Quantum chaos applications: from simple models to quantum computers and Google matrix

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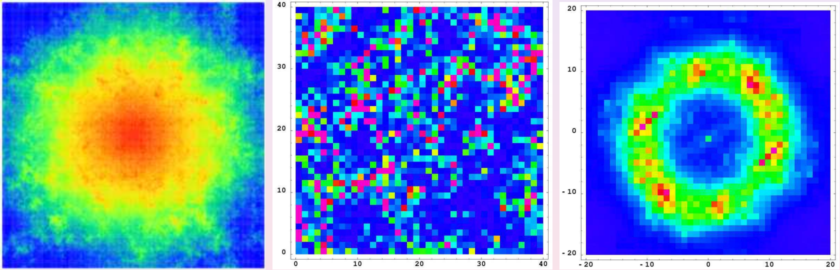


L1: Simple models of classical and quantum chaos

L2: Anderson localization in presence of nonlinearity and interactions

L3: Quantum chaos in many-body systems and quantum computers

L4: Google matrix and directed networks

Anderson localization: introduction & perspectives

1958 => from the talk of P.W.Anderson at Newton Institute, July 21, 2008
see http://www.newton.ac.uk/programmes/MPA/seminars/072117001.html

"Well, In my country," said alice, still panting a little, "you would generally get to somehere else, if you ran very fast for a long time, as we've been doing". "A slow sort of country!", said the queen. "Now *here*, it takes all the running *you* can do, to stay in the same place."

Perspectives: a)localization in new type of systems; b)effects of interactions

Chirikov standard map for soliton dynamics

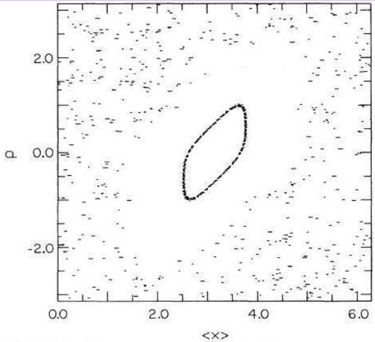


FIG. 1. Two phase-space trajectories with parameters $\beta=25$, $k=0.5$, and $T=2$ (classical K is 2), obtained by numerical in-

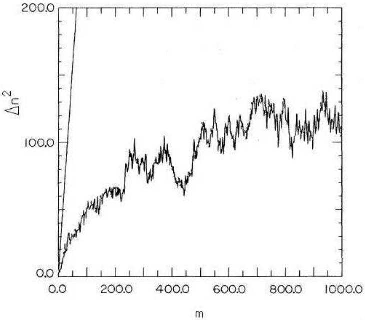


FIG. 4. Plot of the wave packet width in Fourier space Δn^2 vs number of periods m. Here $\beta=10$, $k=2.5$, $T=1$ and classical $K=5$; the initial soliton position and velocity are $x_0=0.2$ and

$$i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - g|\psi|^2 + k\cos x\ \delta_T(t)\right)\psi$$

$$\bar{p} = p + K\sin x\ ,\quad \bar{x} = x + \bar{p}$$

Benvenuto *et al.* (1991)

Nonlinearity and Anderson localization: estimates

$$i\hbar\frac{\partial\psi_n}{\partial t} = E_n\psi_n + \beta|\psi_n|^2\psi_n + V(\psi_{n+1}+\psi_{n-1})\ ; [-W/2 < E_n < W/2]\ \ (DANSE)$$

localization length $\ell \approx 96(V/W)^2$ (1D); $\ln \ell \sim (V/W)^2$ (2D) Amplitudes C in the linear eigenbasis are described by the equation

$$i\frac{\partial C_m}{\partial t} = \epsilon_m C_m + \beta\sum_{m_1 m_2 m_3} U_{mm_1m_2m_3} C_{m_1} C^*_{m_2} C_{m_3}$$

the transition matrix elements are $U_{mm_1m_2m_3} = \sum_n Q^{-1}_{nm} Q_{nm_1} Q^*_{nm_2} Q_{nm_3} \sim 1/\ell^{3d/2}$. There are about ℓ^{3d} random terms in the sum with $U \sim \ell^{-3d/2}$ so that we have $idC/dt \sim \beta C^3$. We assume that the probability is distributed over $\Delta n > \ell^d$ states of the lattice basis. Then from the normalization condition we have $C_m \sim 1/(\Delta n)^{1/2}$ and the transition rate to new non-populated states in the basis $\boldsymbol{m}$ is $\Gamma \sim \beta^2|C|^6 \sim \beta^2/(\Delta n)^3$. Due to localization these transitions take place on a size ℓ and hence the diffusion rate in the distance $\Delta R \sim (\Delta n)^{1/d}$ of $d-$ dimensional $\boldsymbol{m}-$ space is $d(\Delta R)^2/dt \sim \ell^2\Gamma \sim \beta^2\ell^2/(\Delta n)^3 \sim \beta^2\ell^2/(\Delta R)^{3d}$.

At large time scales $\Delta R \sim R$ and we obtain

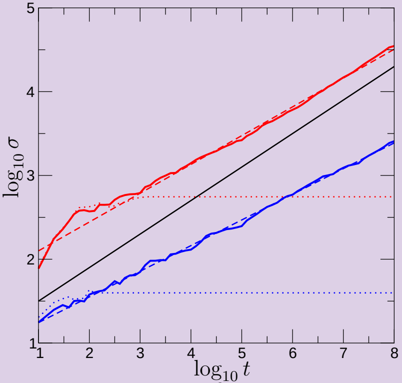
$$\Delta n \sim R^d \sim (\beta\ell)^{2d/(3d+2)} t^{d/(3d+2)};\ (\Delta n)^2 \propto t^\alpha;\ \alpha = 2/(3d+2)$$

Chaos criterion: $S = \delta\omega/\Delta\omega \sim \beta > \beta_c \sim 1$ here $\delta\omega \sim \beta|\psi_n|^2 \sim \beta/\Delta n$ is nonlinear frequency shift and $\Delta\omega \sim 1/\Delta n$ is spacing between exites eigenmodes

DS (1993); Pikovsky, DS (2008) ($d = 1$); García-Mata, DS (2009) ($d \geq 1$)

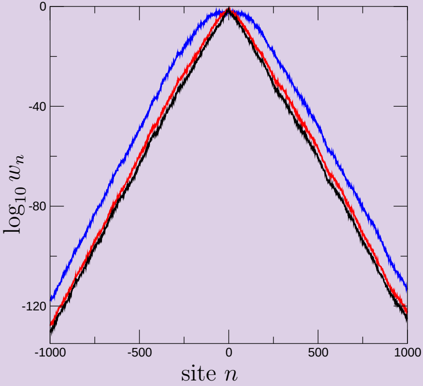
Mulansky, Pikovsky (2009) different nonlinearities

Nonlinearity and Anderson localization (1D)



$W/V = 2, 4$, $\beta = 0, 1$; $\sigma = (\Delta n)^2 \propto t^{\alpha}$;

$\alpha = 2/5$ (theory) 0.34, 0.31 numerics

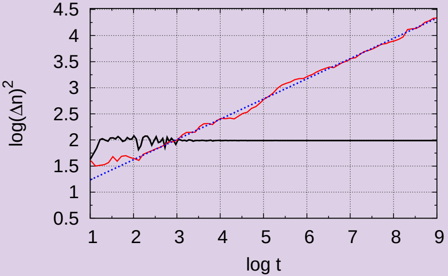


$W/V = 4$, $\beta = 1$, $t = 10^8$, $\beta = 0$

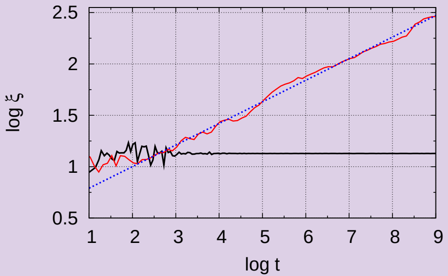
$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n\psi_n + \beta|\,\psi_n\,|^2\psi_n + V(\psi_{n+1} + \psi_{n-1})\ ;[-W/2 < E_n < W/2]$$

Pikovsky, DS (2008)

Kicked nonlinear rotator (1D)



$\alpha_1 = 0.387 \pm 0.003$ (theory 0.4)

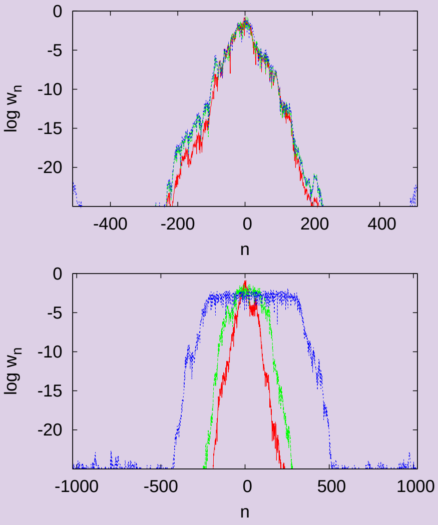


$\nu = 0.210 \pm 0.002$ (theory 0.2); $\xi = (\sum_n |\psi_n|^2)^2 / \sum_n |\psi_n|^4$

$$\psi_n(t+1) = e^{-iT\hat{n}^2/2 - i\beta|\psi_n|^2} e^{-ik\cos\hat{\theta}} \psi_n(t) \;\; ,(k=3, T=2, \beta=0,1)$$

DS (1993); García-Mata, DS (2009)

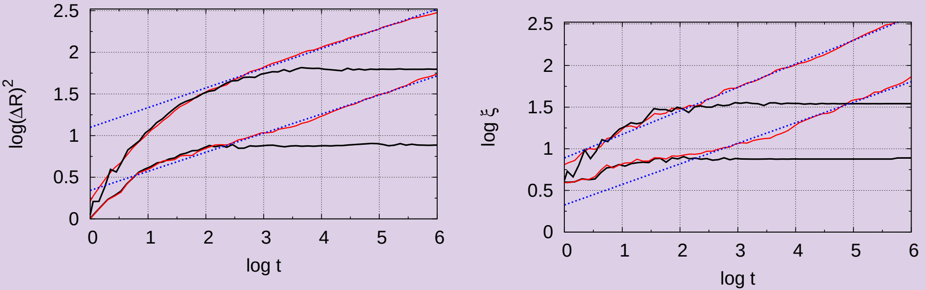
Kicked nonlinear rotator (1D)



$t = 10^3, 10^6, 10^9; \beta = 0; 1$

García-Mata, DS (2009)

Nonlinearity and Anderson localization (2D)



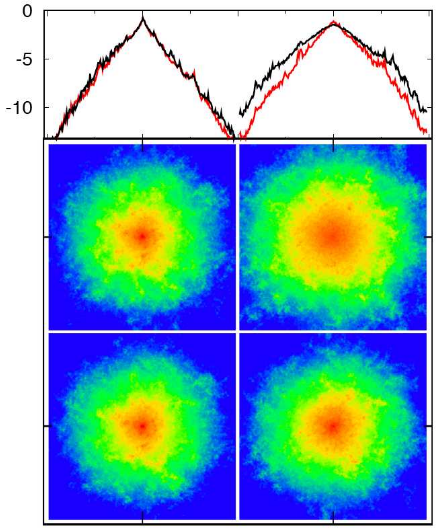
$W/V = 10, 15$, $\beta = 0, 1$; $\alpha_2 = 0.236, 0.229 \pm 0.003$ (theory 0.25)

$\nu = 0.282, 0.247 \pm 0.005$ (theory 0.25); ξ is participation ratio

$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\psi_n|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1})$$

García-Mata, DS (2009)

Nonlinearity and Anderson localization (2D)

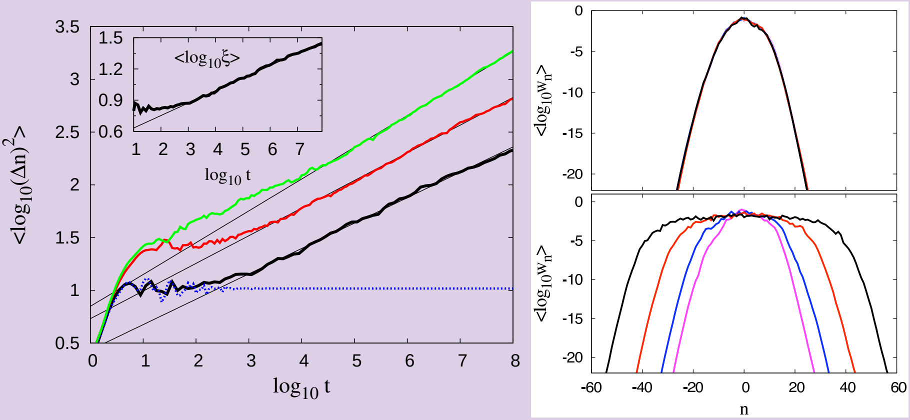


$W = 10$; $\beta = 0$(left), 1(right);
$t = 10^4$ (bottom), 10^6 (middle),
projecton on $x-$axis (top);
256×256 lattice

[also: kicked nonlinear rotator model (1d)]

García-Mata, DS (2009)

Delocalization on disordered Stark ladder



Static field *f* along Stark ladder ($W = 4$): *statistical entanglement*

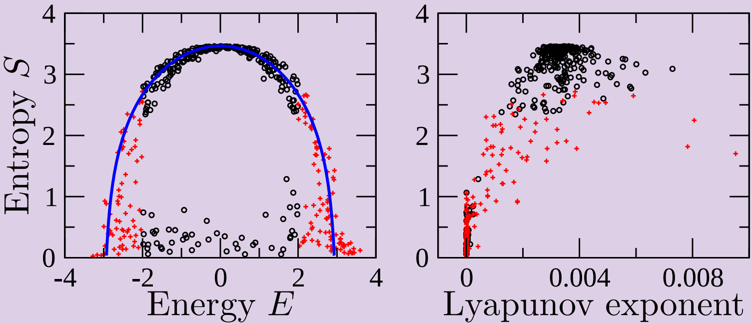
Left: $f = 0, 0.25, 0.5$, $\alpha = 0.30, 0.26, 0.24$, $\beta = 1; 0$ top to bottom; inset IPR at $f = 0.5$;

Right: probabability distribution at $f = 0.5$, $t = 10^2, 10^4, 10^6, 10^8$, $\beta = 0; 1$ (top/bottom)

García-Mata, DS (EPJB 2009)

Dynamical thermalization in DANSE (1D)

starting from Fermi-Pasta-Ulam problem (1955):
regular lattice, delocalized linear modes $\rightarrow$ disorder localized modes

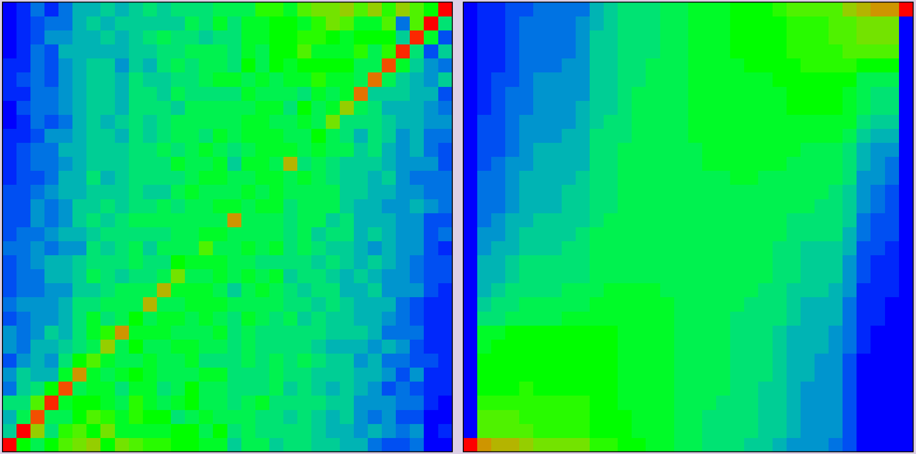


$N = 32, W = 4, \beta = 1, t = 10^7 + 10^6$, initial state: linear eigenmode

Gibbs distribution with temperature T for localized linear modes, $\rho_m = |C_m|^2$:
entropy $S = -\sum_m \rho_m \ln \rho_m, \ \rho_m = Z^{-1} \exp(-\epsilon_m/T), \ Z = \sum_m \exp(-\epsilon_m/T)$,
$E = T^2 \partial \ln Z / \partial T, \ S = E/T + \ln Z$. $\langle \ln Z \rangle \approx \ln N + \ln \sinh(\Delta/T) - \ln(\Delta/T)$, $\Delta \approx 3$

Mulansky, Ahnert, Pikovsky, DS (2009)

Dynamical thermalization in DANSE (1D)

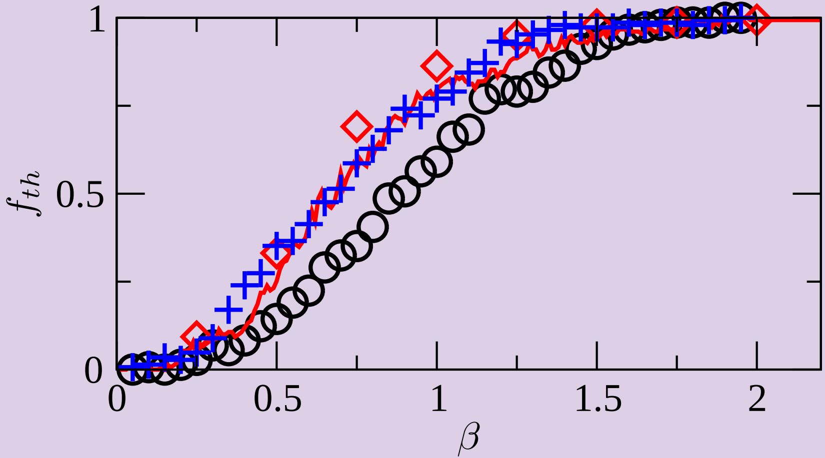


$N = 32, W = 4, \beta = 1, t = 10^6$, initial state: linear eigenmode m', averaged over 8 disoder realisations

Gibbs distribution: time, disorder averaged ρ_m in mode m (y - axis) for initial eigenmode m' (x -axis); left: numerics, right: Gibbs theory

Mulansky *et al.* (2009)

Dynamical thermalization in DANSE (1D)



Fraction of thermalized states: $N = 16$ (circles), 32 (curve), 64($+$) ; $W = 4, t = 10^6$, (diamonds $N = 32, t = 10^7$)

Mulansky *et al.* (2009)

Possible experimental tests & applications

- BEC in disordered potential (Aspect, Inguscio)
- kicked rotator with BEC (Phillips)
- nonlinear wave propagation in disordered media (Segev, Silberberg)
- lasing in random media (Cao)
- energy propagation in complex molecular chains (proteins, Fermi-Pasta-Ulam problem)
- NONLINEAR SPIN-GLASS ? Links to Frenkel-Kontorova model?
- OTHER GROUPS:
  S.Aubry *et al.* PRL **100**, 084103 (2008)
  A.Dhar *et al.* PRL **100**, 134301 (2008)
  S.Fishman *et al.* J. Stat. Phys. **131**, 843 (2008) ...
  S.Flach *et al.* PRL **102**, 024101 (2009) ...
  T.Kottos and B.Shapiro, PRE **83**, 062103 (2011)
  W.-M.Wang *et al.* arXiv:0805.4632[math.DS] (2008)
  see also the participant list of the NLSE Workshop
  at the Lewiner Institute, Technion, June 2008 (http://physics.technion.ac.il/ nlse/)

Experiments on 2D disordered photonic lattices

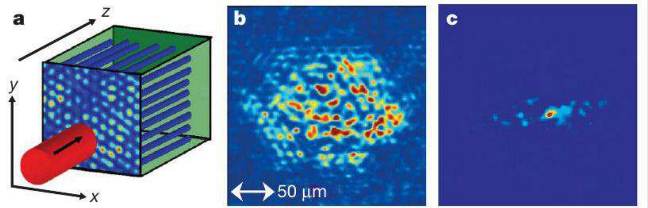
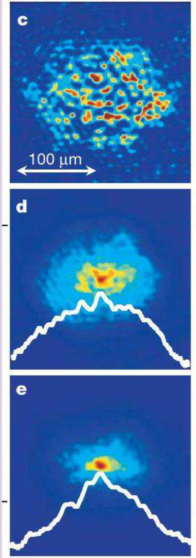


Figure 1 | Transverse localization scheme. a, A probe beam entering a disordered lattice, which is periodic in the two transverse dimensions (x and y) but invariant in the propagation direction (z). In the experiment described here, we use a triangular (hexagonal) photonic lattice with a periodicity of 11.2 μm and a refractive-index contrast of $\sim 5.3 \times 10^{-4}$. The lattice is induced optically, by transforming the interference pattern among three plane waves into a local change in the refractive index, inside a photorefractive SBN:60 ($Sr_{0.6}Ba_{0.4}Nb_2O_6$) crystal. The input probe beam is of 514 nm wavelength and 10.5 μm full-width at half-maximum (FWHM), and it is always launched at the same location, while the disorder is varied in each realization of the multiple experiments. **b,** Experimentally observed diffraction pattern after $L = 10$ mm propagation in the fully periodic hexagonal lattice. **c,** Typical experimentally observed intensity distribution after $L = 10$ mm propagation in one particular realization of the 15% disorder in the lattice.



Segev *et al.* Nature **446**, 52 (2007) (right: disorder growing from top to bottom)

Experiments on 1D disordered photonic lattices

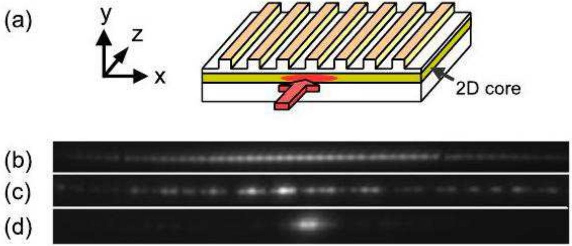


FIG. 1 (color online). (a) Schematic view of the sample used in the experiments. The red arrow indicates the input beam. (b)–(d) Images of output light distribution, when the input beam covers a few lattice sites: (b) in a periodic lattice, (c) in a disordered lattice, when the input beam is coupled to a location which exhibits a high degree of expansion, and (d) in the same disordered lattice when the beam is coupled to a location in which localization is clearly observed.

Silberberg *et al.* PRL **100**, 013906 (2008)

BEC Experiments in 1D incommensurate lattice

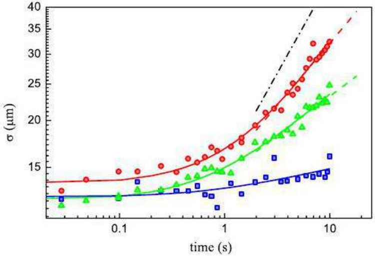


FIG. 1 (color online). Time evolution of the width σ for different initial interaction energies: $E_{\rm int} = 0$ (squares), $E_{\rm int} = 1.8J$ (triangles), and $E_{\rm int} = 2.3J$ (circles). The continuous lines are the fit with Eq. (1). The dashed lines show the fitted asymptotic behavior, while the dash-dotted line shows the expected behavior for normal diffusion. The lattice parameters are $J/h = 180$ Hz, $\Delta/J = 4.9$.

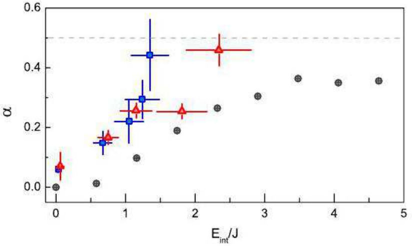


FIG. 2 (color online). Diffusion exponent α vs the initial interaction energy $E_{\rm int}$ in the experiment (triangles and squares) and simulations (circles). The experimental data are for $\Delta/J = 5.3(4)$ and two different values of the tunneling: $J/h = 180$ Hz (triangles) and $J/h = 300$ Hz (squares). The vertical bars are the fitting error of Eq. (1) to the data, while the horizontal bars indicate the statistical error.

^{39}K BEC, 5×10^4 atoms, optical lattice $V(x) = V_1 \cos^2(k_1 x) + V_2 \cos^2(k_2 x)$,
$\lambda_i = 2\pi/k_i = 1064nm, 859nm$

André-Aubry model (or related Harper model) => hopping J in 1st lattice and potential/"disorder" $\Delta \sim V_2$ of 2d lattice, metalic phase at $J/\Delta > 2$

G.Modugno *et al.* PRL **106**, 230403 (2011)

Nonlinearity and localization: open problems

- exponent $\alpha \approx 1/3 < 2/5$, indications on its small decrease at very large times
  Flach *et al.*, Mulansky *et al.* (2009-2011)
- different (higher/lower) nonlinearity exponents $|\psi|^\mu$ still give anomalous spreading
  Mulansky, Pikovsky (2009)
- main part of measure is non-chaotic at small local β
  (zero Lyapunov exponent)
  Pikovsky, Fishman (2011)
- => Arnold diffusion scenario:
  Arnold diffusion in systems with many degrees of freedom
  Chirikov (1979); Chirikov, Vecheslavov (1997)
  spreading over Arnold web of narrow chaotic separatrix layers
  Mulansky *et al.* (2011)

Fast Arnold diffusion and spreading in nonlinear disorderd lattices

- for the Chirikov standard map the width of separatrix layer drops exponentially with perturbation: $w_s \lambda^3 \exp(-\pi\lambda/2)$, $\lambda \sim 1/\sqrt{K}$
  Arnold diffusion along web of chaotic separatrix layers
  Diffusion rate $\ln D \propto \ln w_s^2 \propto -\lambda$
  (Chirikov (1979))
- Chirikov-Vecheslavov (1997): $w_s \sim K^{2.5}$, $D \sim K^{3/2} w_s^2 \sim K^{\nu_D}$, $\nu_D \approx 6.6$!
  multi-particle standard map: $H = |p|^2/2 - K \sum_{i=1}^{N+1} \cos(x_{i+1} - x_i)\delta_1(t)$
  many degrees of freedom: $2 \leq L = N + 1 \leq 15$
- independent computations of weak chaos measure $\mu \propto w_s \propto K^{1.6}$ $(10^{-4} \leq K \leq 0.01)$
  (Mulansky *et al.* (2011))
- slow anomalous spreading:
  $H = \sum_{k=1}^{L} [p_k^2/2 + \eta_k q_k^4/4 + \gamma(q_{k+1} - q_k)^6/6]$; $(0.5 \leq \eta_k \leq 1.5, \gamma \sim 1)$
  Results: $\sigma =< (\Delta k)^2 >\sim t^{\alpha}$, $\alpha \approx 0.55$;
  $\alpha = 8/(9 + 2\nu_D)$; $\nu_D = 6.6 => \alpha = 0.36$
  (Mulansky *et al.* (2011))

Fast Arnold diffusion and spreading in nonlinear disorderd lattices

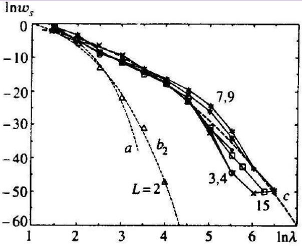
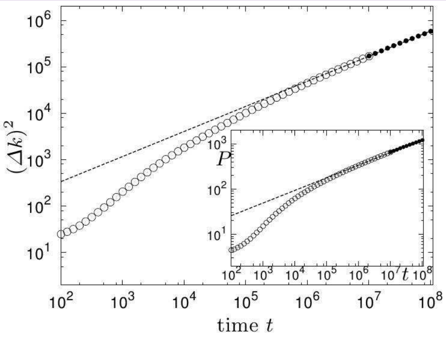


FIG. 1. Summary of numerical data for the model (2.1). Broken solid lines connecting various symbols show computed values of w_s as a function of the adiabaticity parameter $\lambda \equiv 1/\sqrt{K}$ and the resonance dimension $L=N$ indicated by the numbers. Dotted lines represent the theory: (a) small-λ limit, one fitting parameter, Eq. (3.5); (b_2) large-λ limit for $L=2$, two fitting parameters, Eq. (4.9); (c) intermediate asymptotics, three fitting parameters, Eq. (5.8).



- Left: Chirikov-Vecheslavov (1997); Right: Mulansky *et al.* (2011)
- DANSE: H_0 is linear that makes situation more complicated, but most probably there is the same scenario as above

Two Interacting Particles (TIP) effect

Anderson model in d-space + onsite Hubbard interaction U, $V \sim E_F$ is one-particle hopping; exited states $\psi_m(n) \sim \exp(-|n-m|/\ell)/\ell^{d/2}$; $\ell \gg 1$.
Equation in the basis of noninteracting eigenstates $\chi_{m_1 m_2}$:

$$i\partial\chi_{m_1m_2}/\partial t = \epsilon_{m_1m_2}\chi_{m_1m_2} + \sum_{m'_1m'_2} U_{m_1m_2m'_1m'_2}\chi_{m'_1m'_2}$$

$$U_{m_1m_2m'_1m'_2} = U\sum_n \psi_{m_1}(n)\psi_{m_2}(n)\psi^*_{m'_1}(n)\psi^*_{m'_2}(n) \sim U_s \sim U\sqrt{M}/\ell^{2d} \sim U/\ell^{3d/2}$$

Sum runs over $M \sim \ell^d$ coupled states; interaction induced matrix elements U_s, density of coupled states is $\rho_2 \sim \ell^{2d}/V$, TIP transition rate $\Gamma_s \sim {U_s}^2\rho_2 \sim U^2/(V\ell^d)$.

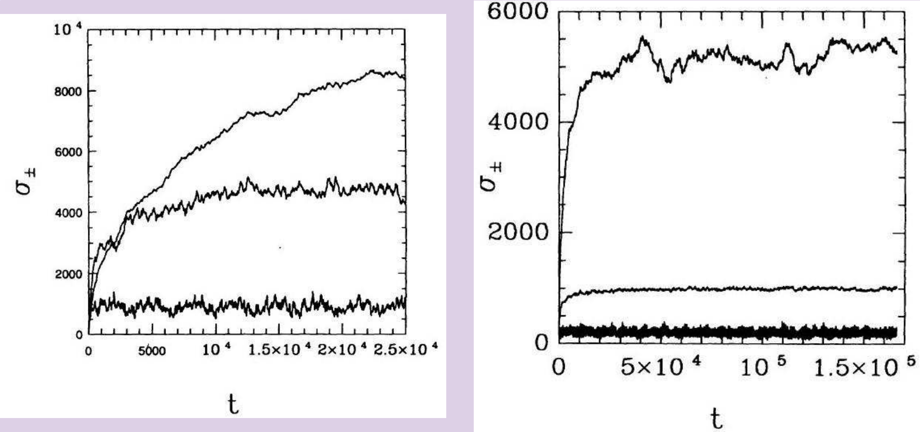
$$\text{Diffusion rate: } D_s \sim \ell^2\Gamma_s \sim U^2/(V\ell^{d-2})$$
$$\Delta n(t^*) \sim \ell^d (D_s t^*)^{d/2} \sim 1/\Delta\omega \sim Vt^*$$
$$\Delta n(t^*)/\ell^2 \sim \ell_2/\ell \sim U^2\ell/V^2 > 1 \;\; (d=1)$$

$$\text{Enhancement factor: } \kappa = \Gamma_s\rho_2 \sim \ell^d(U/V)^2 > 1$$

TIP localization: $\ell_2/\ell \sim (U/V)^2\ell$ (1d); $\ln(\ell_2/\ell) \sim (U/V)^2\ell^2$ (2d);
delocalization for $\kappa \sim (U/V)^2\ell^3 > 1$ (3d)

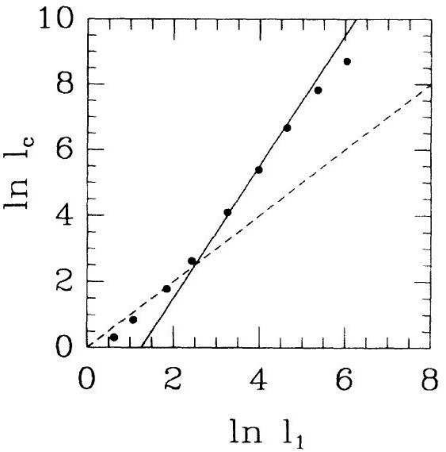
DS (1994),(1996); Y.Imry (1995)
two attractive particles Dorokhov (1990)

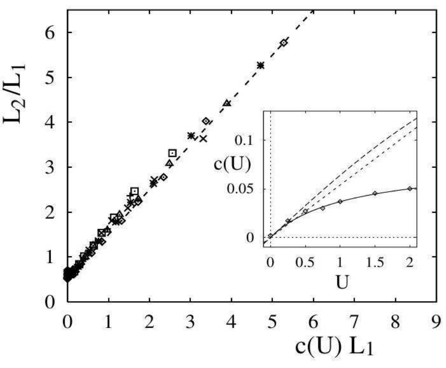
TIP effect: numerical results 1d



Left: TIP 1d $U/V = 1(top), 0(bottom); W/V = 1.4$
Right: two kicked rotator with Hubbard coupling $U = 2(top); 0(bottom)$
$k = 5.7; K = 5.$
DS (1994)

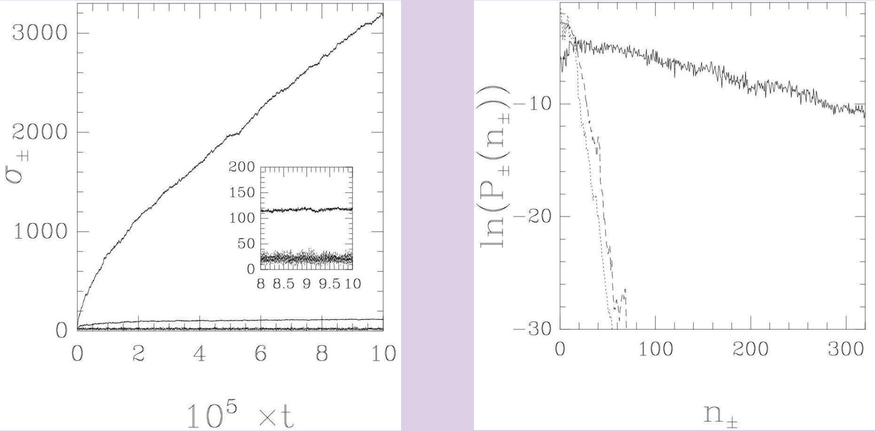
TIP effect: numerical results 1d





Left: bag model (DS (1994)); Right: TIP 1d by Frahm (1999)
Other confirmations: Pichard *et al.* (1995); von Oppen *et al.* (1996)
Continued Confusion: R.Römer, M.Schreiber PRL (1997);
S.Flach *et al.* Pis'ma ZhETF (2011)

TIP effect: numerical results 3d,4d



Left: second moment $\sigma_{\pm}(t)$ as a function on number of kicks in 2-frequency modulated kicked rotators, $k = 0.9$, $\epsilon = 0.75$, 2-frequencies (3d), $U = 2$ (top) and $U = 0$ (bottom); Right: probability distributions in $n_{\pm} = (n_1 \pm n_2)/\sqrt{2}$ at $U = 2, 0$, $\ell_2/\ell \approx 25$. Similar data for 3-frequencies (4d)

Borgonovi, DS (1996)

TIP effect: numerical results 2d

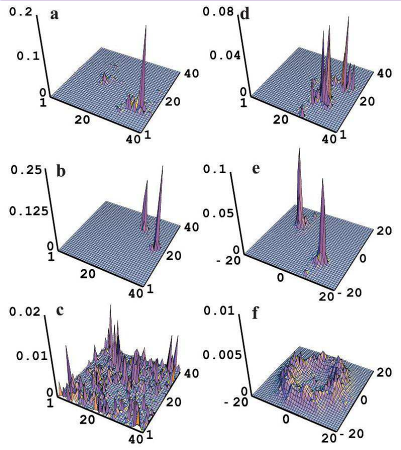


Fig. 1. Probability distributions f and $f_{\rm d}$ for TIP in 2d disordered lattice of size $L = 40$, and interaction of radius $R = 12$ and width $\Delta R = 1$. Left column, one-particle probability f for $W = 8V$: (a) ground state at $U = 0$; (b) ground state with binding energy $\Delta E = -1.05V$ at $U = -2V$; (c) coupled state with $\Delta E = -0.19V$ at $U = -2V$. Right column: (d) f for coupled state, compare to case (c), at $W = 12V$ and $U = -2V$ with $\Delta E \approx -0.19V$; (e) inter-particle distance probability $f_{\rm d}$ related to case (b); (f) $f_{\rm d}$ related to case (c).

probability $f(\mathbf{n}_1) = \sum_{\mathbf{n_2}} F(\mathbf{n_1}, \mathbf{n_2})$ and the probability of inter-particle distance $f_{\rm d}(\mathbf{r}) = \sum_{\mathbf{n_2}} F(\mathbf{r} + \mathbf{n_2}, \mathbf{n_2})$ with $\mathbf{r} = \mathbf{n_1} - \mathbf{n_2}$. The binding energy of an eigenstate in (2) is $\Delta E = E - 2E_{\rm F} \approx E$ since $E_{\rm F} \approx 0$. For the ground state with energy E_g the coupling energy is $\Delta = 2E_{\rm F} - E_g$. The

TIP in 2d with interation U inside a ring of radius $R = 12$ and width $\Delta R = 1$
(see also middle/left figs of page 1 corresponding to bottom panels here)
effective 3d Anderson model (Lages, DS (2000))
Coulomb interactions case has 3d Anderson transition for TIP (DS (2000))

TIP near the Fermi level

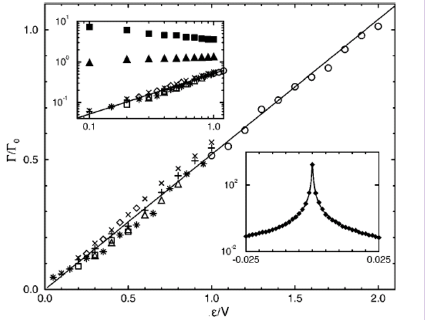


FIG. 1. Energy dependence of the rescaled Breit-Wigner width Γ/Γ_0 in 2D. Direct diagonalization (DD) data at $W/V = 2$: $U/V = 0.6$ with $L = 8$ (○), $L = 15$ (△), $L = 20$ (□); $U/V = 1.5$ and $L = 20$ (◇). Fermi golden rule (FGR) data: $W/V = 2$ with $L = 20$ (+), $L = 25$ (×); $W/V = 1$ with $L = 15$ (∗). The straight line $\Gamma(\epsilon)/\Gamma_0 = C\epsilon/V$ with $C = 0.52$ shows the Imry estimate. Upper inset: the same on a log-log scale with FGR data at higher disorders [$W/V = 6$ (▲) and $W/V = 10$ (■) ($L = 30$)]. Lower inset: ρ_W vs E for $L = 20, W/2 = V = 1, U = 0.6, \epsilon = 0.4$ fitted by ρ_{BW} with $\Gamma = 0.18\Gamma_0$ (solid curve).

Small ϵ energy excitations above the Fermi level:

a)box size $L \ll \ell$

$\rho_2 \sim L^{2d}\epsilon/V$, $\Gamma = C\Gamma_0\epsilon/V$,

$\Gamma_0 = U^2/(VL^d)$, $C = const$

b)box size $L \sim \ell$

$U_s^2 \sim \Delta^2(U/V)^2(1+\epsilon/E_c)^{d/2-2}/g^2$,

with $g = E_c/\Delta > 1$ and for

$\epsilon > E_c \sim V/L^2 > \Delta \sim V/L^d$

$\kappa = \Gamma\rho_2 \sim (U/V)^2(\epsilon/\Delta)^{d/2-1}$

for $L \sim \ell$, $d = 2$ we have κ independent of ϵ for $\epsilon \sim \Delta$.

Problems:

there is no enhancement at E_F,

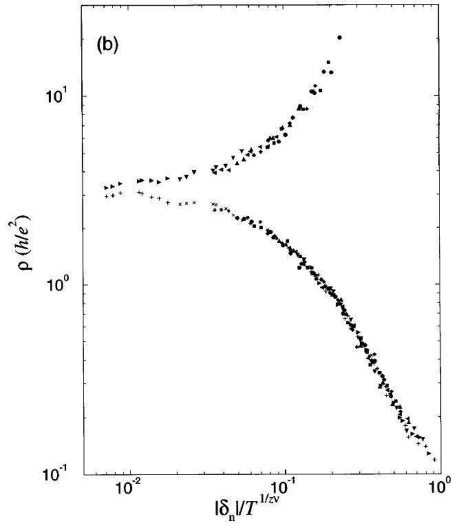
$\kappa \sim 1$

Jacquod, DS PRL (1997)

Here Thouless energy is $E_c = \hbar/t_{dif} \sim L^2/\hbar D$;

$\Delta \sim V/L^d$, conductance $g = E_c/\Delta$

Metal-insulator transition in 2d ?



Demonstrating scaling with temperature in the temperature interval 0.3 to 1K, the linear resistivity is shown as a function of $|\delta_n|/T^b$ for $b = 1/z\nu = 0.83$; electron densities are in the range $7.81 - 10.78 \times 10^{10} cm^{-2}$; $\delta_n = (n_s - n_c)/n_c$.

Kravchenko *et al.* (1996)

Slow Metal (2D)

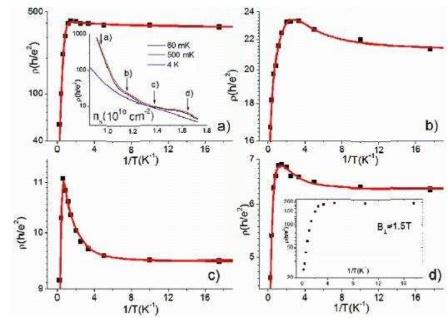


FIG. 2 (color online). Resistivity as a function of inverse temperature $1/T$ at $B = 0$ T (symbols). At all densities, the strongly insulating T dependence at higher temperatures is followed by a decrease in resistance at low T. Device dimensions are $W \times L = 8\ \mu\text{m} \times 0.5\ \mu\text{m}$, spacer $\delta = 40$ nm. Electron densities are indicated by arrows in the inset to (a). Solid lines represent a fit of Eq. (1) to the data. Inset to (a): Resistivity as a function of electron density at $T = 60$ mK, 500 mK, 4 K. Inset to (d): ρ as function of $1/T$ at the same density as (d) but at $B_\perp = 1.5$ T.

TIP diffusion

$D \sim \Gamma_s l^2 \sim U^2/V$ at $(Ul/V)^2 > 1$

vs. usual diffusion $D_0 \sim v_F \ell \sim V$

Thus it is possible to have diffusion with conductance g and resistivity per square ρ_0 (in natural units):

$g \sim 1/\rho_0 \sim D/D_0 \sim (U/V)^2 \ll 1$

With up to $(Ul/V)^2 \sim 1$ and

$g \sim 1/l^2 \ll 1$

Problems: finite particle density, small density of states near the ground state

Experiment suggestion: to measure a charge of quasi-particles from noise fluctuations

M.Baenninger, A.Ghosh, M.Pepper, H.E.Beere, I.Farrer, D.A.Ritchie PRL **100**, 016805 (2008) vs. S.Kravchenko *et al.* RMP **73**,251 (2001)

Many electrons near the Fermi level (Coulomb interactions, no spin)

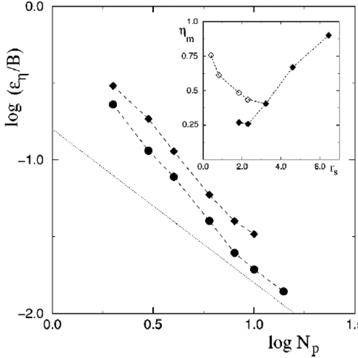


FIG. 4. Dependence of ϵ_η/B on the number of particles N_p, obtained from Fig. 2: $W/V=10$ with $\eta(E_\eta)=0.4$ (full diamond) and $W/V=7$ with $\eta(E_\eta)=0.2$ (●), where $\epsilon_\eta=E_\eta/N_p$. The straight line shows the slope when $E_\eta=$const. The inset gives the dependence of maximal η on r_s for $W/V=7$ and $N_p=6$: $U/V=2$, $8\leqslant L\leqslant 28$ (full diamond), and $L=14$, $0.25\leqslant U/V\leqslant 2$ (◇).

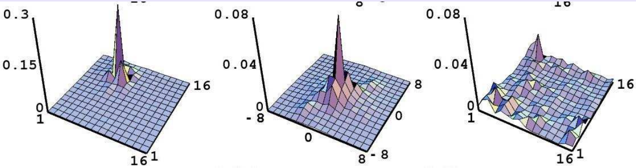
Level-spacing statistics $P(s)$:
$\eta = 1$ Poisson distribution,
$\eta = 0$ Wigner-Dyson distribution
ϵ_η - exitation energy per particle
at a given $\eta = const$ ($B = 4V$)
$r_s = U/(2V\sqrt{\pi\nu})$, $\nu = N_p/L^2 \approx 1/32$
usually $U = 2V$, $r_s \approx 3.2$,
$2 \leq N_p \leq 20$, $8 \leq L \leq 25$

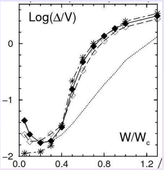
Result: chaotic, ergodic states at temperature going to zero

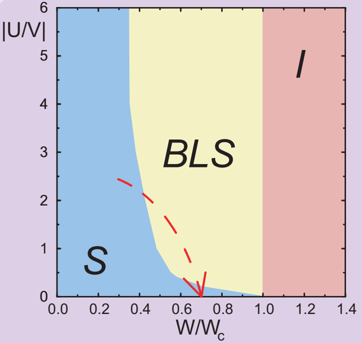
Problems: transport properties ?

DS(2000); Song, DS (2000)

Cooper problem in the vicinity of the Anderson transition







Top (left): $3d$, $W/W_c = 0.5$, $W_c/V = 16.5$, $U/V = -4$ (left/middle); $U/V = 0$ (right); left/right: particle density projected on (x, y) plane; middle: interparticle distance probability

Top (right): large coupling gap Δ, not reproduced by mean field (dashed curve $L = 12$); $U/V = -4$, $L = 10, 12, 14$ (symbols)

Left: Diagram of bi-particle localized (BLS) phase

Result: localized pairs inside noninteracting metallic phase with $g \gg 1$; mean field does not give this BLS phase

Lages, DS PRB **62**, 8665 (2000)

3d Hubbard model of spin-1/2 fermions (projector quantum Monte Carlo)

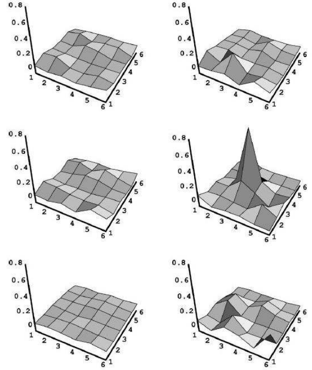


FIG. 1. Distribution of charge density difference for an added pair, $\delta\rho_p$, projected on the (x,y) plane for a $6\times6\times6$ lattice for the same single disorder realization, with $W/t=2$ (left) and $W/t=7$ (right), $N=108$. Top: exact computation for $U=0$, $\xi=70;55$ (left; right). Middle: PQMC calculation for $U/t=-4$, $\xi=48;6.5$ (left; right). Bottom: BdG mean-field calculation for $U/t=-4$, $\xi=132;25$ (left; right). All quantities presented in all figures are in dimensionless units (see text).

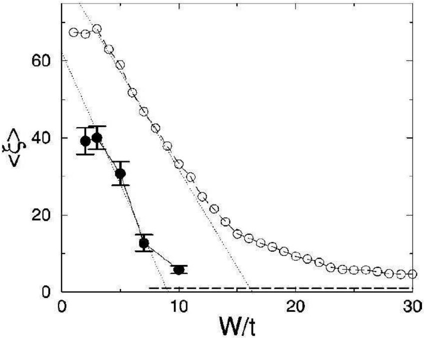


FIG. 2. Inverse participation ratio $\langle\xi\rangle$ averaged over disorder realizations, as a function of disorder strength W for a $6\times6\times6$ lattice, at $U=0$ (open circles) and $U/t=-4$ (solid circles). Dotted lines show linear fits to the data, the dashed line represents $\xi=1$ (see text), and error bars indicate statistical errors.

Srinivasan, Benenti, DS (2002) (up to $N=110$ fermions; $t=V$)

Superconductor-Insulator Transition: experiment

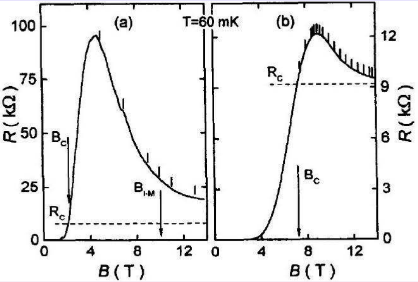


Fig.1. Magnetoresistance of the film in state 1 (a) and in state 2 (b). The critical R_c and B_c values at $T = 0$ are indicated. Also shown is the position of metal-insulator transition, B_{I-M}, determined from Fig.2. The temperature dependences of the resistance are analyzed at fields marked by vertical bars

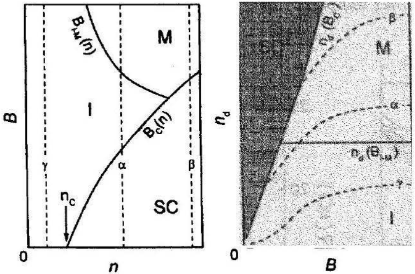


Fig.4. Schematic phase diagram of the observed transitions in the (n, B) and (B, n_d) planes. The evolution of states α, β, γ with magnetic field is shown by dashed lines. In shaded area the value n_d is not defined

V.F.Gantmakher *et al.* Pis'ma ZhETF **68**, 337 (1998)

Superinsulator in a static electric field

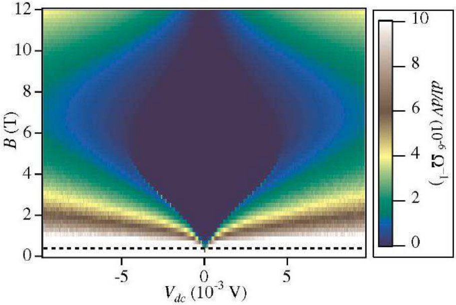


FIG. 4 (color). Two-dimensional map of the dI/dV values in the $B - V_{\rm dc}$ plane. For the sample of Fig. 2 (Ja5), we have measured dI/dV traces as a function of $V_{\rm dc}$ at B intervals of 0.2 T and at $T = 0.01$ K. The color scale legend on the right-hand side shows the various colors used to represent the values of dI/dV. The horizontal dashed line denotes B_c ($= 0.4$ T) of this sample.

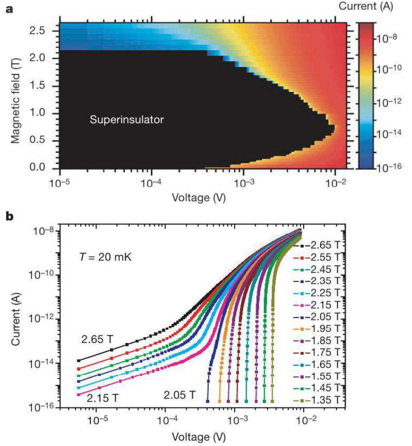
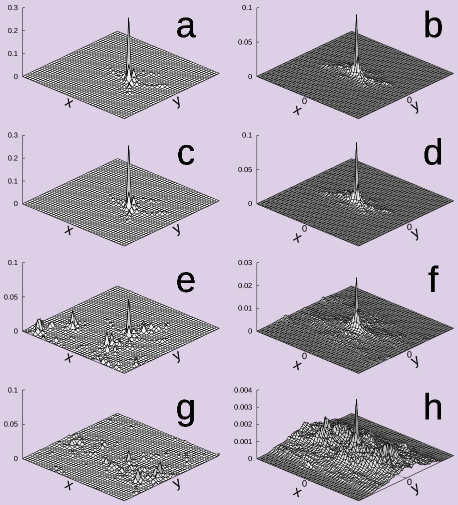


Figure 3 | Magnetic-field-tuned transition to superinsulating state. a, The two-dimensional colour map of the current values in the B–V plane. The colour scale on the right-hand side represents current. The black domain in the map corresponds to the superinsulating state. The border between the

Left: Shahar *et al.* PRL **94**, 017003 (2005).

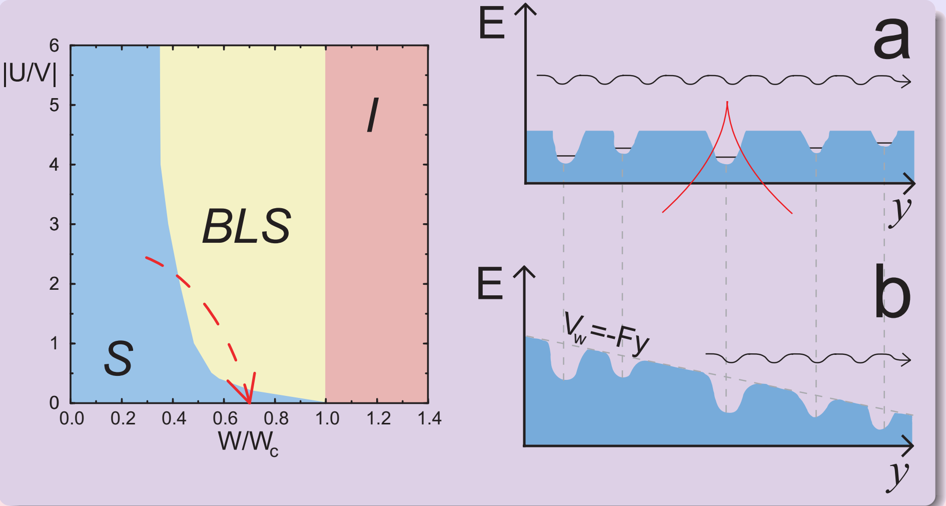
Right: Baturina *et al.* Nature **452**, 613 (2008).

Superinsulator as BLS phase



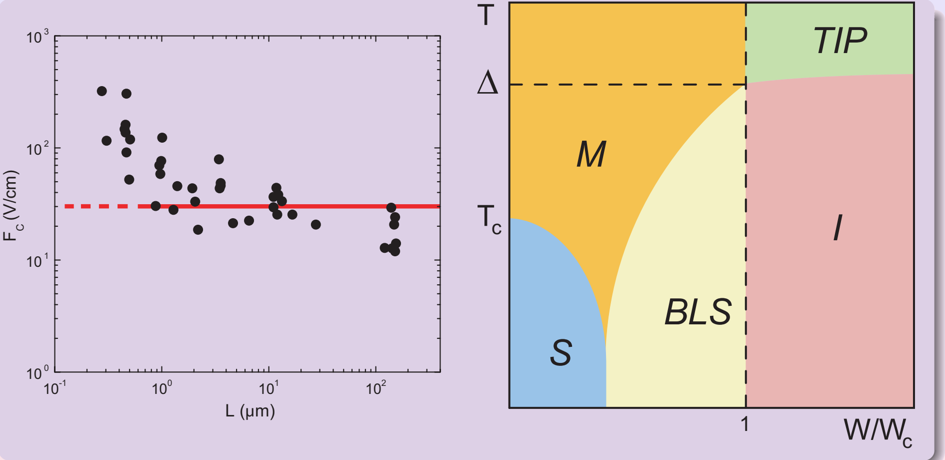
Delocalization of two interacting particles with attractive Hubbard interaction $U = -2V$ by a static electric force F. The generalized Cooper problem is considered on 2D lattice of size $L \times L = 40 \times 40$ at disorder strength $W = 5V$, a static field F is directed along $y-$axis. Probability is shown for a lowest energy eigenstate with a maximum probability $f(y) = \sum_x f(x, y)$ at $y = L/2$. Left panels show one-particle probability $f(x, y)$ and right panels show interparticle distance probability $f_d(x, y)$. The static electric force, directed along $y-$axis, is $F = 0(a, b)$, $F = 0.003V(c, d)$, $F = 0.016V(e, f)$, $F = 0.052V(g, h)$. (Lages, DS (2011))

Superinsulator as BLS phase



field breaking of BLS pairs: $F_c \approx \Delta/\ell$, $V_c = F_c L$ at $L > \ell$ (Lages, DS (2011))

Superinsulator as BLS phase



Left: F_c vs L at $L > \ell \sim 0.5\mu m$ (Kowal, Ovadyahu Physica C **468**, 322 (2011))
Right: phase diagram BLS-TIP-SIT (Lages, DS (2011))

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