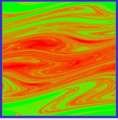
Turbulent flows on complex networks



Dima Shepelyansky (CNRS, Toulouse)
www.quantware.ups-tlse.fr/dima

Collaboration: A.D.Chepelianskii, L.Ermann, B.Georgeot, O.Giraud (CNRS, France)
A.O.Zhirov, O.V.Zhirov (NSU - BINP, Novosibirsk)

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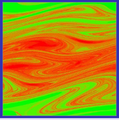

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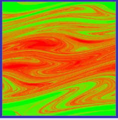
"Through mechanisms still only partially understood, wind transfers energy and momentum to surface water waves." A.C.Newell and V.E.Zakharov (PRL 1992)

Turbulent flows on complex networks



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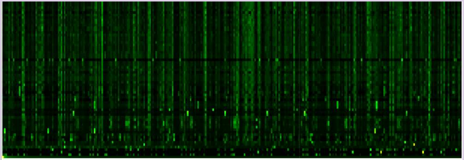
"Through mechanisms still only partially understood, wind transfers energy and momentum to surface water waves." A.C.Newell and V.E.Zakharov (PRL 1992)

Through mechanisms still only partially understood,
Google finds instantaneously the page you need.

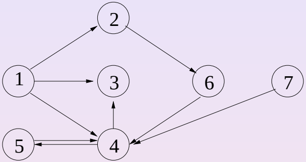


Story started in 1998 (now $N \sim 10^{11}$ nodes)

S. Brin and L. Page, Computer Networks and ISDN Systems **30**,107 (1998).

Directed networks

Weighted adjacency matrix



$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

For a directed network with N nodes the adjacency matrix $\mathbf{A}$ is defined as $A_{ij} = 1$ if there is a link from node j to node i and $A_{ij} = 0$ otherwise. The weighted adjacency matrix is

$$S_{ij} = A_{ij} / \sum_k A_{kj}$$

In addition the elements of columns with only zeros elements are replaced by $1/N$.

How Google works

Google Matrix and Computation of PageRank

$\mathbf{p} = \mathbf{S}\mathbf{p} \Rightarrow$ **p**= stationary vector of **S**; can be computed by iteration of **S**.

To remove convergence problems:

- Replace columns of 0 (dangling nodes) by $\frac{1}{N}$:

  In our example, $\mathbf{S} = \begin{pmatrix} 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & 0 & 1 & 1 & 1 \\ 0 & 0 & \frac{1}{7} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \end{pmatrix}$.

- To remove degeneracies of the eigenvalue 1, replace **S** by **Google matrix**

  $\mathbf{G} = \alpha \mathbf{S} + (1-\alpha)\frac{\mathbf{E}}{N}$; $Gp = \lambda p$ => Perron-Frobenius operator

- α models a random surfer with a random jump after approximately 6 clicks (usually $\alpha = 0.85$); PageRank vector => p at $\lambda = 1$ ($\sum_j p_j = 1$).

Models of real networks

Real networks are characterized by:

- **small world property**: average distance between 2 nodes $\sim \log N$
- **scale-free property**: distribution of the number of incoming or outcoming links $P(k) \sim k^{-\nu}$ $(\nu \sim 2.1(in); 2.7(out))$

Can be explained by a twofold mechanism:

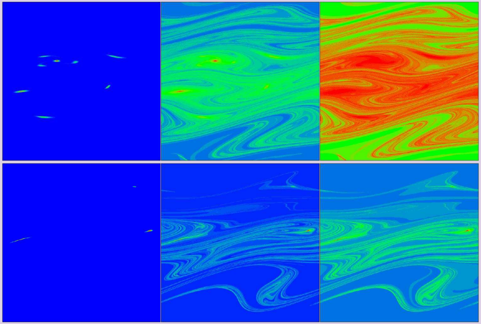
- Constant growth: new nodes appear regularly and are attached to the network
- Preferential attachment: nodes are preferentially linked to already highly connected vertices.

PageRank vector for large WWW:

- $p_j \sim 1/j^{\beta}$, where j is the ordered index
- number of nodes N_n with PageRank p scales as $N_n \sim 1/p^{\nu}$ with numerical values $\nu = 1 + 1/\beta \approx 2.1$ and $\beta \approx 0.9$.

Google matrix of dynamical attractors

Ulam networks $\rightarrow$ no gap, sensitivity to α (Ulam method (1960))



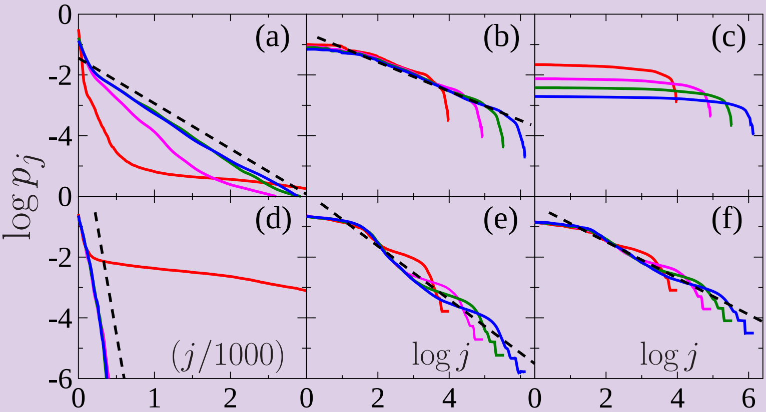
PageRank p_j for the Google matrix generated by the Chirikov typical map at $T = 10$, $k = 0.22$, $\eta = 0.99$ (set $T10$, top row) and $T = 20$, $k = 0.3$, $\eta = 0.97$ (set $T20$, bottom row) with $\alpha = 1, 0.95, 0.85$ (left to right). The phase space region $0 \leq x < 2\pi; -\pi \leq y < \pi$ is divided on $N = 3.6 \cdot 10^5$ cells.

Chirikov typical map (1969) with dissipation

$\bar{y} = \eta y + k \sin(x + \theta_t)$, $\bar{x} = x + \bar{p}$

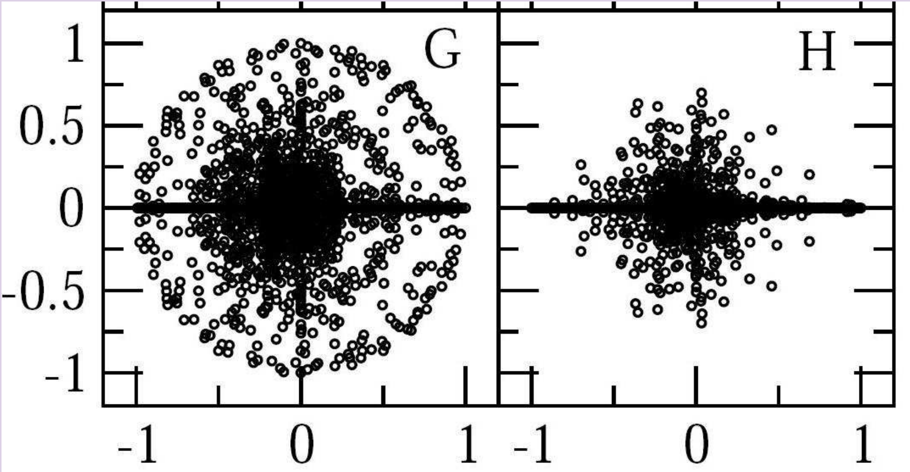
$\theta_t = \theta_{t+T}$ are random phases periodically repeated after T iterations,
chaos border $k_c \approx 2.5/T^{3/2}$, Kolmogorov-Sinai entropy $h \approx 0.29k^{2/3}$;
grid of $N = N_x \times N_p$ cells with $N_c \sim 10^4$ trajectories which generates links (transition probabilities) from one cell to another; effective noise of cell size;
maximum $N = 22500; 1.44 \cdot 10^6$ [DS, Zhirov PRE 81, 036213 (2010)]

PageRank distribution



Differential distribution of number of nodes with PageRank distribution p_j for $N = 10^4$, $9 \cdot 10^4$, $3.6 \cdot 10^5$ and $1.44 \cdot 10^6$ curves, the dashed straight lines show fits $p_j \sim 1/j^{\beta}$ with β: 0.48 (b), 0.88 (e), 0.60 (f). Dashed lines in panels (a),(d) show an exponential Boltzmann decay (see text, lines are shifted in j for clarity). In panels (a),(d) the curves at large N become superimposed. Panel order as in color Fig. above.

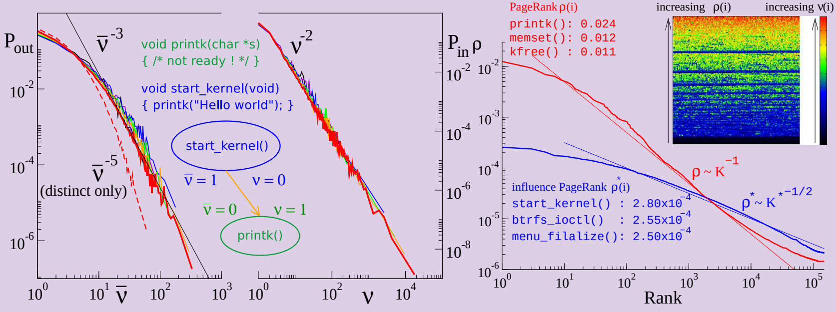
Absence of spectral gap in real WWW



Spectrum of Google matrix for two British universities at $\alpha = 1$, $N \approx 13000$.
[Georgeot, Giraud, DS PRE 81, 056109 (2010)]

Linux Kernel Network

Procedure call network for Linux

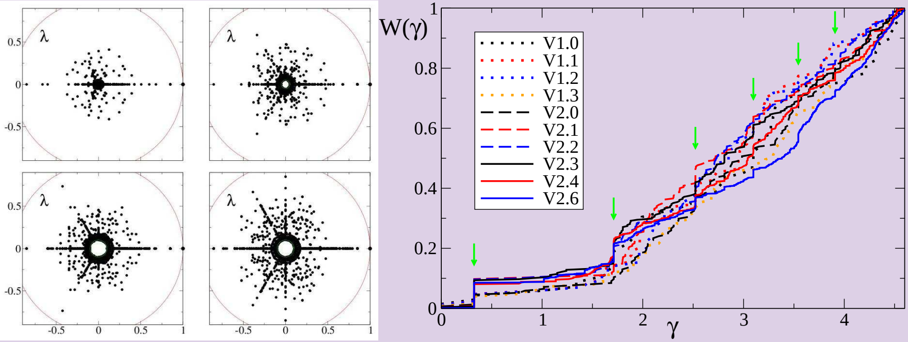


Links distribution (left); PageRank and inverse PageRank (CheiRank) distribution (right) for Linux versions up to 2.6.32 with $N = 285509$ ($\rho \sim 1/j^{\beta}, \beta = 1/(\nu - 1)$).

[Chepelianskii arxiv:1003.5455]

Fractal Weyl law for Linux Network

Sjöstrand Duke Math J. 60, 1 (1990), Zworski *et al.* PRL 91, 154101 (2003) → quantum chaotic scattering;
Ermann, DS EPJB 75, 299 (2010)→ Perron-Frobenius operators

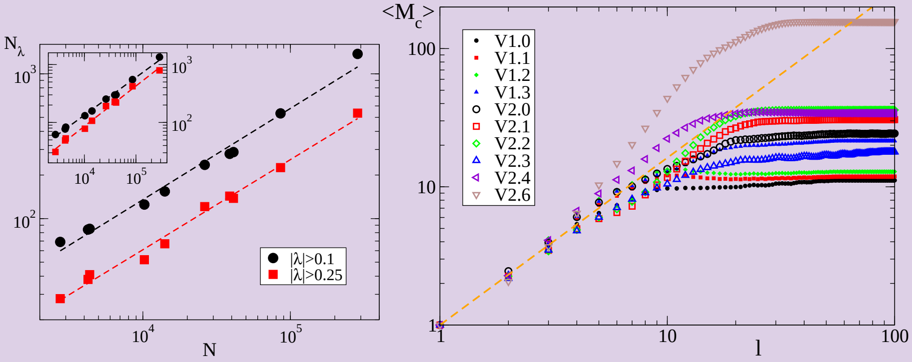


Spectrum of Google matrix (left); integrated density of states for relaxation rate $\gamma = -2\ln|\lambda|$ (right) for Linux versions, $\alpha = 0.85$.

[Ermann, Chepelianskii, DS arxiv:1005.1395]

Fractal Weyl law for Linux Network

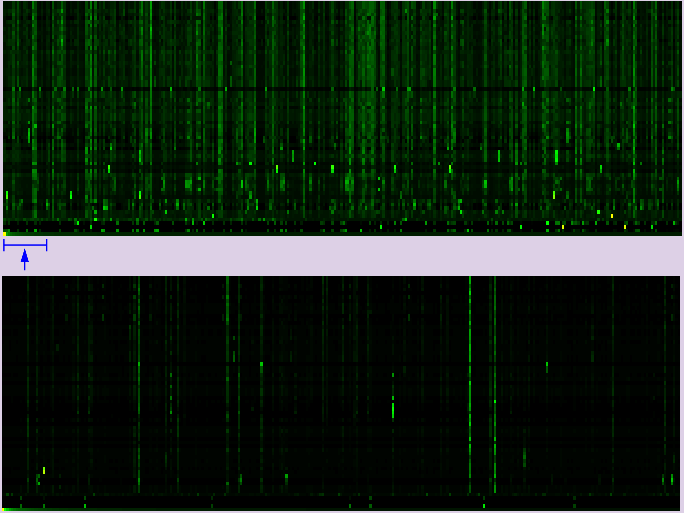
Number of states $N_\lambda \sim N^\nu, \ \ \nu = d/2 \ \ \ (N \sim 1/\hbar^{d/2})$



Number of states N_λ with $|\lambda| > 0.1; 0.25$ vs. N, lines show $N_\lambda \sim N^\nu$ with $\nu \approx 0.65$ (left); average mass $< M_c >$ (number of nodes) as a functon of network distance l, line shows the power law for fractal dimension $< M_c > \sim l^d$ with $d \approx 1.21$ (right).

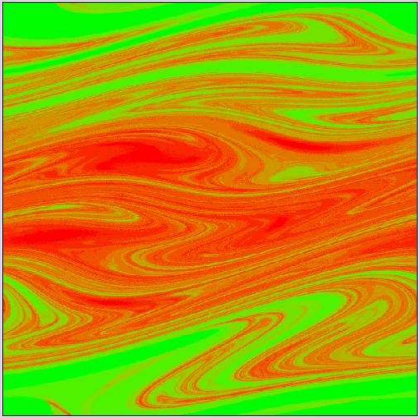
[Ermann, Chepelianskii, DS arxiv:1005.1395]

Fractal Weyl law for Linux Network



Coarse-grained probability distribution $|\psi_i(j)|^2$ for the eigenstates of the Google matrix of Linux Kernel version 2.6.32.

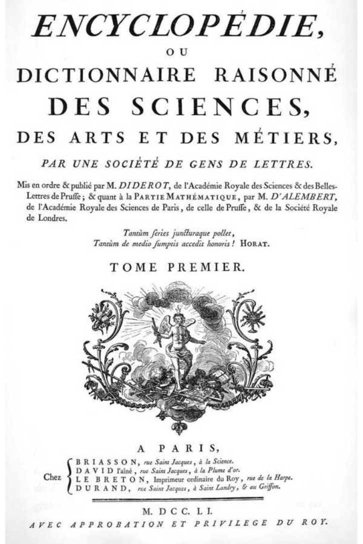
Google Matrix of Turbulence



PROPOSAL:

- Ulam method applied to the Navier-Stokes equation (J.Bec talk)
- cells in phase-space
- Google matrix **G** of transition probabilities
- PageRank and spectral properties of **G**

From Encyclopédie (1751) to Wikipedia (2009)



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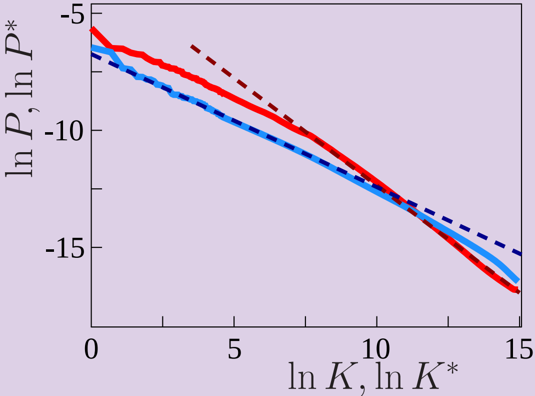
"The library exists *ab aeterno.*"
Jorge Luis Borges *The Library of Babel, Ficciones*

Two-dimensional ranking of Wikipedia articles

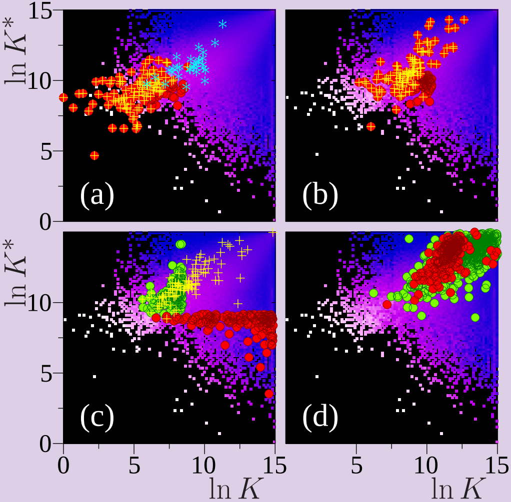
Wikipedia English articles $N = 3282257$ dated Aug 18, 2009



Dependence of probability of PagRank P (red) and CheiRank P^* (blue) on corresponding rank indexes K, K^*; lines show slopes $\beta = 1/(\nu - 1)$ with $\beta = 0.92; 0.57$ respectively for $\nu = 2.09; 2.76$.

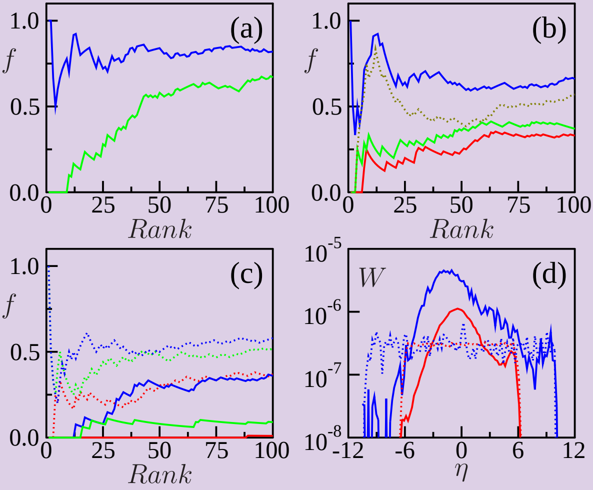
[Zhirov, Zhirov, DS arxiv:1006.4270]

Two-dimensional ranking of Wikipedia articles



Density distribution in plane of PageRank and CheiRank indexes ($\ln K$, $\ln K^*$): (a)100 top countries from 2DRank (red), 100 top from SJR (yellow), 30 Dow-Jones companies (cyan); (b)100 top universities from 2DRank (red) and Shanghai (yellow); (c)100 top personalities from PageRank (green), CheiRank (red) and Hart book (yellow); (d)758 physicists (green) and 193 Nobel laureates (red).

Two-dimensional ranking of Wikipedia articles



Overlap fraction f with (a)SJR countries ranking, (b)Shanghai universities ranking; (c)Hart personalities ranking (K blue, K_2 green, K^* red, black curve from Google); (d)slice of probability in 2D plane.

Wikipedia ranking of universities, personalities

Universities:

PageRank: 1. Harvard, 2. Oxford, 3. Cambridge, 4. Columbia, 5. Yale, 6. MIT, 7. Stanford, 8. Berkeley, 9. Princeton, 10. Cornell.

2DRank: 1. Columbia, 2. U. of Florida, 3. Florida State U., 4. Berkeley, 5. Northwestern U., 6. Brown, 7. U. Southern CA, 8. Carnegie Mellon, 9. MIT, 10. U. Michigan.

CheiRank: 1. Columbia, 2. U. of Florida, 3. Florida State U., 4. Brooklyn College, 5. Amherst College, 6. U. of Western Ontario, 7. U. Sheffield, 8. Berkeley, 9. Northwestern U., 10. Northeastern U.

Personalities:

PageRank: 1. Napoleon I of France, 2. George W. Bush, 3. Elizabeth II of the United Kingdom, 4. William Shakespeare, 5. Carl Linnaeus, 6. Adolf Hitler, 7. Aristotle, 8. Bill Clinton, 9. Franklin D. Roosevelt, 10. Ronald Reagan.

2DRank: 1. Michael Jackson, 2. Frank Lloyd Wright, 3. David Bowie, 4. Hillary Rodham Clinton, 5. Charles Darwin, 6. Stephen King, 7. Richard Nixon, 8. Isaac Asimov, 9. Frank Sinatra, 10. Elvis Presley.

CheiRank: 1. Kasey S. Pipes, 2. Roger Calmel, 3. Yury G. Chernavsky, 4. Josh Billings (pitcher), 5. George Lyell, 6. Landon Donovan, 7. Marilyn C. Solvay, 8. Matt Kelley, 9. Johann Georg Hagen, 10. Chikage Oogi.

Wikipedia ranking of physicists

Physicists:

PageRank: 1. Aristotle, 2. Albert Einstein, 3. Isaac Newton, 4. Thomas Edison, 5. Benjamin Franklin, 6. Gottfried Leibniz, 7. Avicenna, 8. Carl Friedrich Gauss, 9. Galileo Galilei, 10. Nikola Tesla.

2DRank: 1. Albert Einstein, 2. Nikola Tesla, 3. Benjamin Franklin, 4. Avicenna, 5. Isaac Newton, 6. Thomas Edison, 7. Stephen Hawking, 8. Gottfried Leibniz, 9. Richard Feynman, 10. Aristotle.

CheiRank: 1. Hubert Reeves, 2. Shen Kuo, 3. Stephen Hawking, 4. Nikola Tesla, 5. Albert Einstein, 6. Arthur Stanley Eddington, 7. Richard Feynman, 8. John Joseph Montgomery, 9. Josiah Willard Gibbs, 10. Heinrich Hertz.

Nobel laureates:

PageRank: 1. Albert Einstein, 2. Enrico Fermi, 3. Richard Feynman, 4. Max Planck, 5. Guglielmo Marconi, 6. Werner Heisenberg, 7. Marie Curie, 8. Niels Bohr, 9. Paul Dirac, 10. J.J.Thomson.

2DRank: 1. Albert Einstein, 2. Richard Feynman, 3. Werner Heisenberg, 4. Enrico Fermi, 5. Max Born, 6. Marie Curie, 7. Wolfgang Pauli, 8. Max Planck, 9. Eugene Wigner, 10. Paul Dirac.

CheiRank: 1. Albert Einstein, 2. Richard Feynman, 3. Werner Heisenberg, 4. Brian David Josephson, 5. Abdus Salam, 6. C.V.Raman, 7. Peter Debye, 8. Enrico Fermi, 9. Wolfgang Pauli, 10. Steven Weinberg.

Google Matrix Applications

practically to everything ....

more data at

http://www.quantware.ups-tlse.fr//QWLIB/2drankwikipedia/