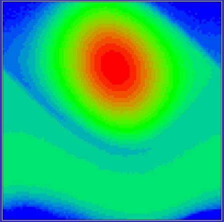
# Synchronization theory of microwave induced zero-resistance states



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arxiv:0905.0593 (2009); arXiv:1302.2778 (2013)
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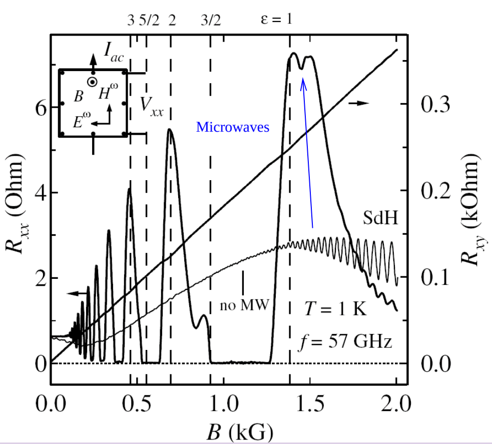
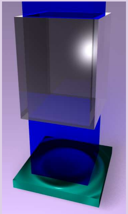
# Zero resistance states discovery in 2002




- Under microwave irradiation 4-terminal $R_{xx}$ vanishes
- High mobility two dimensional electron gas $\ell \approx 140\mu m$
- Temperature of about 1K



Waveguide

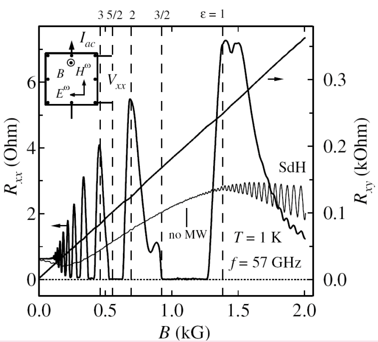
Microwave field
f ~ 50 GHz

Measure sample
DC resistance

- R.G. Mani, J.H. Smet, K. von Klitzing, V. Narayanamurti, W.B. Johnson and V. Umansky, Nature **420**, 646 (2002).
- M.A.Zudov, R.R.Du, L. N. Pfeiffer and K. W. West PRL **90**, 046807 (2003).

# Main experimental features

- Zero resistance states have a $1/B$ periodic structure




- main control parameter is

$$j = \frac{\omega}{\omega_c} = \frac{\omega}{eB/m}$$

- $R_{xx}$ has a peak if $j = n - \delta$ n integer
- $R_{xx}$ is zero if $j = n + \delta$ $\delta \simeq 1/4$
- high harmonics up to $n \simeq 10$
- Arrhenius law dependence on temperature with activation energy $\simeq 20$ K

- Length scales at equilibrium

| $\lambda_{Fermi}$ | $<$ | $\lambda_T$ (at 1 K) | $\ll$ | $r_c$ | $\ll$ | Mean free path |
|---|---|---|---|---|---|---|
| $\simeq 50$ nm | | $\frac{\hbar}{\sqrt{mT}} \simeq 100$ nm | | $\frac{v_F}{\omega_c} \simeq 1\mu$m | | $\ell \simeq 100\ \mu$m |

# Experimental features $\rightarrow$ theory arguments

**EXPERIMENT**

- **weak field:** $\epsilon \propto \Delta v_{osc}/v_F \sim 0.01 - 0.05 \ \ (\sim 1V/cm)$
- **high Landau levels (high filling factors)** $\nu \approx 60$
- **high harmonics** $j = \omega_c/\omega \geq 1$**: no such transitions in oscillator**
- **high mobility, mean free path** $l_e \approx 140\mu m$**, small angle scattering**
  $\rightarrow$ **smooth potential,** $\omega_c \gg \omega_{pot}$
  $\rightarrow$ **due to adiabatic theorem transport is very weak in the bulk with smooth potential**
  $\rightarrow$ **effect of scattering on sharp disks in the bulk**

**CONJECTURE**

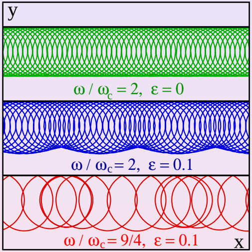
- **main contribution to transport comes from ballistic transmission along edges or scattering on sharp disks inside bulk**
- **ZRS = microwave stabilization of edge transport or trapping on disks**

**LINKS**

- **edge transport in quantum Hall effect**
  **B.I.Halperin PRB 25, 2185 (1982) , M.Büttiker PRB 38, 9375 (1988)**

# Classical theory of edge transport under irradiation

**Newton equations of motion (model 1, specular wall $I_{wall}$, small angle scattering $I_S$)**




$$d\mathbf{v}/dt = \omega_{\mathbf{c}} \times \mathbf{v} + \epsilon\omega \cos \omega t - \gamma(v)\mathbf{v} + I_{wall} + I_S \quad (1)$$

$\epsilon = e\mathbf{E}/(m\omega v_F)$ describes microwave driving field $\mathbf{E}$,

velocity $\mathbf{v}$ is measured in units of Fermi velocity $v_F$,

$\gamma(v) = \gamma_0(|\mathbf{v}|^2 - 1)$ describes relaxation processes to the Fermi surface

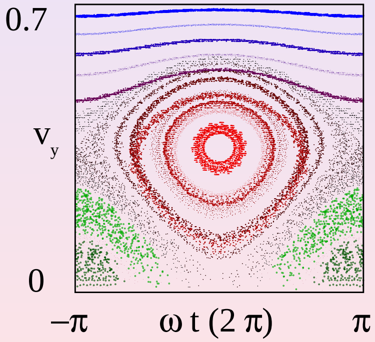
random angle scattering on microwave period with amplitude $\pm\alpha$

**Direct real space analysis of trajectories is complicated,**
**Construct Poincaré section !**

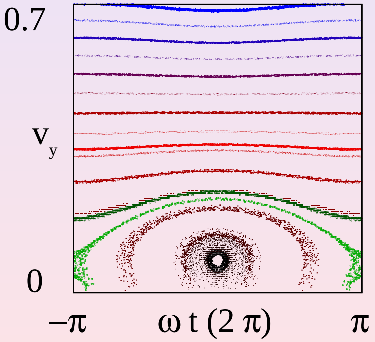
# Poincaré section (Newton equations)

Abscissa : phase of the microwave field $\omega t (\mathrm{mod}\ 2\pi)$ at the moment of collision with the wall
Ordinate : $v_y$ velocity at the moment of collision (divided by Fermi velocity)
(Electric field $\epsilon = (0, 0.02)$, no noise and no dissipation)

$j = \omega/\omega_c = 9/4$ (model 1)

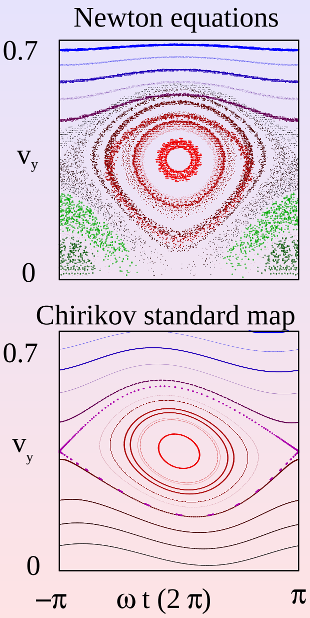



$j = \omega/\omega_c = 2.05$ (model 1)




Appearance of a nonlinear resonance

# Chirikov standard map




Approximate description of the nonlinear resonance

velocity change at wall collision:
double wall velocity

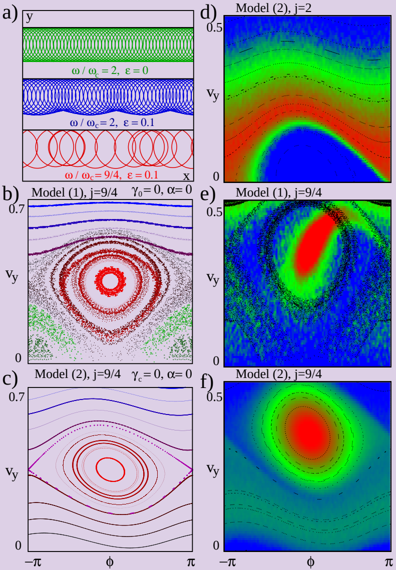
small angles near wall: time between collisions $\Delta t = 2(\pi - v_y)/\omega_c$

this leads to the Chirikov standard map :

$$\begin{cases} v_y(n+1) = v_y(n) + 2\epsilon \sin\phi(n) + I_{cc} \\ \phi(n+1) = \phi(n) + 2(\pi - v_y(n+1))\omega_{/}\omega_c \end{cases}$$

model 2, $I_{cc}$ describes noise and dissipation

# Phase space portrait, with noise and dissipation




Left Column: Dynamics without dissipation

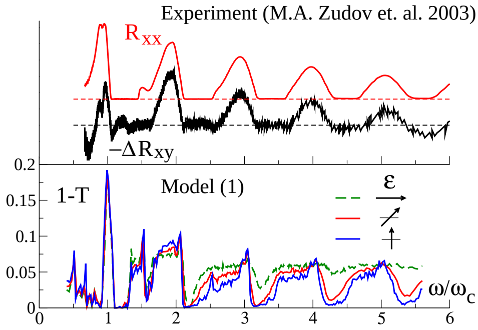
Right Column: Color scale show the density of propagating particles on the Poincaré section in presence of noise and dissipation (red → maximum) Black points show trajectories without noise and dissipation.

$\omega/\omega_c = 2$ microwave repels particles from the edge (d)

$\omega/\omega_c = 9/4$ particles are trapped in the resonance (e,f)

Here $\gamma_0 = 10^{-3}$ (e), $\gamma_c = 10^{-2}$ (d,f) and $\alpha \simeq 5 \times 10^{-3}$.

# Stabilization of edge transport (1/B dependence)



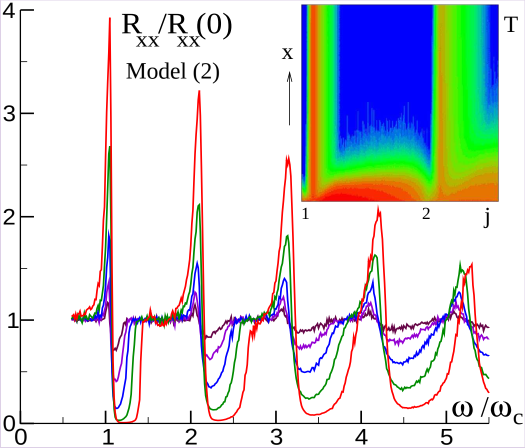

Top: $R_{xx}$ and $-\Delta R_{xy} = \frac{H}{ne} - R_H$ as a function of $\omega/\omega_c$ (experiment)

Bottom: Transmission along sample edge as a function of $\omega/\omega_c$

(model 1)

For $l_e \gg r_c$ the billiard model of a Hall bar gives $R_{xx} \propto -\Delta R_{xy} \propto 1 - T$
Microwave field is $\epsilon = 0.05$, relaxation $\gamma_0 = 10^{-3}$ and noise amplitude $\alpha = 3 \times 10^{-3}$.
Transmission without microwaves is $T \simeq 0.95$, $N = 5000$ orbits.

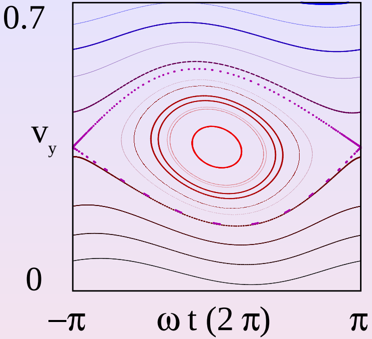
# Stabilization of edge transport (Dependence on microwave field)




Growth of ZRS peaks and dips (model 2) as a function of microwave field amplitude $\epsilon = 0.00375, 0.0075, 0.015, 0.03, 0.06$.

Insert shows transmission probability $T$ at distance $x$ along the edge for $\epsilon = 0.02$ $(0 < x < 10^3 v_F/\omega)$.

# Position and width of the resonance




$$\begin{cases} v_y(n+1) = v_y(n) + 2\epsilon \sin\phi(n) \\ \phi(n+1) = \phi(n) + 2(\pi - v_y(n+1))\omega/\omega_c \end{cases}$$

A phase shift by $\phi \to \phi + 2\pi$ does not change the behavior of map. Hence the phase space structure is periodic in $j = \omega/\omega_c$ with period unity which naturally yields high harmonics. The resonance is centered at $v_y = \pi(1 - m\omega_c/\omega)$ where $m$ is the integer part of $\omega/\omega_c$.

The chaos parameter of the map is $K = 4\epsilon\omega/\omega_c$ and the resonance separatrix width $\delta v_y = 4\sqrt{\epsilon\omega_c/\omega}$.

The energy barrier of the resonance: $E_r/E_F = (\delta v_y)^2/2E_F = 16\epsilon\omega_c/\omega$.

# Activation energy and escape rates

Typical spread square width in velocity angle during the relaxation time $1/\gamma_c$ is $D_s = \alpha^2/\gamma_c$. The resonance square width is $(\delta v_y)^2 = 16\epsilon\omega_c/\omega$ and escape probability from the resonance is

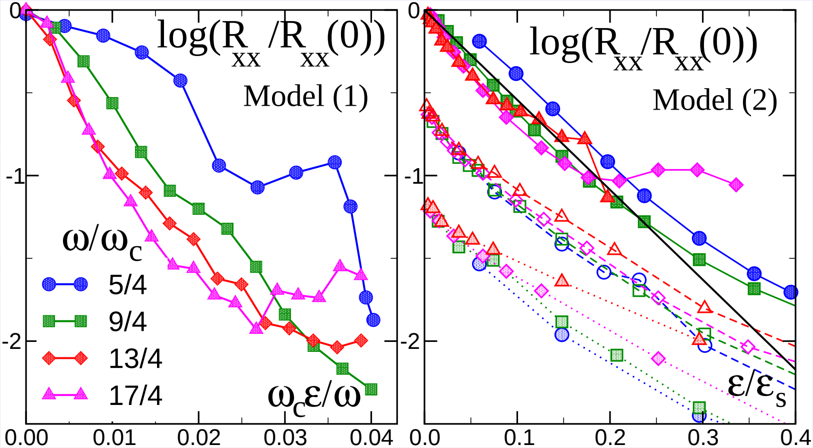
$$W \sim \exp(-(\delta v_y)^2/D_s) \sim \exp(-A\epsilon\omega_c/(D_s\omega))$$

with $R_{xx}/R_{xx}(0) \sim 1 - T \sim W; \quad \epsilon_s = \omega D_s/\omega_c; \quad A \approx 16.$

Arrhenius law with activation energy equal to the energy height of the nonlinear resonance $E_r = 16\epsilon\omega_c E_F/\omega$ where $E_F$ is the Fermi energy. This dependence appears as an additional damping factor in ZRS amplitude:

$$R_{xx} \propto \exp(-A\epsilon\omega_c/(D_s\omega))\exp(-16\epsilon\omega_c E_F/\omega T_e)$$

# ZRS parameter dependence




Dependence of rescaled $R_{xx}$ on rescaled microwave field $\epsilon$ for models (1) (left) and (2) (right). Left: parameters as in Fig. 2 and $\epsilon$ is varied. Right: $\gamma = 0.01$, $\alpha = 0.02$ (full), $\gamma = 0.01$, $\epsilon = 0.03$ (dashed), $\epsilon = 0.03$, $\alpha = 0.02$ (dotted), the straight line shows theory with $A = 12.5$; symbols are shifted for clarity and $\epsilon_s = \omega D_s/\omega_c$.

# Experimental dependence of ZRM minima on $T$ and $j = \omega/\omega_c$

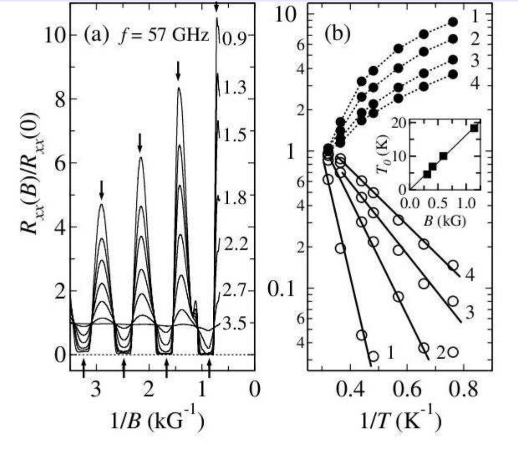



FIG. 3. (a) $R_{xx}(B)/R_{xx}(0)$ under MW ($f = 57$ GHz) illumination, plotted vs $1/B$ at different $T$ from 0.9 to 3.5 K. Upward

Experiment $\rightarrow$ Activation energy

$$E_r \propto \frac{\omega_c}{\omega}$$

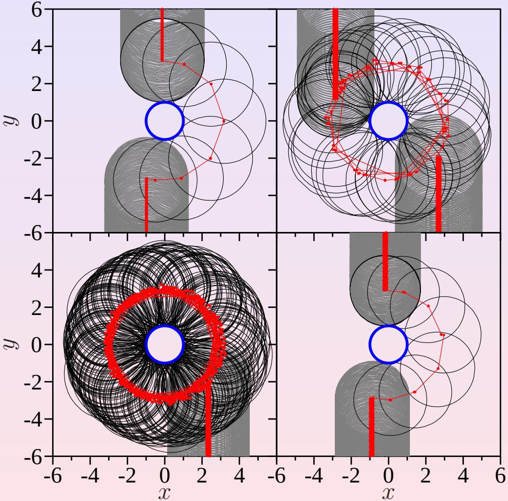
Theory predicts

$$E_r = 16\epsilon E_F \frac{\omega_c}{\omega}$$

For $\epsilon = 0.01$ we obtain

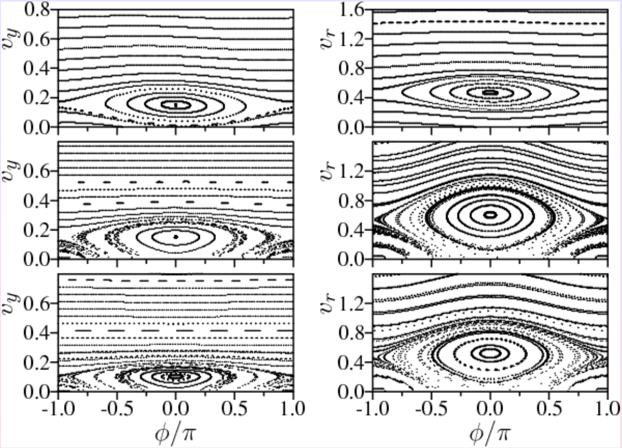
$$E_r \simeq 20 \text{ K}$$

# Microwave induced scattering on disk



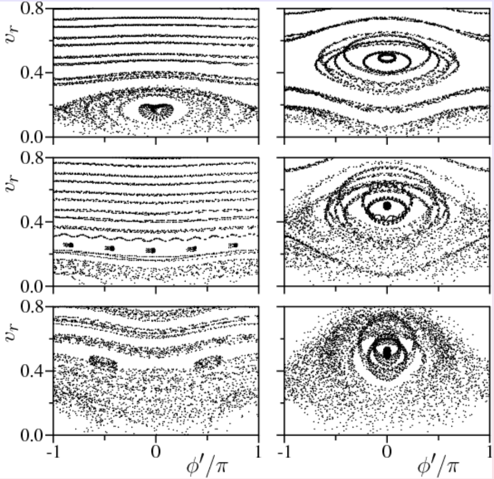

Top left: $\epsilon = 0$, $j = 9/4$;
top right: temporary captured path at $\epsilon = 0.04$, $j = 9/4$;
bottom left: path captured forever at $\epsilon = 0.04$, $j = 9/4$;
bottom right: no capture at $\epsilon = 0.04$, $j = 2$;
dissipation at disk collisions $\gamma_d = 0.01$.

# Poincaré sections for scattering on disk



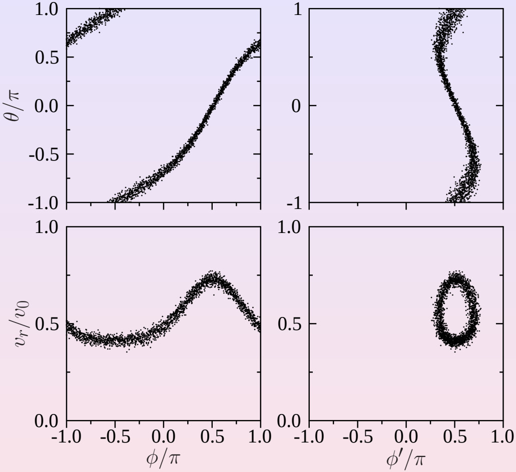

Left: wall model W1,W2; right: disk model DR1 with radial field; $\epsilon = 0.01$, $j = 2.1$ (top, middle) and $j = 3.1$ (bottom). Top panels are obtained with the Chirikov standard map, middle and bottom panels are obtained from Newton equations; no dissipation.

# Poincaré sections for scattering on disk



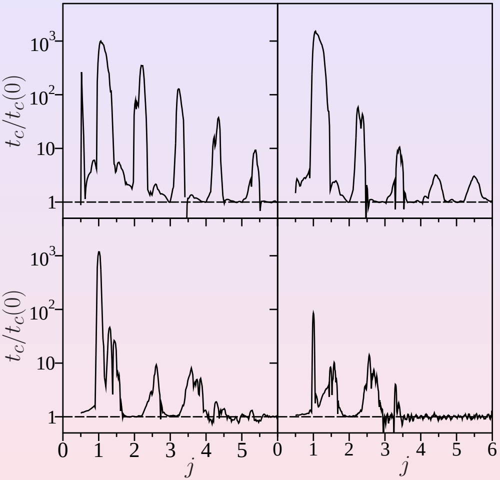

Linearly polarized field. Top: $j = 2.1$ (left), 2.25 (right), $\epsilon = 0.01$.
Middle: $j = 2.75$ (left), 2.25 (right), $\epsilon = 0.02$.
Bottom: $j = 2.75$ (left), 2.25 (right), $\epsilon = 0.04$. No dissipation.

# Phase synchronization at disk collisons



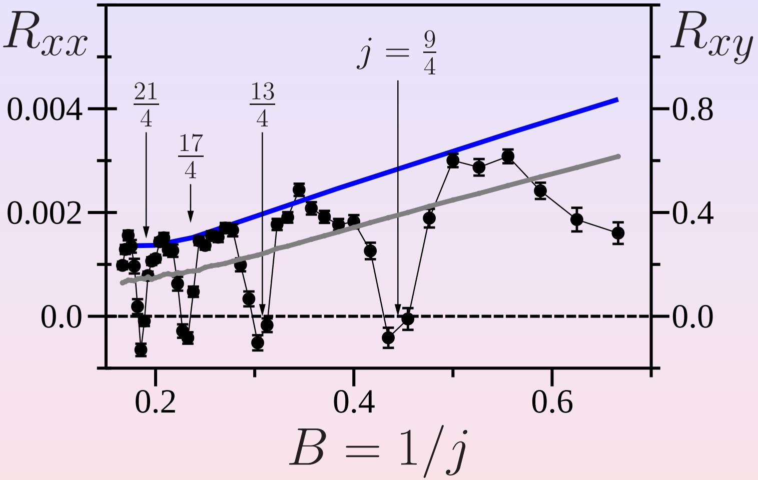

Angle $\theta$ and radial velocity $v_r$ vs. microwave phase $\phi = \omega t$ at the moments of disk collision (left), $v_r$ vs phase $\phi' = \phi - \theta$ (phase in rotating frame) (right) Here $j = 2.25$, $\epsilon = 0.04$, static filed $\epsilon_s = 0.001$, dissipation rate at disk collision $\gamma_d = 0.01$, no noise.

# Capture time on disk



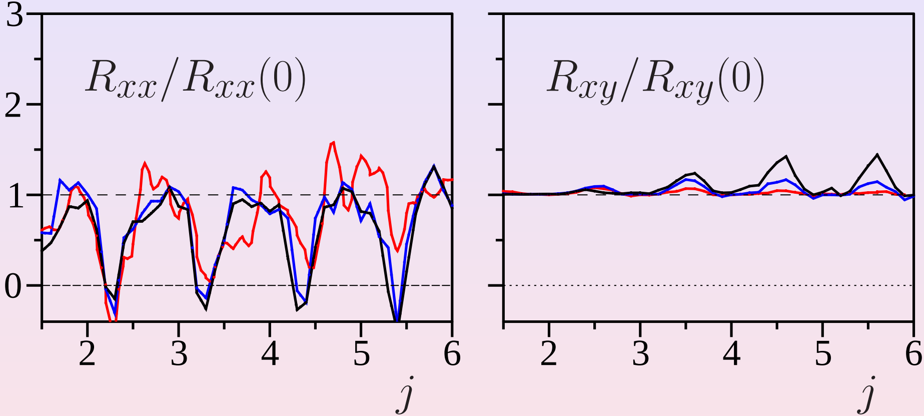

Trapping times at disk as a function of $j$; $t_c$ with microwave, $t_c(0)$ without microwave. Dissipation rate at disk collision $\gamma_d = 0.01$, no noise.

# Scattering on many disks



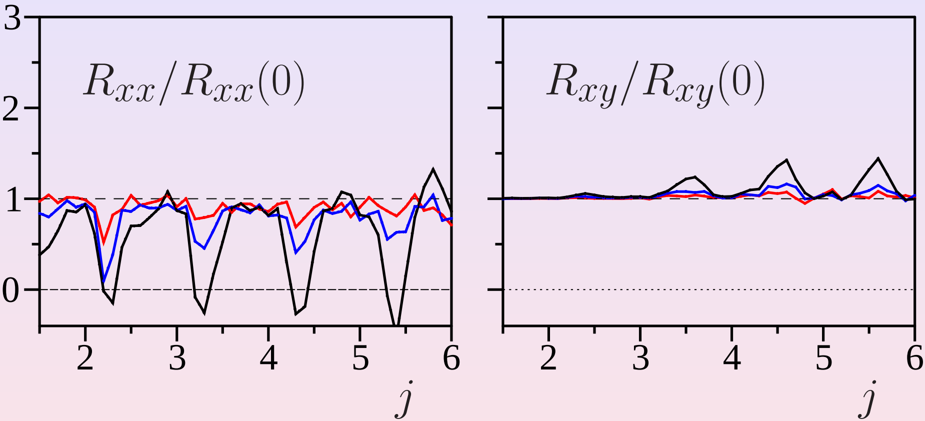

Dependence of $R_{xx}$ and $R_{xy}$ on magnetic field $B = 1/j$ in a.u.. Blue/gray curves show $R_{xx}/R_{xy}$ in a dark. Black curve with points show $R_{xx}$ vs. $B$ at microwave field $\epsilon = 0.04$. Here $\epsilon_s = 0.001$, $\gamma_d = 0.01$, noise amplitude $\alpha_i = 0.005$, $\tau_i = 1/\omega$; averaging over 200 orbits on time $t = 10^6/\omega$.

# Scattering on many disks



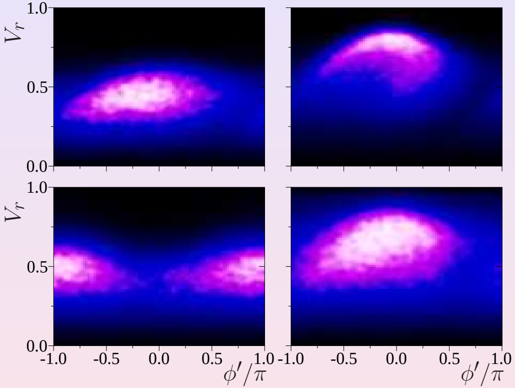

Rescaled $R_{xx}$ and $R_{xy}$ vs. $j$ at noise $\alpha_i = 0.005$ (black), 0.01 (blue), 0.02 (red). Here $\epsilon = 0.04$, $\epsilon_s = 0.001$.

# Scattering on many disks




Rescaled $R_{xx}$ and $R_{xy}$ vs. $j$ at $\epsilon = 0.01$ (red), 0.02 (blue), 0.04 (black); noise $\alpha_i = 0.005$.

# Scattering on many disks




Phase space $(v_r, \phi')$ at disk collisions at $\epsilon = 0.04$: $j = 2.1$ (top left), $j = 2.25$ (top right), $j = 2.75$ (bottom left), $j = 3.25$ (bottom right); $\phi' = \phi - \theta$ (other parameters as in Fig. above).

# Synchronization scales for disk scattering

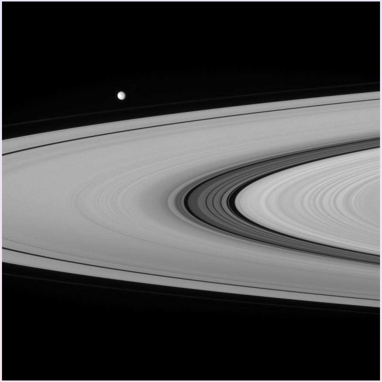
Characteristics of nonlinear resonance:

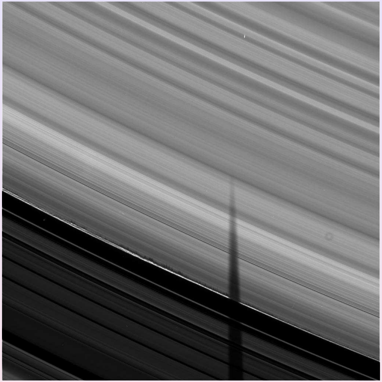
$$
\begin{aligned}
E_r &= 8\epsilon\rho E_F/(\rho + j - 1);\ \rho = 1 + r_c/r_d\ ; \\
v_{res} &= \pi\rho\delta j/(j + \rho - 1);\ \delta v = 4\sqrt{\epsilon\rho/(2(j + \rho - 1))}; \\
\delta j_\epsilon &= \delta v(\rho + j - 1)/(2\pi\rho);\ j = \omega/\omega_c\ ,
\end{aligned}
$$

where $\delta j_\epsilon$ is a shift of resonance produced by a finite separatrix half width $\delta v/2$. For our numerical simulations we have $\rho = 1 + r_c/r_d = 1 + j$ with $v_{res} = \pi(j + 1)\delta j/(2j)$, $\delta v = 2\sqrt{\epsilon(j + 1)/j}$ and $\delta j_\epsilon = (2/\pi)\sqrt{\epsilon j/(j + 1)}$.

Quantum regime: magnetic length $r_c/\sqrt{n_L}$ gives effective disk radius $r_d$ ?

# Rings of Saturn analogy





Images of Cassini-Huygens mission: Rings of Saturn and Mimas (left), mountains at the edge of size of Mont Blanc (right). The edge of B ring is of $10m$ width on a distance of $117580km$, ring width of $30m$, Cassini Divison of $4620km$, ice clumps of $10m$ size. Synchronization mechanism of edge formation DS, Pikovsky, Schmidt, Spahn MNRAS 395, 1934 (2009).

# Conclusions

- Microwaves can stabilize edge trajectories against small angle disorder scattering
- Microwaves creats trapping around impurities and gives practically zero displacement along static field with $R_{xx} \to 0$
- Nonlinear resonance described by the Chirikov standard map $\to$ at $j$ resonances
- Importance of relaxation processes to the Fermi surface
- Nonlinear resonance width $\to$ activation energy
- Microscopic theory for relaxation to the Fermi surface ?
- Extension to describe other low magnetic field resistance oscillations (ZDRS, PIRO, ...) ?
- Experiments with electrons on helium surface
  $\to$ laboratory modelling of rings of Saturn ?
- Towards Quantum synchronization theory for scattering on impurities, edges in presence of microwave field