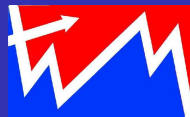
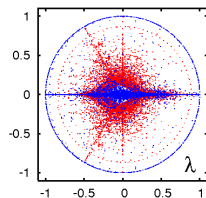
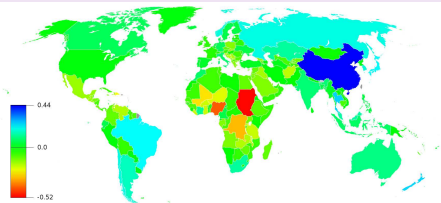
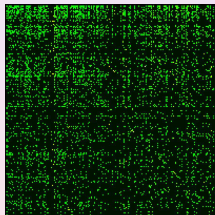


# Influence of petroleum and gas trade of Russia on EU economies from the Google matrix analysis of UN COMTRADE data

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with S.Coquide, J.Lages (U Besancon), L.Ermann (CNEA TANDAR)

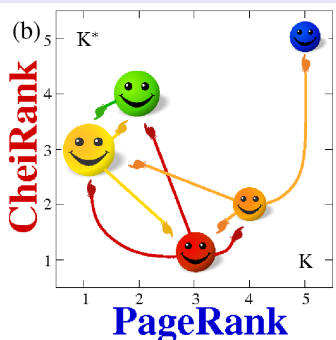
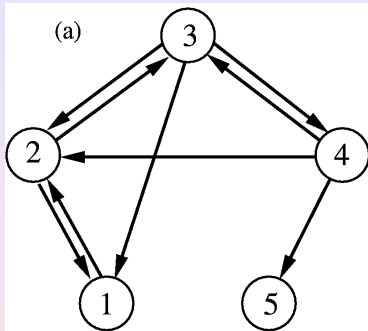


- \* Markov (1906) → Brin and Page (1998) → Google matrix and search engines
- \* Applications: multiproduct world trade networks (UN COMTRADE), Wikipedia Ranking of World Universities (WRWU) ...

Support: EC FET Open NADINE (2012-2015), APLIGOOGLE-CNRS (2016-2017), LABEX NEXT (2017-2020) + thanks to UN COMTRADE

# Google matrix construction rules

Markov chains (1906) and Directed networks



For a directed network with  $N$  nodes the adjacency matrix  $\mathbf{A}$  is defined as  $A_{ij} = 1$  if there is a link from node  $j$  to node  $i$  and  $A_{ij} = 0$  otherwise. The weighted adjacency matrix is

$$S_{ij} = A_{ij} / \sum_k A_{kj}$$

In addition the elements of columns with only zeros elements are replaced by  $1/N$ .

# Google matrix construction rules

## Google Matrix and Computation of PageRank

$\mathbf{P} = \mathbf{S}\mathbf{P} \Rightarrow \mathbf{P}$  = stationary vector of  $\mathbf{S}$ ; can be computed by iteration of  $\mathbf{S}$ .

To remove convergence problems:

- Replace columns of 0 (dangling nodes) by  $\frac{1}{N}$ :

$$\mathbf{S} = \begin{pmatrix} 0 & 1/2 & 1/3 & 0 & 1/5 \\ 1 & 0 & 1/3 & 1/3 & 1/5 \\ 0 & 1/2 & 0 & 1/3 & 1/5 \\ 0 & 0 & 1/3 & 0 & 1/5 \\ 0 & 0 & 0 & 1/3 & 1/5 \end{pmatrix} \quad \mathbf{S}^* = \begin{pmatrix} 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/3 & 0 & 1 & 0 \\ 0 & 1/3 & 1/2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- To remove degeneracies of  $\lambda = 1$ , replace  $\mathbf{S}$  by **Google matrix**

$$\mathbf{G} = \alpha \mathbf{S} + (1 - \alpha) \frac{\mathbf{E}}{N}; \quad \mathbf{G}\mathbf{P} = \lambda \mathbf{P} \Rightarrow \text{Perron-Frobenius operator}$$

- $\alpha$  models a random surfer with a random jump after approximately 6 clicks (usually  $\alpha = 0.85$ ); **PageRank vector**  $\Rightarrow \mathbf{P}$  at  $\lambda = 1$  ( $\sum_j P_j = 1$ ).
- **CheiRank vector  $\mathbf{P}^*$** :  $\mathbf{G}^* = \alpha \mathbf{S}^* + (1 - \alpha) \frac{\mathbf{E}}{N}$ ,  $\mathbf{G}^* \mathbf{P}^* = \mathbf{P}^*$   
( $\mathbf{S}^*$  with inverted link directions)  $\rightarrow \mathbf{K} - \mathbf{K}^*$  **PageRank-CheiRank index**  
Chepelianskii arXiv:1003.5455 (2010) ...

# Computation algorithms

## \* PageRank vector by power iteration:

multiplication of initial random vector by  $G$  matrix; convergence to  $\lambda = 1$  eigenvector as  $\alpha^t$ , about  $t = 200$  iterations are enough for double precision convergence (all eigenvalues have  $|\lambda| \leq \alpha < 1$  except  $\lambda = 1$ ); on average there are only about 10-20 nonzero links for each node (about 20 multiplications of vector by a line of matrix)

→ small-world structure of real networks or six degrees of separation (Milgram Psychology Today (1967));

\* **Arnoldi algorithm**: eigenvalues with largest  $|\lambda|$  and related selected eigenvectors corresponding to quasi-isolated communities.

\* **REGOMAX Reduced Google matrix**: description of interactions of subset of selected nodes in a huge network

\* **GPU codes for reduced Google matrix**: 100 times acceleration compared to one-processor computer; collaboration with

Denis Demidov (Russian Academy of Sciences, Kazan; see <https://github.com/ddemidov> for GPU oriented codes)

What is the central bank of Wikipedia ?

by D.Demidov, K.M.Frahm, DS Physica A v.542, p.123199 (2020)

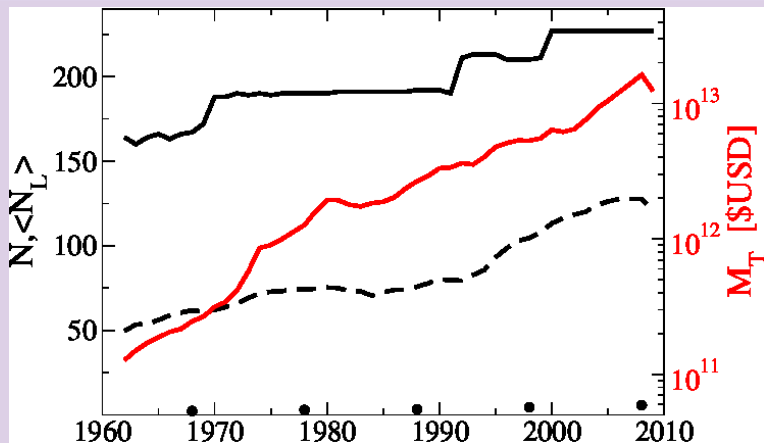
# Directed networks analyzed

- \* **Wikipedia editions:** EN (2009)  $N = 3282257$ ;  
24 editions Wiki2013:  $N = 4212493$  EN,  $N = 1532978$  DE,  $N = 1352825$  FR  
24 editions Wiki2017:  $N = 5416537$  EN,  $N = 2057898$  DE,  $N = 1866546$  FR
- \* **Entier Twitter (2009):**  $N = 41$  millions
- \* **Entier Phys. Rev. citation network(1893-2009):**  $N = 460422$
- \* **World Trade Network (WTN) from UN COMTRADE about 50 years:**  $N = 227$   
for all commodities; multiproduct trade with 61 products  $N = 13847$ ; available  
with 5000 products and  $N \approx 1$  million
- \* **Bitcion network transactions (beginning 2009 till April 2013):**  $N = 6297009$
- \* **Linux Kernel network:**  $N = 285509$
- \* **UK university networks till 2006:** U Oxford, Cambridge  $N \approx 200000$
- \* **Network of protein-protein interactions for cancer:**  $N \approx 3000$

see Ermann, Frahm, DS Rev Mod Phys (2015)

ALL PUBS: <http://www.quantware.ups-tlse.fr/dima/subjgoogle.html>

# World trade network (WTN) of United Nations COMTRADE 1962-2010

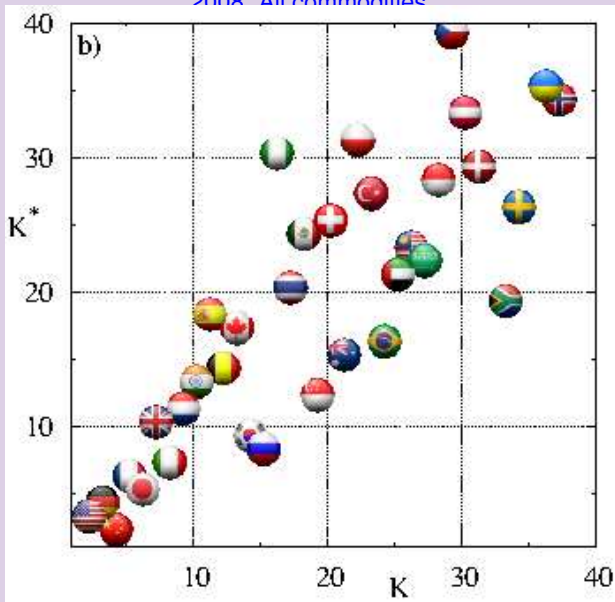


Number of countries (black), links (dashed/points) and mass volume in USD (red)

Leonardo Ermann, DS arxiv:1103.5027 (2011); EPJB (2015)

# Ranking of World Trade

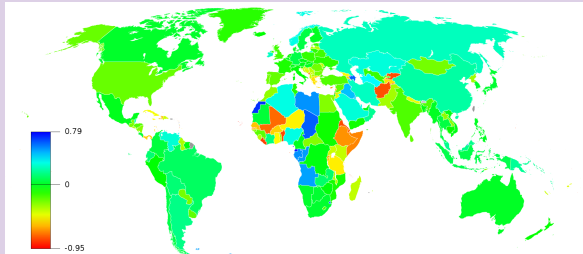
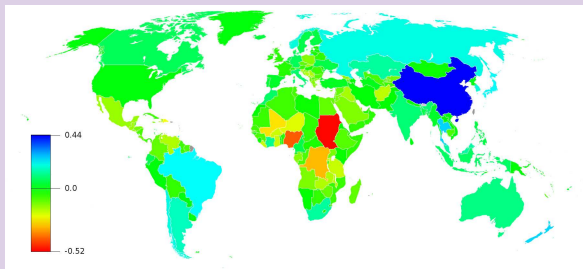
2008: All commodities



# CheiRank-PageRank balance (2008)

$B_c = (P_c^* - P_c)/(P_c^* + P_c)$  (top - CheiRank-PageRank;  
bottom -Export-Import volume; multiproduct world trade

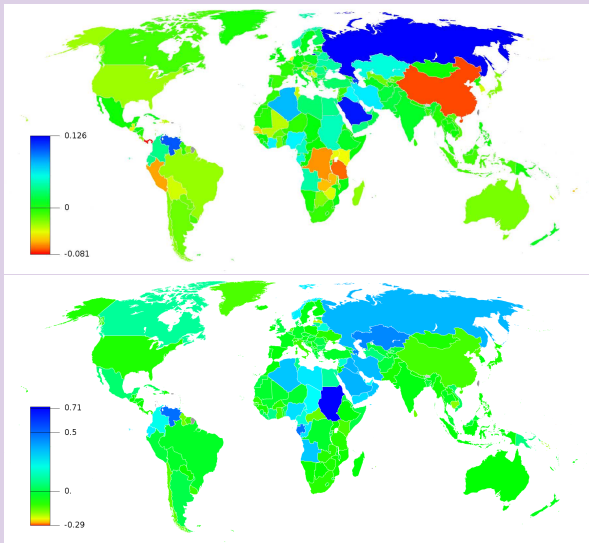
$N_c = 227$  countries,  $N_p = 61$  products,  $N = 13847$  ==> UN COMTRADE)





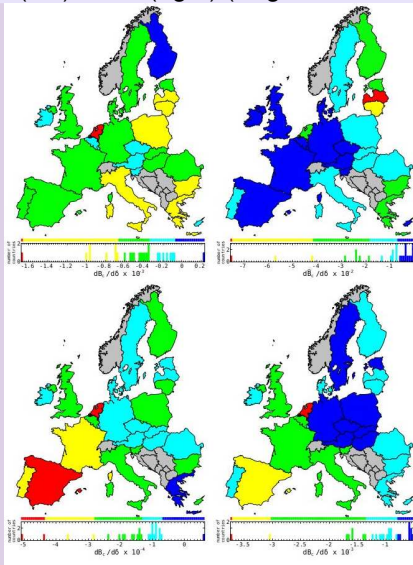
# Sensitivity to petroleum price (2008)

$$B_c = (P_c^* - P_c) / (P_c^* + P_c), \text{ color} \Rightarrow dB_c / d\delta_{\text{petroleum}}$$

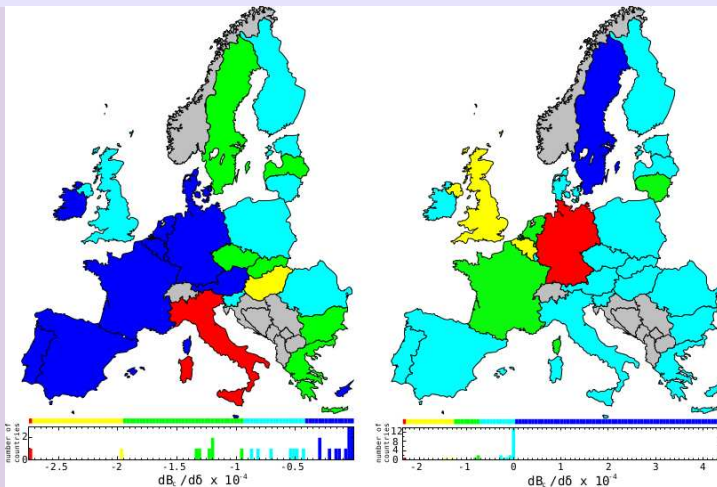


# EU sensitivity to petroleum price (2016)

Top: RU (Page-Chei-Rank - left); RU (Import-Export - right)  
Bottom: Saudi Arabia (left), USA (right) (Page-Chei-Rank)

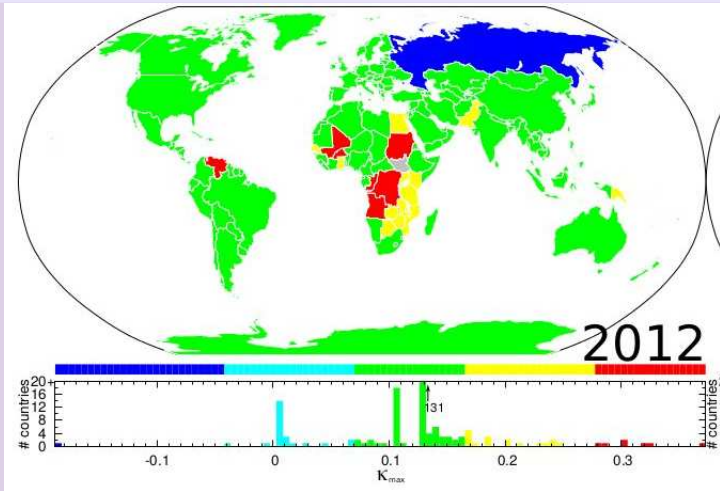


# EU sensitivity to gas price (2016)



Balance derivative  $dB_c/d\delta$  from gas price increase from Russia (left), Norway (right)

# Model of crisis contagion in the world trade network of (UN COMTRADE 2012)



Blue - stable; red - bankruptcy

# Main references

- Ref.1 L.Ermann and D.L.Shepelyansky, "Google matrix of the world trade network", Acta Physica Polonica A v.120(6A), pp. A158-A171 (2011)
- Ref.2 L.Ermann and D.L.Shepelyansky, "Google matrix analysis of the multiproduct world trade network", Eur. Phys. J. B v.88, p.84 (2015)
- Ref.3 C.Coquide, L.Ermann, J.Lages and D.L.Shepelyansky, "Influence of petroleum and gas trade on EU economies from the reduced Google matrix analysis of UN COMTRADE data", Eur. Phys. J. B v.92, p.171 (2019)
- Ref.4 C.Coquide, J.Lages and D.L.Shepelyansky, "Crisis contagion in the world trade network ", submitted to Springer Series 17 Feb (2020) (arXiv:2002.07100[q-fin.TR])

Articles available at

<http://www.quantware.ups-tlse.fr/dima/subjgoogle.html>