Chaotic enhancement of dark matter density in binary systems and galaxies

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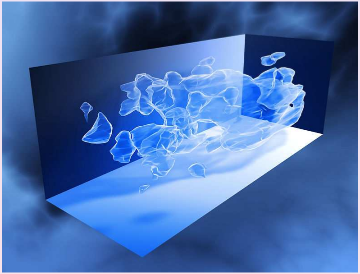
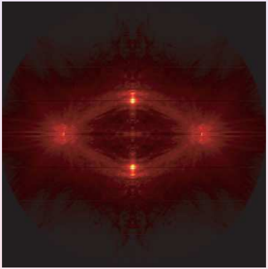
I.B.Khriplovich, DS arXiv:0906.2480[astro-ph] (2009)
J.Lages, DS MNRAS Lett. **430**, L25 (2013)
G.Rollin, J.Lages, DS arXiv:1211.0903[astro-ph.EP] (2014)





* capture of dark matter in the Solar system
* chaotic dynamics of dark matter in the Solar system, binary systems, galaxies
* parameters $\rho_g \sim 4 \cdot 10^{-25} g/cm^3$, $u \sim 220 km/s$,
  $f(v)dv \sim 4(v^2 dv/u^3) exp(-3v^2/2u^2)$

Halley comet map

Chirikov, Vecheslavov (1988-89): 46 appearances from history/numerics
map description of comet dynamics; Petrosky (1986)

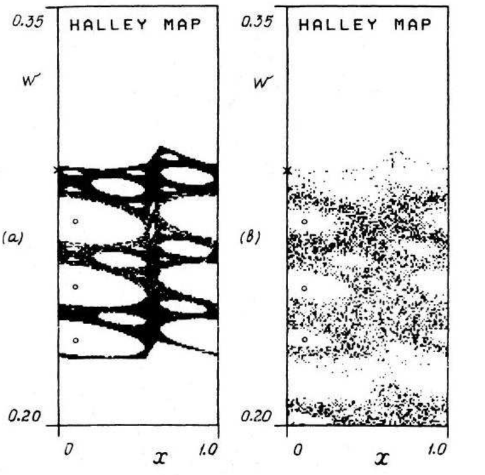


Fig. 3a and b. Phase trajectory of map (3) in the STA (6). Initial conditions (crosses) $w_1 = 0.29164$; $x_1 = 0$ (in 1986, see Table 1): **a** Jupiter's perturbation only, $N = 1.5\ 10^5$ iterations; **b** perturbation by both Jupiter and Saturn, $N = 4000$

Halley map ($w = -2E$)

$\bar{w} = w + F(x)$,

$\bar{x} = x + \bar{w}^{-3/2}$;

$x = t/T_J$ Jupiter phase at perihelion,

$F_{max} \sim 5M_J/M_S = 5m_p/M$; $q \leq 4r_p$

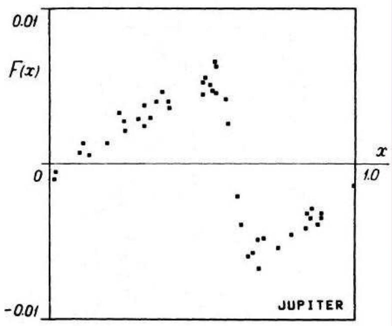


Fig. 1. The full perturbation of comet Halley vs. Jupiter's phase

diffusive life time: 10^7 years

Capture of dark matter by the Solar system

capture => inverse process to ionization

Let us now estimate the capture cross-section σ assuming that for all DMPs the dynamics is described by the Kepler map with fixed $\beta \sim 1$. Then only DMPs with energies $|w| = v^2 r_p / k m_p M = v^2/v_p^2 < \beta m_p/M$ are captured under the condition that $q < r_p$ (here v_p is the velocity of the planet). The value of q can be expressed via the DMP parameters at infinity, where its velocity is v and its impact parameter is r_d, and hence $q = (v r_d)^2/2kM$.[13] Since $q \sim r_p$ we obtain the cross-section

$$\sigma \sim \pi r_d^2 \sim \frac{2\pi k M r_p}{v^2} \sim 2\pi r_p^2 \left(\frac{v_p}{v}\right)^2 \sim \frac{2\pi r_p^2 M}{\beta m_p}, \tag{9}$$

where the last relation is taken for those typical velocities, $v^2 \sim \beta v_p^2 m_p/M$, at which the capture of DMPs takes place (for $q \approx 1.4 r_p$ we have $\beta \approx 5$). Then Eqs. (4) and (9) give the captured mass Δm_p of (7) with an additional numerical factor $\beta \sim 1$.

According to the above estimates, DMPs captured by Jupiter have typical velocities at infinity $v \sim (\beta m_p/M)^{1/2} v_p \sim 1\,\mathrm{km/s}$ for typical $\beta \sim 5$ and $m_p/M \approx 10^{-3}$, $v_p \approx 13\,\mathrm{km/s}$. This value of v is in good agreement with the numerical simulations of Ref. 5, which give typical captured DMP velocities for Jupiter of $1\,\mathrm{km/s}$.

Khriplovich, DS (2009)

Dark matter dynamical map

dynamics after capture

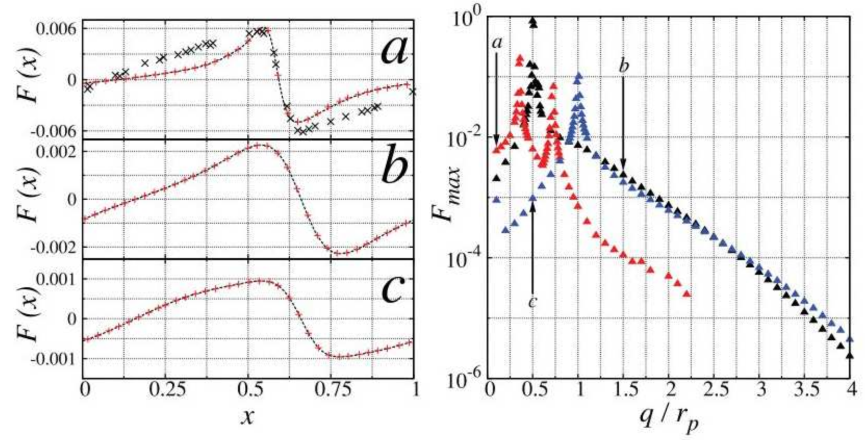
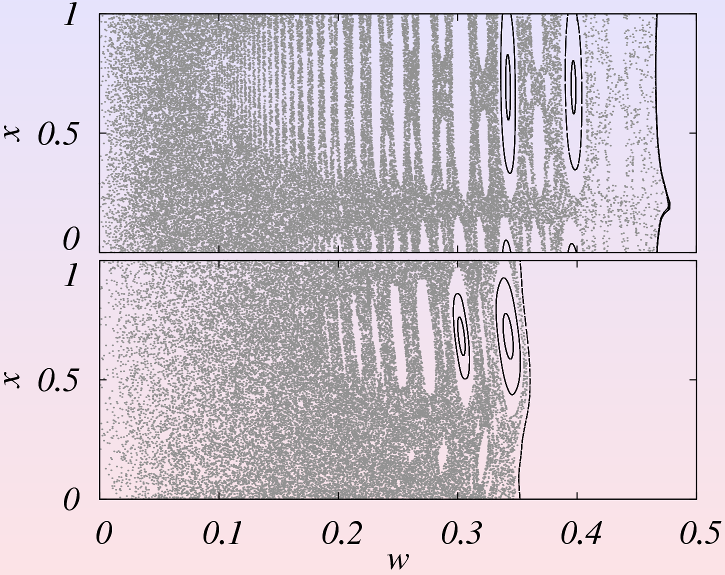


Figure 1. Left-hand panel: dependence of the kick function $F(x)$ on Jupiter phase x for DMP orbit parameters shown by pluses: (a) $q = 0.11$, $\theta = 2.83$, $\varphi = 1.95$ of the Halley comet case; here the crosses show data for the Halley comet with all SS planets taken from fig. 1 of Chirikov & Vecheslavov

Poincaré sections



top: dark map for Halley comet (fig1a); bottom: Kepler map $J = 0.007$

Capture cross section

capture mechanism

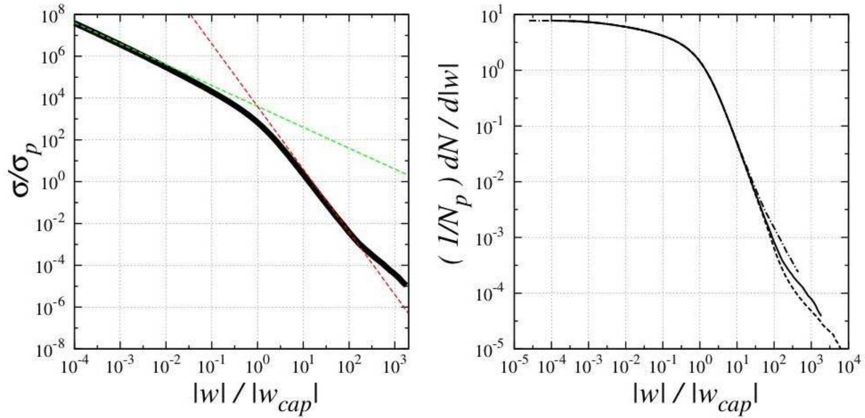


FIG. 2: Left panel: Capture cross section σ as a function of the rescaled DMP energy at infinity $|w|/|w_{cap}|$. The parameter $\sigma_p = \pi r_p^2$ is the area of the circular Jovian orbit. The slopes of the dashed lines are -1 and -3 in log-log scale. Right panel: $dN/d|w|$ as a function of captured DMP energy $|w|$. dN is the number of captured DMPs coming from infinity with energy $|w|$ in the interval $d|w|$. The parameter N_p is the typical number of DMPs passing through the planet orbit in the absence of any Keplerian potential. Data are shown for three different planets: Jupiter $w_{cap} \sim M_{♃}/M_{\odot} \simeq 9.5 \times 10^{-4}$ (solid line), Saturn $w_{cap} \sim M_{♄}/M_{\odot} \simeq 2.8 \times 10^{-4}$ (dashed line), and a fictitious planet $w_{cap} \sim M_4/M_{\odot} \simeq 4 \times 10^{-3}$ (dot-dashed line).

Distribution of captured dmp

distribution of captured dmp

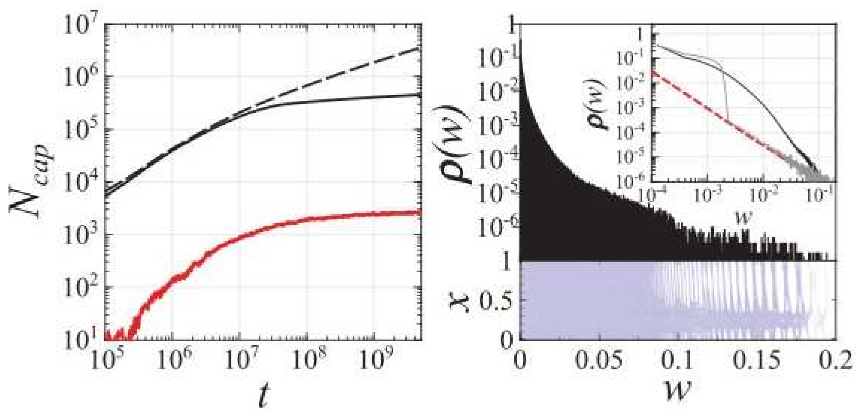


Figure 3. Left-hand panel: the number N_{cap} of captured DMP, as a function of time t in years, for energy range $w > 0$ (dashed curve), $w > 4 \times 10^{-5}$ corresponding to half distance between Sun and Alpha Centauri System (black curve), $w > 1/20$ corresponding to $r < 100$ au (red curve); DMP are injected at constant flow $f(v)$ at all angles. Right-hand panel: the top part

Surface density of dark matter in the Solar system

space density of dmp, radial dependence

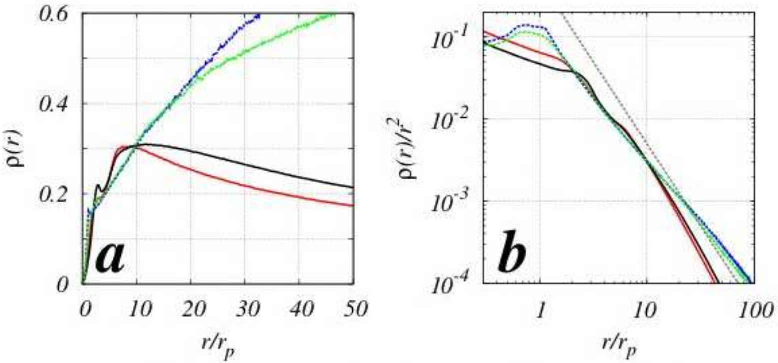
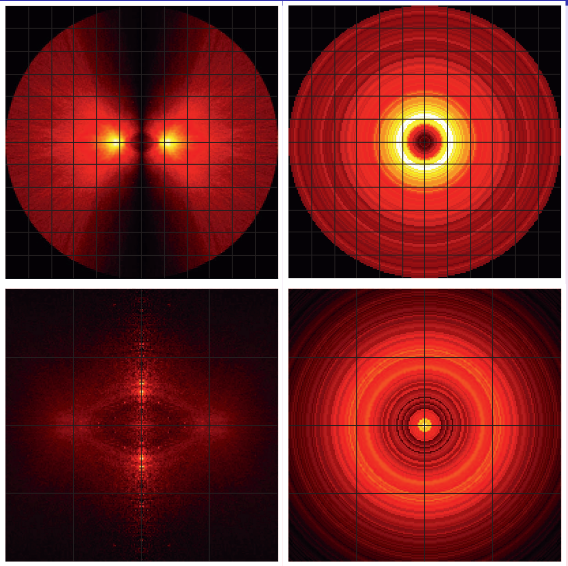


Figure 4. (a) Stationary radial density $\rho(r) \propto dN/dr$ from the Kepler map at $J = 0.005$ with $u = 17$ at time t_S (red curve) and $u = 0.035$ at time $t_u \approx 4 \times 10^8 T_p$(black curve); data from the dark map at $m_p/M = 10^{-3}$ are shown by the blue curve at $u = 17$ and time t_S for the Sun-Jupiter case, and by the green curve at $u = 0.035$ and t_S for the SMBH; the normalization is fixed as $\int_0^{6r_p} \rho dr = 1$, $r_p = 1$. *(b)* Volume density $\rho_v = \rho/r^2$ from

Space density of dark matter in binary system



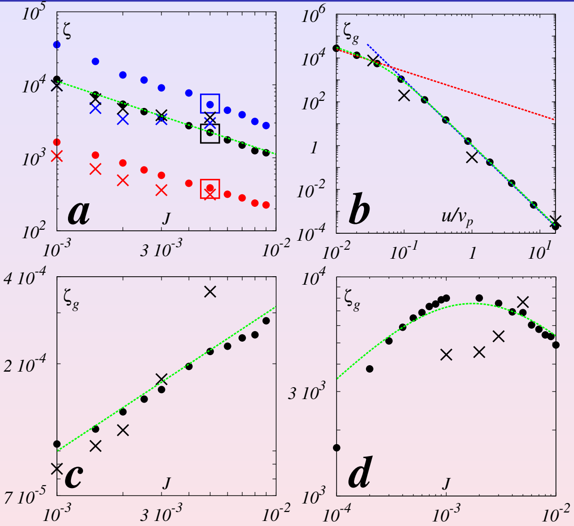
$u = 0.035$ after t_S, $m_p/M = 1/1000$; top: $dN/dzdr_\rho$, bottom: dN/dV, planes $x = 0 (r < 6)$ (left), $z = 0 (r < 2)$ (right)

Chaotic enhancement

Lages and Shepelyansky (2013). We compute the total mass of DMP flow crossing the range $q \leqslant 4r_p$ during time t_S: $M_{tot} = \int_0^\infty dv v f(v) \sigma \rho_g t_S \approx 35 \rho_g t_S k r_p M/u$ where we use the cross-section $\sigma = \pi r_d^2 = 8\pi k M r_p / v^2$ for injected orbits with $q \leqslant 4r_p$, $w = v^2$, k is the gravitational constant. For SS at $u/v_p \approx 17$ we have $M_{tot} \approx 0.5 \cdot 10^{-6} M$.

From the numerically known fractions η_{ri} of previous Section and the fraction of captured orbits $\eta_{AC} = N_{AC}/N_{tot}$ we find the mass $M_{ri} = \eta_{ri} \eta_{AC} M_{tot}$ inside the volume $V_i = 4\pi r_i^3/3$ of radius $r < r_i$ ($r_i = 0.2r_p; r_p; 6r_p$). Here N_{tot} is the total number of injected orbits during the time t_S while the number of orbits injected in the range $|w| < J$ (only those can be captured) is $N_J = N_{tot} (\int_0^J dw f(w)/w)/(\int_0^\infty dw f(w)/w)$. For $J \ll u^2$ we have $\kappa = N_{tot}/N_J = 2u^2/(3J) \approx 3.8 \times 10^4$ for $u/v_p = 17$ and $\kappa = 1$ for $u/v_p = 0.035$ at $J = 0.005$. Thus for $u/v_p = 17$ the number of orbits, injected at $0 < |w| < J$, $N_J = 4 \times 10^{11}$ corresponds to the total number of injected orbits $N_{tot} \approx 1.5 \times 10^{16}$. Finally we obtain the global density enhance-

Chaotic enhancement



(a) $\zeta = \rho_v(r_i)/\rho_{gJ}$, $u/v_p = 17$ (Jupiter), ρ_{gJ} is galactic density for $0 < |w| < J$, $r_i/r_p = 0.2, 1, 6$; (b) $\zeta_g = \rho_v(r_i)/\rho_g$, Kepler map at $J = 0.005$, lines $1/u; 1/u^3$; (c) $\zeta_g(J)$ at $u/v_p = 17$; (d) same as (c) at $u/v_p = 0.035$. Here $J = 5m_p/M$.

Formula of chaotic enhancement

All these results can be summarized by the following formula for chaotic enhancement factor of DMP density in a binary system:

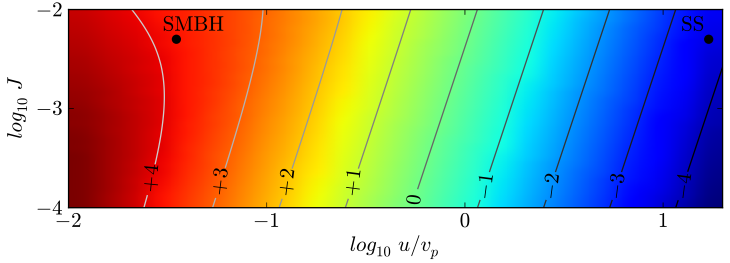
$$\zeta_g = A\sqrt{J}(v_p/u)^3/[1 + BJ(v_p/u)^2] \, , J = 5m_p/M \, . \qquad (3)$$

Here ζ_g is given for DMP density at $r_i = r_p$ and $A \approx 15.5$, $B \approx 0.7$. This formula gives a good description of numerical data of Fig. 6. For $u^2 \gg J$ we have $\zeta_g \ll 1$ but we still have enhancement $\zeta = 0.72\zeta_g(u/v_p)^3/J^{3/2} \approx 0.72A/J \gg 1$. The color representation of dependence (3) is shown in Fig. 7.

Formula of chaotic enhancement

ple estimates. The total captured mass $M_{cap} \approx M_{AC}$ is accumulated during the diffusive time t_d and hence $M_{cap} \sim v_p^2 J t_d M_{tot}/(\pi u^2 t_S) \sim \rho_g \tau_d J (v_p/u)^3$ where $\tau_d = t_d/T_p$ and we omit numerical coefficients. This mass is concentrated inside a radius $r_{cap} \sim 1/J$ so that at $r \sim 1/J$ the volume density is $\rho_v(r = 1/J) \sim M_{cap}/r_{cap}^3 \sim \rho_g J^2 w_{ch}^2 (v_p/u)^3 \sim \rho_{gJ} J^{1/2} w_{ch}^2 \sim \rho_{gJ} J^{1.3}$, where we use a relation $\tau_d \sim w_{ch}^2/J^2 \sim 1/J^{6/5}$. (Our modeling of injection process in the Kepler map with a constant injection flow in time, counted in number of map iterations, indeed, shows that the number of absorbed particles scales as $N_K \sim \tau_d \sim J^{-6/5}$ at small J.) It is important to stress that $\rho_v(r = 1/J) \ll \rho_{gJ}$ in contrast to a naive expectation that $\rho_v(r = 1/J) \sim \rho_{gJ}$. Using our empirical density decay $\rho_v \propto 1/r^\beta$ with $\beta \approx 2.25$ for the Kepler map we obtain $\zeta \propto 1/J^{0.95}$ being close to the dependence $\zeta \sim 1/J$ and $\zeta_g \sim J^{1/2}/(u/v_p)^3$ from (3) at $u^2 \gg J$. For the dark map we have $\beta \approx 1.5$ but $w_{ch} \sim const$ due to sharp variation of $F(x)$ with x that again gives $\zeta \sim 1/J$. We think that it is difficult to obtain exact analytical derivation of the relation $\zeta \sim 1/J$ due to contributions of different q values (which have different τ_d) and different kick shapes in (1) that affect τ_d and the structure of chaotic component. In the regime $(u/v_p)^2 \ll J$ all energy range of scattering flow is absorbed by one kick and M_{cap} is increased by a factor $(u/v_p)^2/J$ leading to increase of ζ_g by the same factor giving $\zeta_g \propto v_p/(u\sqrt{J})$ in agreement with (3).

Chaotic enhancement factor



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