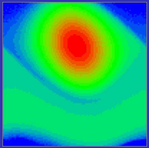
Microwave stabilization of edge transport and zero-resistance states arxiv:0905.0593



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www.lps.u-psud.fr/Collectif/gr_07/
www.quantware.ups-tlse.fr/dima

Discussions: H. Bouchiat (Orsay)
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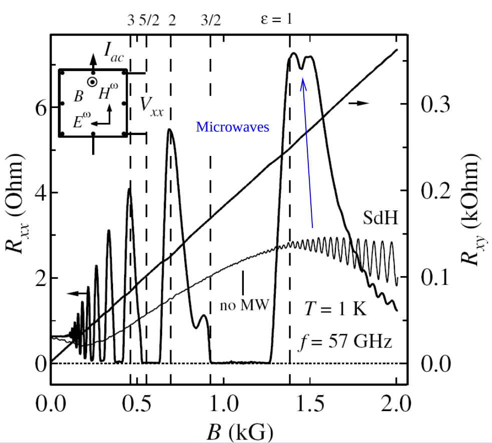
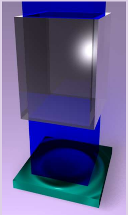
A. A. Bykov (Novosibirsk)
A. S. Pikovsky (Potsdam)

Zero resistance states discovery in 2002



- Under microwave irradiation 4-terminal R_{xx} vanishes
- High mobility two dimensional electron gas $\ell \approx 140\mu m$
- Temperature of about 1K



Waveguide

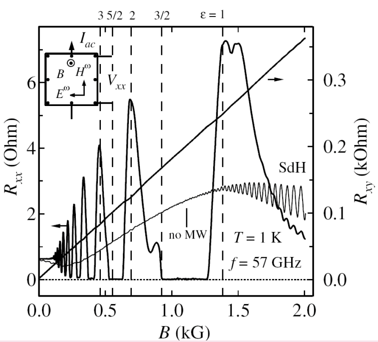
Microwave field
f ~ 50 GHz

Measure sample
DC resistance

- R.G. Mani, J.H. Smet, K. von Klitzing, V. Narayanamurti, W.B. Johnson and V. Umansky, Nature **420**, 646 (2002).
- M.A.Zudov, R.R.Du, L. N. Pfeiffer and K. W. West PRL **90**, 046807 (2003).

Main experimental features

- Zero resistance states have a $1/B$ periodic structure



- main control parameter is

$$j = \frac{\omega}{\omega_c} = \frac{\omega}{eB/m}$$

- R_{xx} has a peak if $j = n$ n integer
- R_{xx} is zero if $j = n + \delta$ $\delta \simeq 1/2$
- high harmonics up to $n \simeq 10$
- Arrhenius law dependence on temperature with activation energy $\simeq 20$ K

- Length scales at equilibrium

λ_{Fermi}	$<$	λ_T (at 1 K)	$\ll$	r_c	$\ll$	Mean free path
$\simeq 50$ nm		$\frac{\hbar}{\sqrt{mT}} \simeq 100$ nm		$\frac{v_F}{\omega_c} \simeq 1\mu$m		$\ell \simeq 100\ \mu$m

Existing theories

Many theoretical proposals

- V.I. Ryzhii, Sov. Phys. Solid State **11**, 2078 (1970)
- A.C. Durst, S. Sachdev, N. Read, and S. M. Girvin, Phys. Rev. Lett. **91**, 086803 (2003)
- M.G. Vavilov and I.L. Aleiner, Phys. Rev. B **69**, 035303 (2004)
- J. Iñarrea and G. Platero, Phys. Rev. Lett. **94**, 016806 (2005)
- I.A. Dmitriev, A.D. Mirlin and D.G. Polyakov, Phys. Rev. Lett. **91**, 226802 (2003)
- P.W. Anderson and W.F. Brinkman, cond-mat/0302129
- I. A. Dmitriev, M. G. Vavilov, I. L. Aleiner, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. B **71**, 115316 (2005).

all related to bulk mechanisms

Existing theories rely on a “switching” mechanism

A.C. Durst, S. Sachdev, N. Read, and S. M. Girvin (PRL 2003)

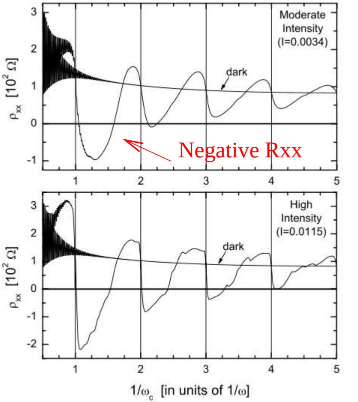


FIG. 3. Calculated radiation-induced resistivity oscillations. We plot ρ_{xx} vs $1/\omega_c$ at fixed ω for $\mu = 50\omega$, $k_BT = \omega/4$, $\gamma = 0.08\omega$, and three values of radiation intensity (in units of $m^*\omega^3$): $I = 0$ (dark), $I = 0.0034$ (upper panel), and $I = 0.0115$ (lower panel). For computational purposes, the energy spectrum is cutoff at 20 Landau levels above and below the chemical potential. The high-frequency oscillations seen at small $1/\omega_c$ are the familiar SdH oscillations with period $1/\mu$.

A.V. Andreev, I. L. Aleiner and A.J. Millis (PRL 2003)

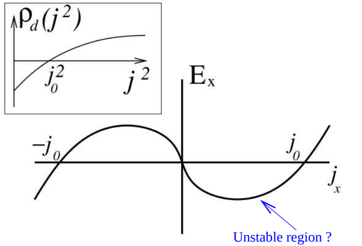


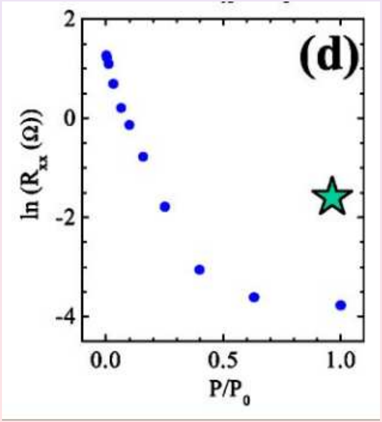
FIG. 1. Assumed dependence of the dissipative (parallel to current) component of the local electric field E_x on the current density j_x. Inset: dependence of the dissipative resistivity on the square of the current.

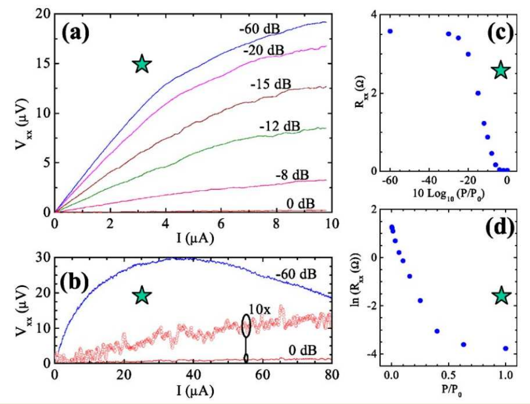
Origin of high harmonics ? Short range scattering on weak dissorder (but not too weak !)

Smooth dependence in experiments !

R. G. Mani, V. Narayanamurti, K. von Klitzing, J. H. Smet, W. B. Johnson and V. Umansky

Phys. Rev. B 70, 155310 (2004)





Experimental features $\rightarrow$ theory arguments

EXPERIMENT

- **weak field:** $\epsilon \propto \Delta v_{osc}/v_F \sim 0.01 - 0.05 \ \ (\sim 1V/cm)$
- **high Landau levels (high filling factors)** $\nu \approx 60$
- **high harmonics** $j = \omega_c/\omega \geq 1$**: no such transitions in oscillator**
- **high mobility, mean free path** $l_e \approx 140 \mu m$**, small angle scattering**
  $\rightarrow$ **smooth potential,** $\omega_c \gg \omega_{pot}$
  $\rightarrow$ **due to adiabatic theorem transport is very weak in the bulk**

CONJECTURE

- **main contribution to transport comes from ballistic transmission along edges**
- **ZRS = microwave stabilization of edge transport**

LINKS

- **edge transport in quantum Hall effect**
  B.I.Halperin PRB 25, 2185 (1982) , M.Büttiker PRB 38, 9375 (1988)

Transport/Elastic life time in 2DEG

- Transport time $\tau \simeq \ell / v_F \simeq 0.5$ nano seconds

  Determined by the mobility at zero magnetic field.

- Elastic life time $\tau_q \simeq 0.01$ nano seconds

  Determined by the decay of the Shubnikov-de Haas oscillations as a function of $1/B$

  $\longrightarrow$ Life time of a plane-wave like state

- $$\tau^{-1} = \int W(\theta)(1 - \cos(\theta))d\theta \text{ and } \tau_q^{-1} = \int W(\theta)d\theta$$

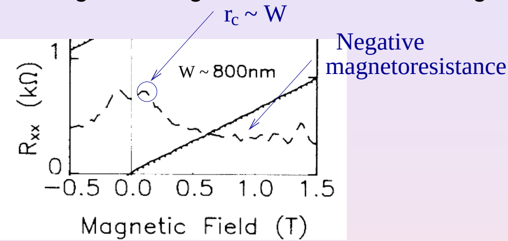
  $W(\theta)$ is the rate of scattering at angle θ

- $\tau \simeq 50\tau_q \gg \tau_q$ implies

  Scattering occurs at small angle θ

Negative magnetoresistance in 2DEG

- Negative magnetoresistance and magneto-size effects in ballistic junctions



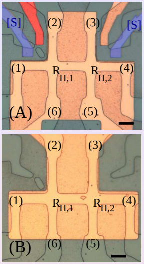
M. L. Roukes et. al. PRL 1987

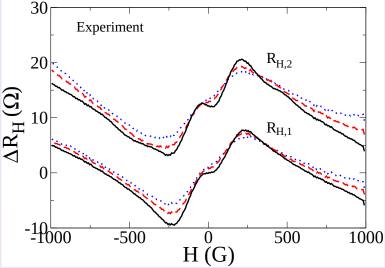
- Theory $\rightarrow$ Billiard model proposed by C.W.J Beenakker, H. van Houten, PRL 1989

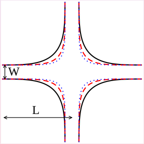
- Negative magnetoresistance due to guiding along sample edges !

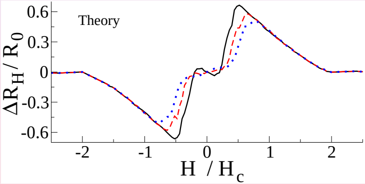
- R_{xx} does not drop to zero because guiding is not perfect.

Billiard model for a Hall probe

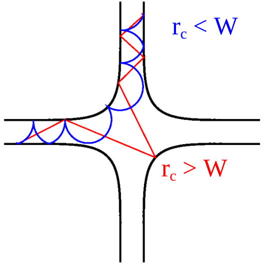








$$\Delta R_H = R_H - \frac{H}{ne}$$

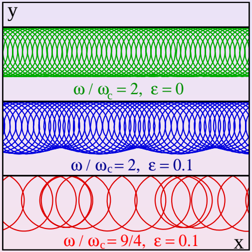


Recent experiments in Groupe Meso : PRL **102**, 086810 (2009) →

The billiard model gives reliable predictions for equilibrium/non equilibrium transport

Classical theory of edge transport under irradiation

Newton equations of motion (model 1, specular wall I_{wall}, small angle scattering I_S)



$$d\mathbf{v}/dt = \omega_{\mathbf{c}} \times \mathbf{v} + \epsilon\omega \cos \omega t - \gamma(v)\mathbf{v} + I_{wall} + I_S \quad (1)$$

$\epsilon = e\mathbf{E}/(m\omega v_F)$ describes microwave driving field $\mathbf{E}$,

velocity $\mathbf{v}$ is measured in units of Fermi velocity v_F,

$\gamma(v) = \gamma_0(|\mathbf{v}|^2 - 1)$ describes relaxation processes to the Fermi surface

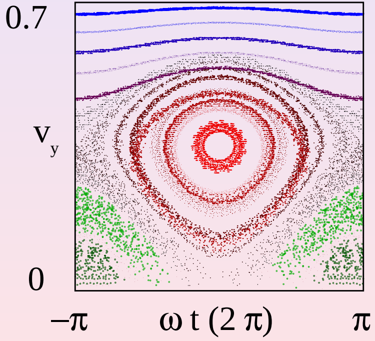
random angle scattering on microwave period with amplitude $\pm\alpha$

Direct real space analysis of trajectories is complicated,
Construct Poincaré section !

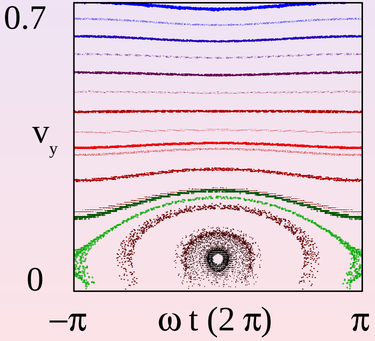
Poincaré section (Newton equations)

Abscissa : phase of the microwave field $\omega t (\mathrm{mod}\ 2\pi)$ at the moment of collision with the wall
Ordinate : v_y velocity at the moment of collision (divided by Fermi velocity)
(Electric field $\epsilon = (0, 0.02)$, no noise and no dissipation)

$j = \omega/\omega_c = 9/4$ (model 1)

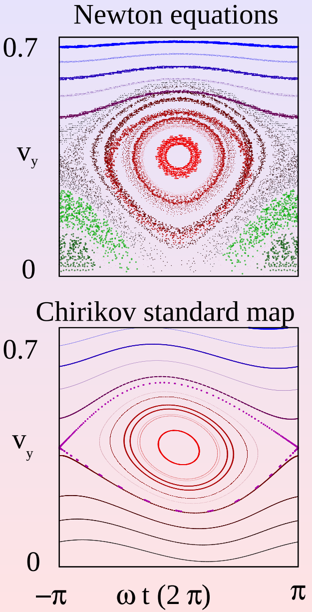


$j = \omega/\omega_c = 2.05$ (model 1)



Appearance of a nonlinear resonance

Chirikov standard map



Approximate description of the nonlinear resonance

velocity change at wall collision:
double wall velocity

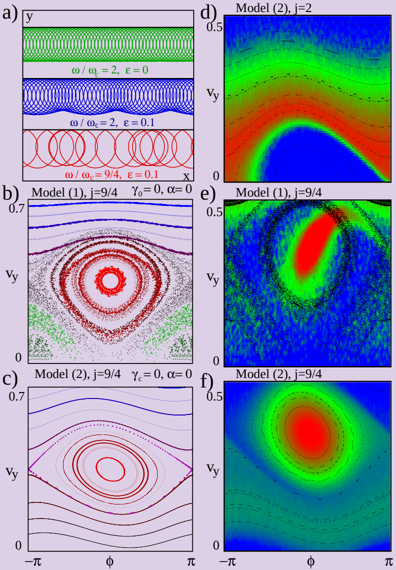
small angles near wall: time between collisions $\Delta t = 2(\pi - v_y)/\omega_c$

this leads to the Chirikov standard map :

$$\begin{cases} v_y(n+1) = v_y(n) + 2\epsilon \sin\phi(n) + I_{cc} \\ \phi(n+1) = \phi(n) + 2(\pi - v_y(n+1))\omega_{/}\omega_c \end{cases}$$

model 2, I_{cc} describes noise and dissipation

Phase space portrait, with noise and dissipation



Left Column: Dynamics without dissipation

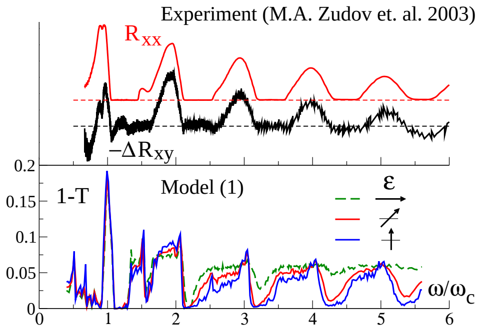
Right Column: Color scale show the density of propagating particles on the Poincaré section in presence of noise and dissipation (red → maximum) Black points show trajectories without noise and dissipation.

$\omega/\omega_c = 2$ microwave repels particles from the edge (d)

$\omega/\omega_c = 9/4$ particles are trapped in the resonance (e,f)

Here $\gamma_0 = 10^{-3}$ (e), $\gamma_c = 10^{-2}$ (d,f) and $\alpha \simeq 5 \times 10^{-3}$.

Stabilization of edge transport (1/B dependence)



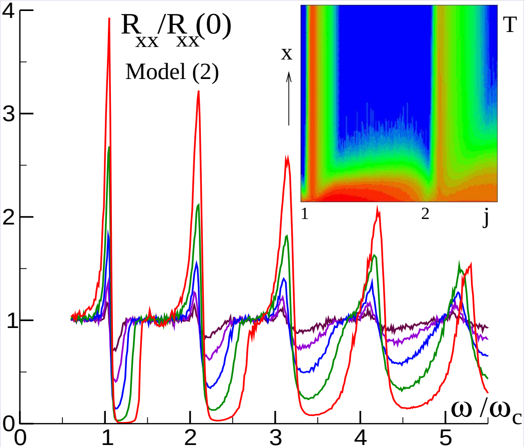
Top: R_{xx} and $-\Delta R_{xy} = \frac{H}{ne} - R_H$ as a function of ω/ω_c (experiment)

Bottom: Transmission along sample edge as a function of ω/ω_c

(model 1)

For $l_e \gg r_c$ the billiard model of a Hall bar gives $R_{xx} \propto -\Delta R_{xy} \propto 1 - T$
Microwave field is $\epsilon = 0.05$, relaxation $\gamma_0 = 10^{-3}$ and noise amplitude $\alpha = 3 \times 10^{-3}$.
Transmission without microwaves is $T \simeq 0.95$, $N = 5000$ orbits.

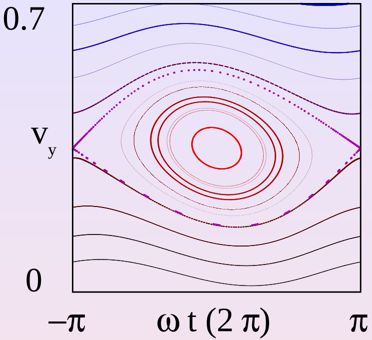
Stabilization of edge transport (Dependence on microwave field)



Growth of ZRS peaks and dips (model 2) as a function of microwave field amplitude $\epsilon = 0.00375, 0.0075, 0.015, 0.03, 0.06$.

Insert shows transmission probability T at distance x along the edge for $\epsilon = 0.02$ ($0 < x < 10^3 v_F/\omega$).

Position and width of the resonance



$$\begin{cases} v_y(n+1) = v_y(n) + 2\epsilon \sin\phi(n) \\ \phi(n+1) = \phi(n) + 2(\pi - v_y(n+1))\omega/\omega_c \end{cases}$$

A phase shift by $\phi \to \phi + 2\pi$ does not change the behavior of map. Hence the phase space structure is periodic in $j = \omega/\omega_c$ with period unity which naturally yields high harmonics. The resonance is centered at $v_y = \pi(1 - m\omega_c/\omega)$ where m is the integer part of ω/ω_c.

The chaos parameter of the map is $K = 4\epsilon\omega/\omega_c$ and the resonance separatrix width $\delta v_y = 4\sqrt{\epsilon\omega_c/\omega}$.

Activation energy and escape rates

Typical spread square width in velocity angle during the relaxation time $1/\gamma_c$ is $D_s = \alpha^2/\gamma_c$. The resonance square width is $(\delta v_y)^2 = 16\epsilon\omega_c/\omega$ and escape probability from the resonance is

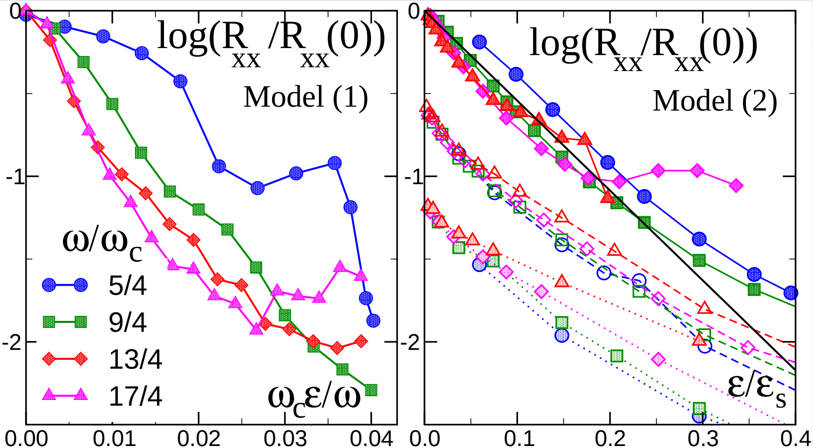
$$W \sim \exp(-(\delta v_y)^2/D_s) \sim \exp(-A\epsilon\omega_c/(D_s\omega))$$

with $R_{xx}/R_{xx}(0) \sim 1 - T \sim W; \quad \epsilon_s = \omega D_s/\omega_c; \quad A \approx 16.$

Arrhenius law with activation energy equal to the energy height of the nonlinear resonance $E_r = 16\epsilon\omega_c E_F/\omega$ where E_F is the Fermi energy. This dependence appears as an additional damping factor in ZRS amplitude:

$$R_{xx} \propto \exp(-A\epsilon\omega_c/(D_s\omega)) \exp(-16\epsilon\omega_c E_F/\omega T_e)$$

ZRS parameter dependence



Dependence of rescaled R_{xx} on rescaled microwave field ϵ for models (1) (left) and (2) (right). Left: parameters as in Fig. 2 and ϵ is varied. Right: $\gamma = 0.01$, $\alpha = 0.02$ (full), $\gamma = 0.01$, $\epsilon = 0.03$ (dashed), $\epsilon = 0.03$, $\alpha = 0.02$ (dotted), the straight line shows theory with $A = 12.5$; symbols are shifted for clarity and $\epsilon_s = \omega D_s/\omega_c$.

Experimental dependence of ZRM minima on T and $j = \omega/\omega_c$

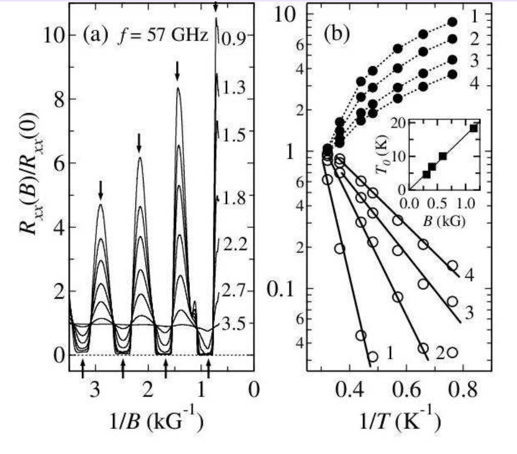


FIG. 3. (a) $R_{xx}(B)/R_{xx}(0)$ under MW ($f = 57$ GHz) illumination, plotted vs $1/B$ at different T from 0.9 to 3.5 K. Upward

Experiment $\rightarrow$ Activation energy

$$E_r \propto \frac{\omega_c}{\omega}$$

Theory predicts

$$E_r = 16\epsilon E_F \frac{\omega_c}{\omega}$$

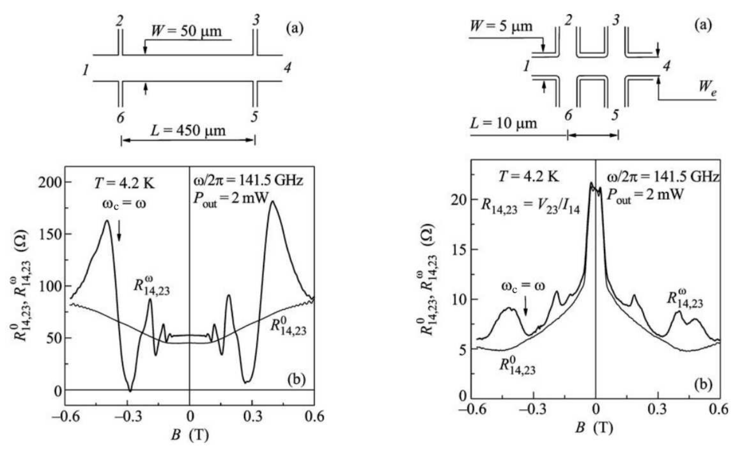
For $\epsilon = 0.01$ we obtain

$$E_r \simeq 20 \text{ K}$$

Experiments in micrometric Hall bars

Dips → Long time scale : Trapping in the resonance

Peaks → Short time scale when resonance ejects particles



A. Bykov, JETP Letters **89**, 575 (2009)

Conclusions

- Microwaves can stabilize edge trajectories against small angle disorder scattering
- Non linear resonance described by the Chirikov standard map $\longrightarrow$ high j resonances
- Importance of relaxation processes to the Fermi surface
- Non linear resonance height $\longrightarrow$ activation energy
- Microscopic theory for relaxation to the Fermi surface ?
- Extension to describe other low magnetic field resistance oscillations (ZDRS, PIRO, ...) ?