkov EPL **37**, 121 (1997);
P.Jacquod, DLS PRL **79**, 1837 (1997); B.Georgeot, DLS PRE **62**, 3504 (2000)

Dyn-thermalization in many-body Q-systems

Example of dynamically thermalized eigenstate of quantum computer



Example: Quantum computer with $n_q = 24$ qubits. One quantum eigenstate: Occupation numbers n_i vs. rescaled exitation energies $\epsilon_i = \delta_i$. Left: $J/J_c \approx 0.3$, $T = 0.12\delta$, $\delta E = 0.64\delta$, $S = 1.84$. Right: $J/J_c \approx 2.4$, $T = 0.19\delta$, $\delta E = 1.16\delta$, $S = 12.5$. Full curves: Fermi-Dirac thermal distribution with given temperature T. G.Benenti *et al.* EPJD **17**, 265 (2001)

QC Hamiltonian: $H = \sum_i \Gamma_i \sigma_i^z + \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x$;
$\Gamma_i = \Delta_0 + \delta_i$, $-\delta < 2\delta_i < \delta$, $-J < J_{ij} < J$; $\rightarrow J_c \approx 4\delta/n_q$

B.Georgeot, DLS PRE **62**, 3504 (2000), DLS Physica Scripta **T90**, 112 (2001)