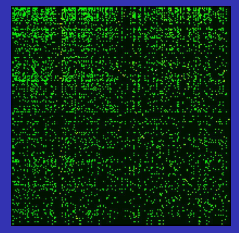
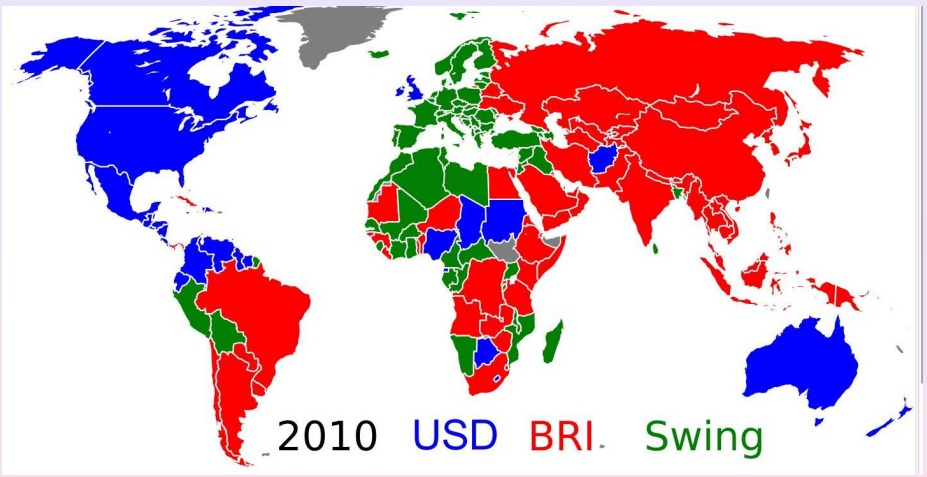
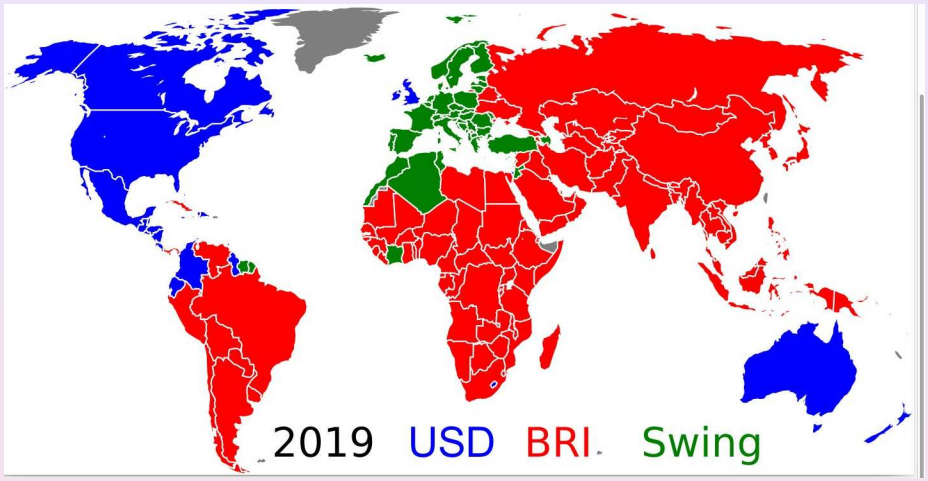
Opinion formation in the World Trade Network

Dima Shepelyansky (Lab Phys Theor CNRS, Toulouse)
www.quantware.ups-tlse.fr/dima



with C.Coquide, J.Lages (U at Besancon) and K.Frahm (LPT)



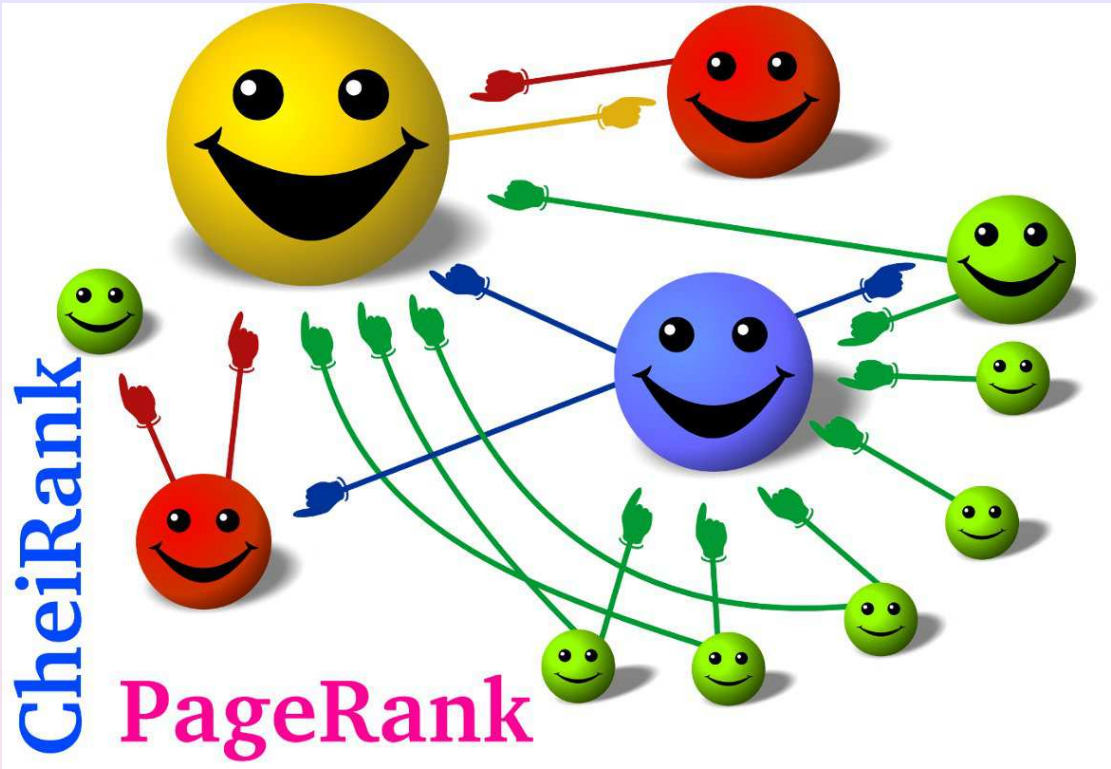


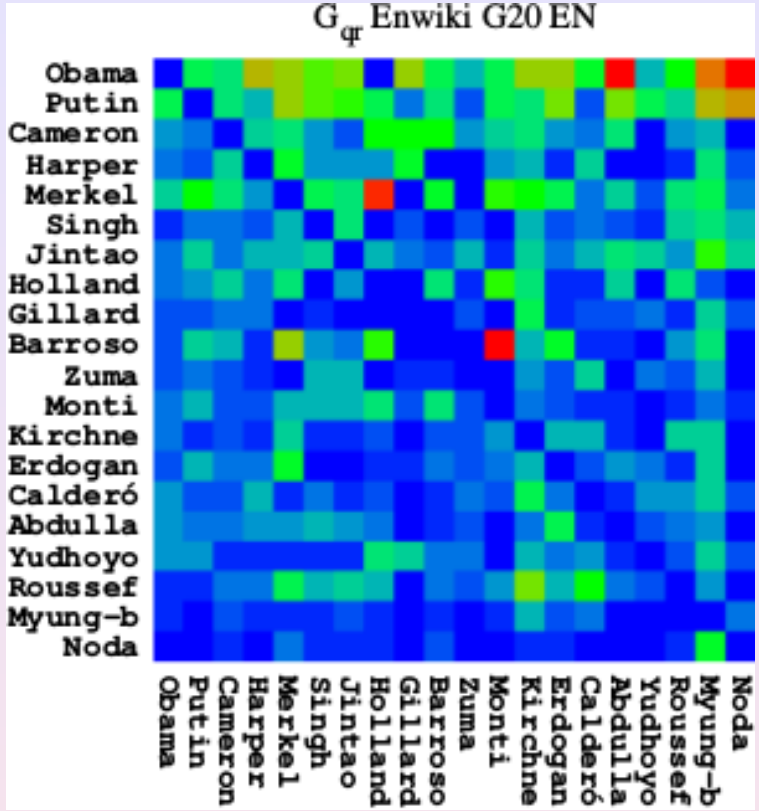
* Markov chains (1906) → Brin and Page (1998) → PageRank algorithm for WWW
* World Trade Network (WTN) from UN COMTRADE database
* Opinion formation in WTN: currency battle of USD, hypothetical BRICS (BRI), EURO
* Reduced Google matrix (REGOMAX) of directed networks (brief introduction)

"War is too important to be left to the generals" G.Clemenceau
"Trade is too important to be left to the economists"

Support: ANR FR NANOX-MTDINA

(1906) Markov vs Wigner (1955)





1945: Nuclear physics $\rightarrow$ Wigner (1955)$\rightarrow$ Random Matrix Theory
1991: WWW, small world social networks $\rightarrow$ Markov (1906) $\rightarrow$ Google matrix;
reduced Google matrix (REGOMAX) - Wikipedia-G20

Despite the importance of large-scale search engines on the web, very little academic research has been done on them.

S.Brin and L.Page, Comp. Networks ISDN Systems **30**, 107 (1998)

Import and Export transfers between countries

194 countries for 2010-2020 years

trade between $N = 194$ countries over the decade 2010–2020. We use the aggregated money matrix element $M_{cc'}$ which gives the volume of goods, expressed in USD, exported from the country c' to the country c during a given year. The total volume of commodities imported by and exported from the country c is then $M_c = \sum_{c'} M_{cc'}$ and $M_c^* = \sum_{c'} M_{c'c}$, respectively. The total volume of goods exchanged in a given year is $M = \sum_c M_c = \sum_c M_c^*$. The quantities

$$S_{cc'} = \frac{M_{cc'}}{M_{c'}^*} \quad \text{and} \quad S_{cc'}^* = \frac{M_{c'c}}{M_{c'}} \tag{1}$$

give, respectively, the fraction of the total volume exported from the country c', which is effectively imported by the country c, and the fraction of the total volume imported by the country c', which is effectively exported from the country c. These quantities measure the relative importance of the country c in the exports and the imports of the country c'. The ImportRank and the ExportRank, i.e.,

$$P_c = \frac{M_c}{M} \quad \text{and} \quad P_c^* = \frac{M_c^*}{M} \tag{2}$$

Opinion formation concept for WTN

* Spin systems: neighbours of local spin determine its orientation (your friends influence your opinion)
* Two specific groups with fixed opinion (Ising spin up or down)
* Other spins/countries have random initial opinions (trade currency preference TCP)
* Asynchronous Monte Carlo Determination of Trade Currency Preferences (spin σ_c takes the sign of Z_c); convergence to steady-state

For this country, we compute the following TCP score

$$Z_c = \sum_{c' \neq c} \sigma_{c'} (S_{c'c} + S^*_{c'c}) \frac{(P_{c'} + P^*_{c'})}{2}, \qquad (3)$$

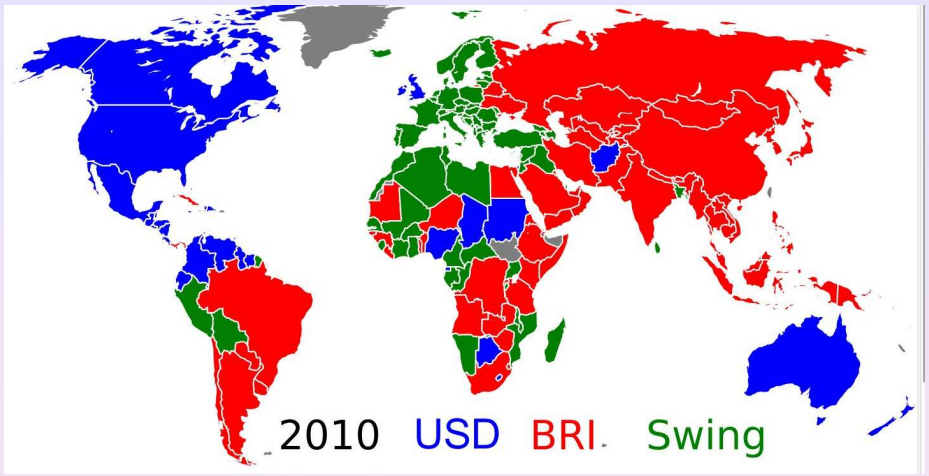
where the sum is performed over all the countries c', which are economical partners

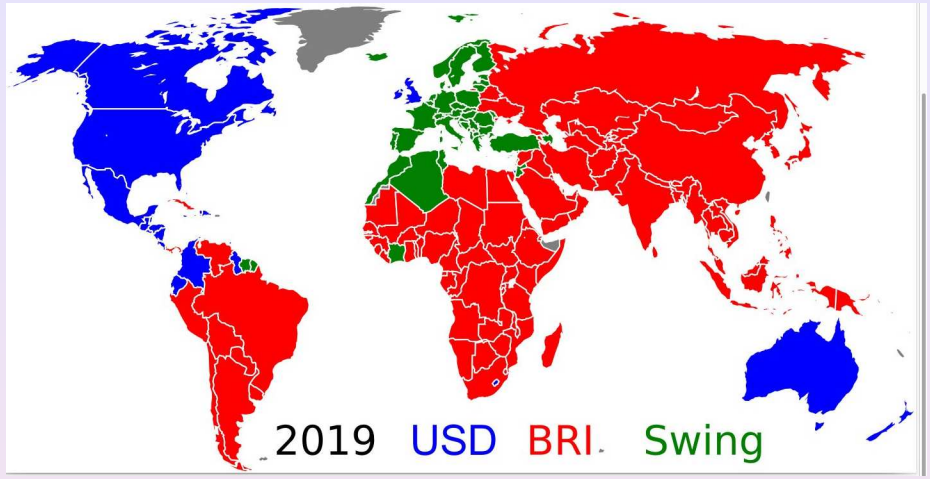
* Certain similarity with associative memory formation model (Hopfield PNAS **79(2)**, 2554 (1982))
* One can use $S_{cc'}$, $S^*{}_{cc'}$ and P_c, $P^*{}_c$ from WTN Google matrix (without significant difference)

Core groups of fixed opinions in WTN

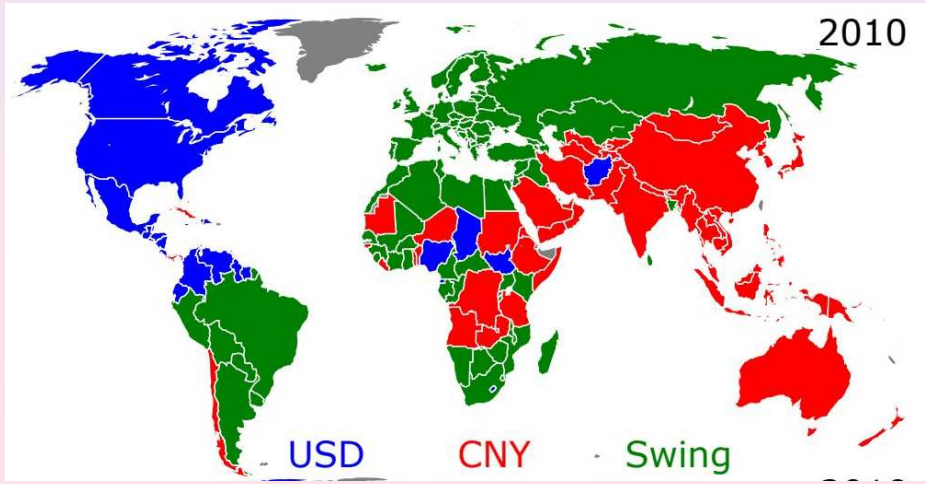
Core Group	Country Name	Currency
Anglo-Saxon	United States of America United Kingdom Canada Australia New Zealand	USD
BRICS+	China India Russia United Arab Emirates Brazil Saudi Arabia South Africa Argentina Egypt Iran Ethiopia	BRI
EU9	Germany France Netherlands Italy Belgium Spain Austria Portugal Luxembourg	EUR

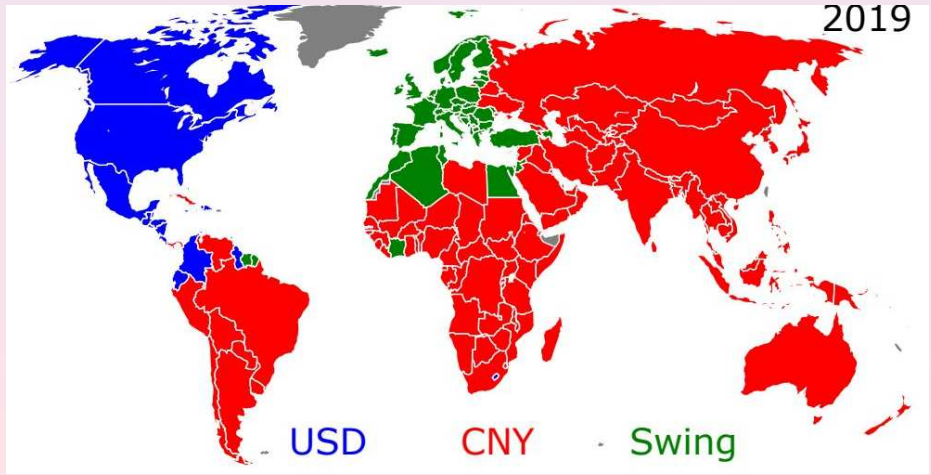
TCP of Anglo-Saxon USD vs BRICS+ (BRI)





USA (USD) versus China (Yuan CNY)





Two clusters of fixed opinions in WTN

Anglo-Saxon vs BRICS+

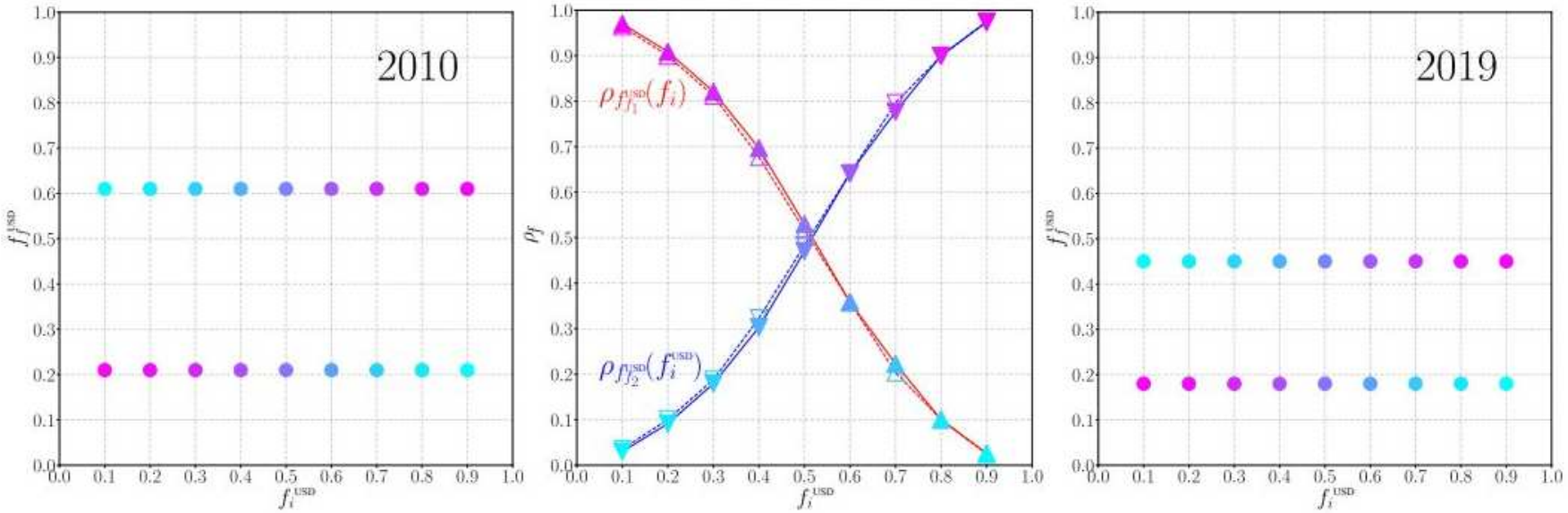


Figure Final fraction f_f^{USD} of countries with a trade currency preference for USD versus the initial fraction f_i^{USD} of these countries for years 2010 (left panel) and 2019 (right panel). There are two possible final fractions for each considered year: $f_{f_1}^{\mathrm{USD}} = 0.21$ and $f_{f_2}^{\mathrm{USD}} = 0.61$ in 2010, and $f_{f_1}^{\mathrm{USD}} = 0.18$ and $f_{f_2}^{\mathrm{USD}} = 0.45$ in 2019. The color of the points represents the ratio of the Monte Carlo process with the corresponding final state $\rho_{f_f}(f_i)$, low ratio in cold blue and high ration in violet. The central panel shows the evolution of $\rho_{f_f}(f_i)$ with f_i. The red (blue) curve and the up (down) triangles denote the minimal (maximal) final state. The full (empty) symbols correspond to the year 2019 (2010).

Fractions of countries - trade volumes

Anglo-Saxon vs BRICS+

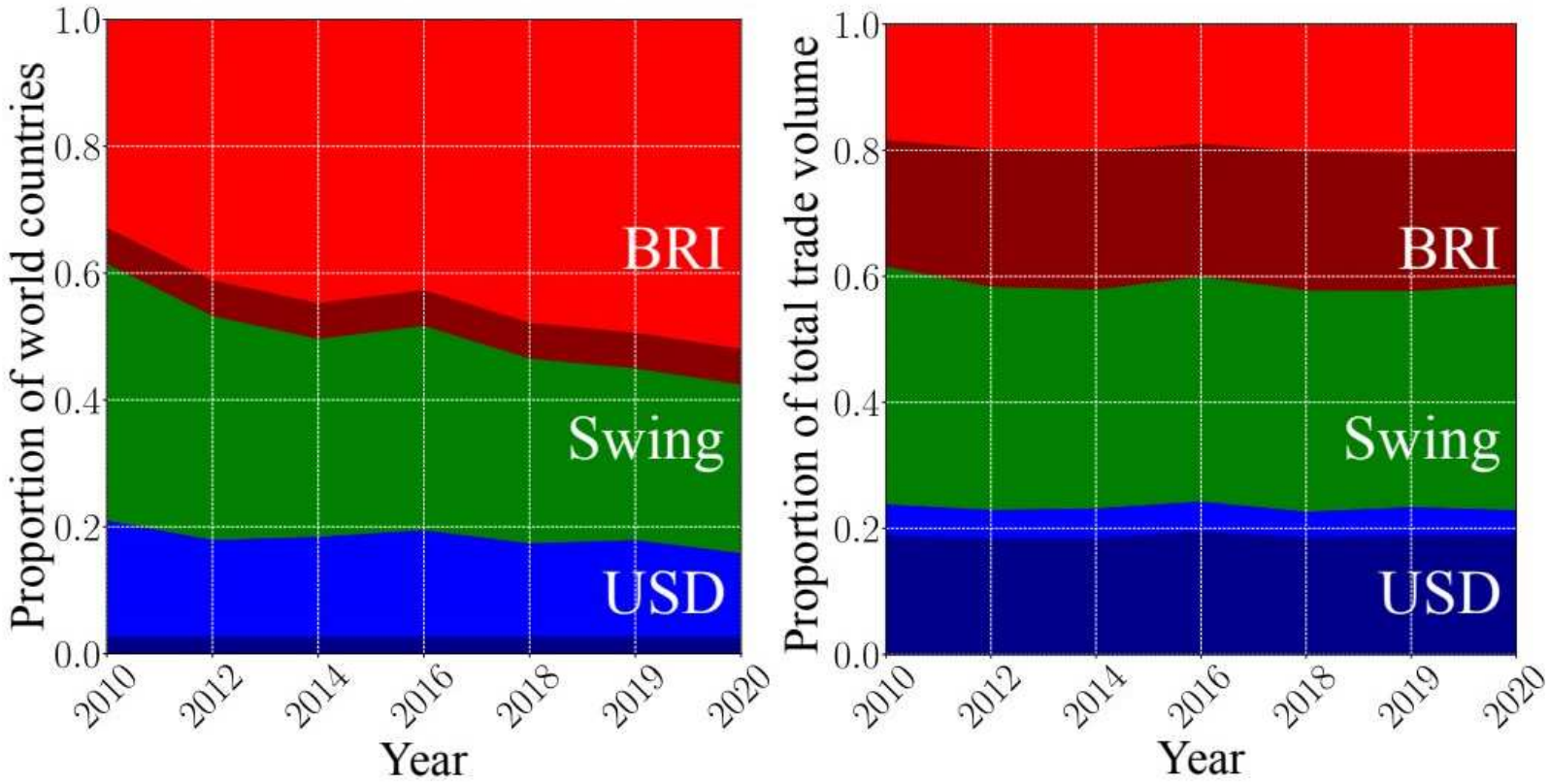


Figure 1. Time evolution of the size of the trade currency preference groups. The width of a given band corresponds to the corresponding fraction of world countries in a TCP group (**left panel**) and to the corresponding fraction of the total trade volume generated by this group (**right panel**). The USD group is colored in blue, the BRI group in red, and the swing group in green. Within the BRI (USD) group, the proportion corresponding to the BRICS+ (the ANGL countries) is shown in dark red (dark blue).

Case of 3 currencies in WTN

Anglo-Saxon (USD - 5) - BRICS+ (BRI - 11) - EU9 (EUR - 9)

5. Three Currencies Case

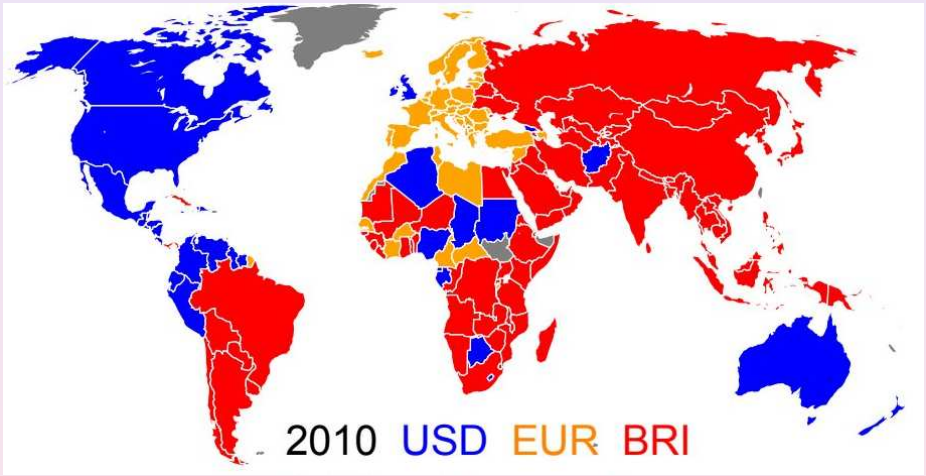
Let us now consider three currencies, namely, USD, EUR and BRI. We keep the USD and the BRI core groups, namely, the ANGL group and the BRICS+ (see Table 1). The EUR core group is the EU9 group constituted by Austria, Belgium, France, Germany, Italy, Luxemburg, Netherlands, Portugal and Spain [32]. Hence, the countries of the ANGL group keep trading in USD, the countries of the EU9 group in EUR and the BRICS+ in BRI. For each country c, we compute the following TCP-score for each currency $¢ \in \{\text{USD}, \text{EUR}, \text{BRI}\}$

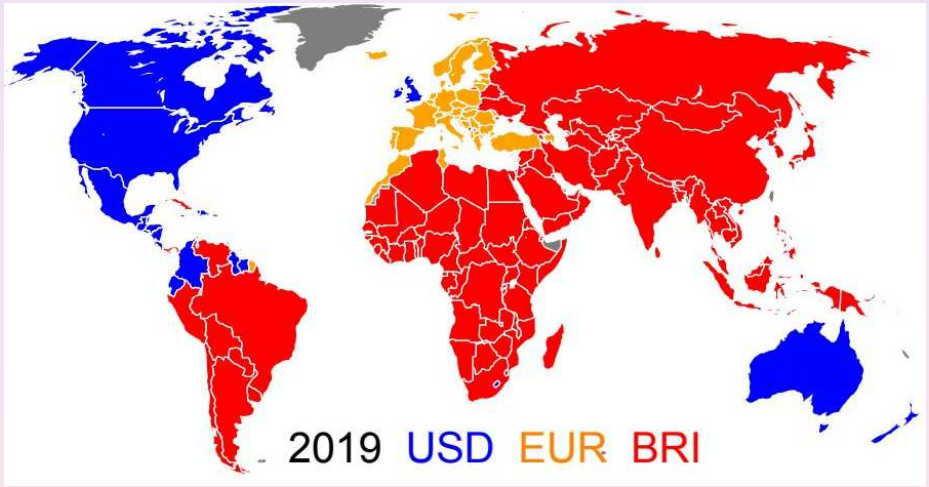
$$Z_{c,¢} = \frac{\sum_{c' \neq c}^{(¢)} \left(S_{c'c} + S^*_{c'c}\right)\left(P_{c'} + P^*_{c'}\right)}{\sum_{c' \neq c} \left(S_{c'c} + S^*_{c'c}\right)\left(P_{c'} + P^*_{c'}\right)}, \tag{4}$$

where the sum $\sum_{c' \neq c}^{(¢)}$ is performed over all the countries who are commercial partners of the country c and who prefer to trade with the currency $¢$. For a given country c, the sum of these scores over the different considered currencies is equal to one, i.e., $\sum_{¢} Z_{c,¢} = 1$. The country c then adopts the trade currency for which the TCP-score is the maximum, i.e., $¢_1$ such as $Z_{c,¢_1} = \max_{¢} Z_{c,¢}$. The asynchronous Monte Carlo procedure (see Section 2.1) is performed with the above defined TCP-score (4). For a given year among those considered (2010–2020), the final steady state distribution of the TCPs is unique and does not depend on the initial distribution of the TCPs. By contrast with the two currencies model (see

WTN map for 3 currencies)

Anglo-Saxon (USD - 5) - BRICS+ (BRI - 11) - EU9 (EUR - 9)





Fractions for 3 currencies case

Anglo-Saxon (USD - 5) - BRICS+ (BRI - 11) - EU9 (EUR - 9)

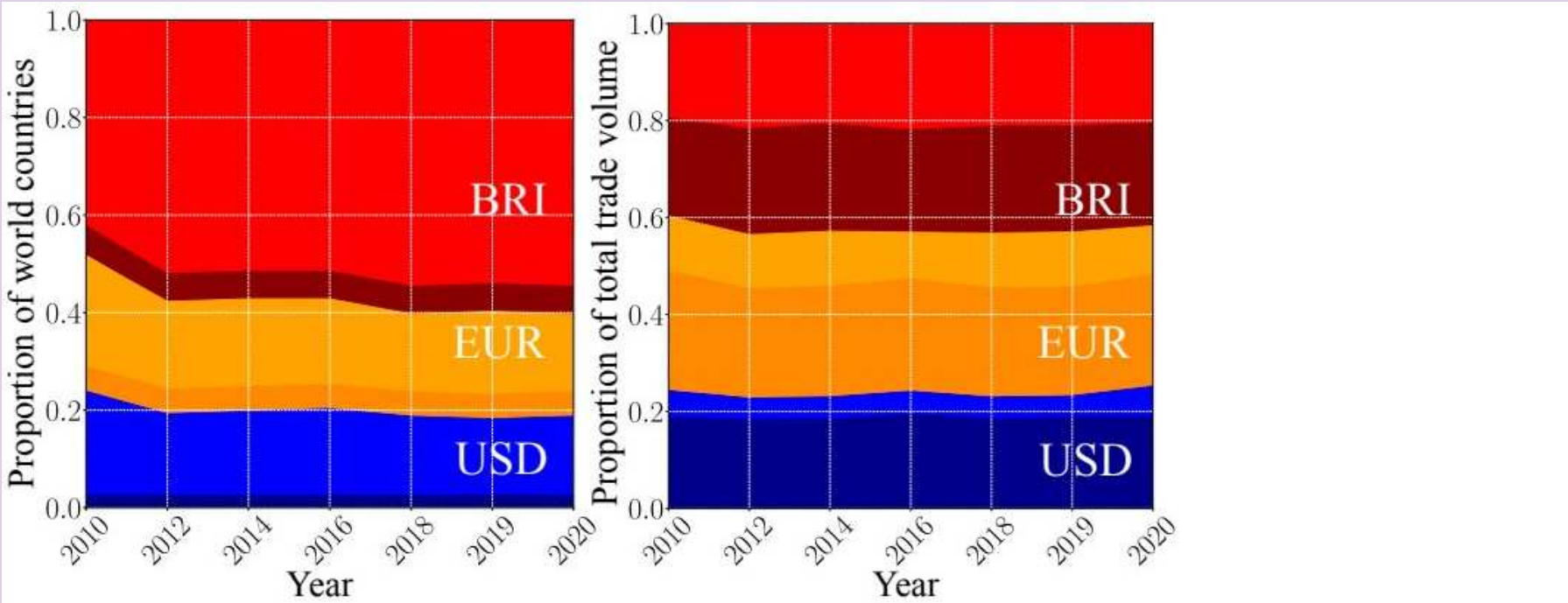
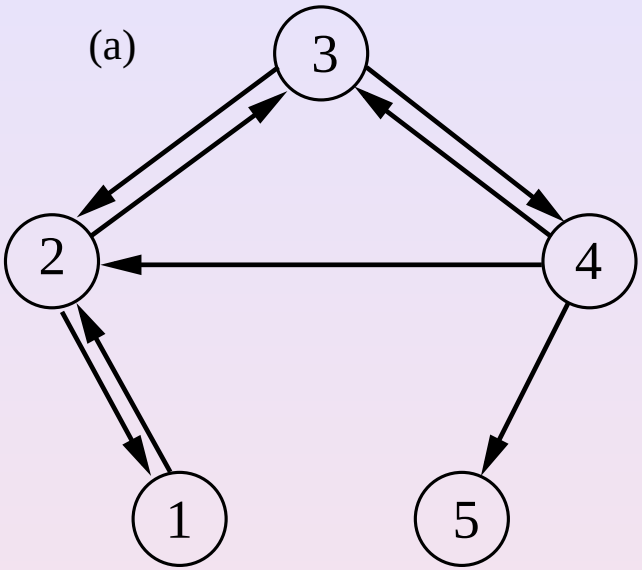
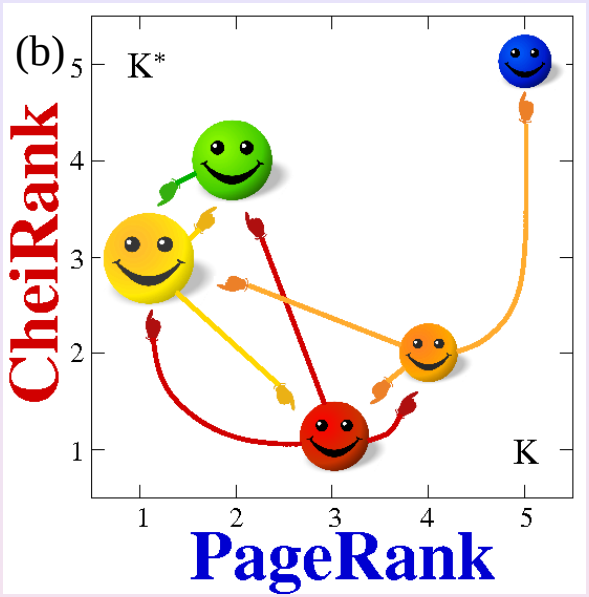


Figure Time evolution of the size of the trade currency preference groups. The width of a given band corresponds to the corresponding fraction of world countries in a TCP group (**left panel**) and to the corresponding fraction of the total trade volume generated by this group (**right panel**). The USD group is colored in blue, the BRI group in red and the EUR group in gold. Within the BRI (USD) [EUR] group, the proportion corresponding to the BRICS+ (the ANGL countries) [the EU9 group] is shown in dark red (dark blue) [dark gold].

Google matrix construction rules

Markov chains (1906) and Directed networks





For a directed network with N nodes the adjacency matrix $\mathbf{A}$ is defined as $A_{ij} = 1$ if there is a link from node j to node i and $A_{ij} = 0$ otherwise. The weighted adjacency matrix is

$$S_{ij} = A_{ij} / \sum_k A_{kj}$$

In addition the elements of columns with only zeros elements are replaced by $1/N$.

Google matrix construction rules

Google Matrix and Computation of PageRank

P $= \mathbf{SP} \Rightarrow$ **P**= stationary vector of **S**; can be computed by iteration of **S**.

To remove convergence problems:

- Replace columns of 0 (dangling nodes) by $\frac{1}{N}$:

$$S = \begin{pmatrix} 0 & 1/2 & 1/3 & 0 & 1/5 \\ 1 & 0 & 1/3 & 1/3 & 1/5 \\ 0 & 1/2 & 0 & 1/3 & 1/5 \\ 0 & 0 & 1/3 & 0 & 1/5 \\ 0 & 0 & 0 & 1/3 & 1/5 \end{pmatrix} \qquad S^* = \begin{pmatrix} 0 & 1/3 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/3 & 0 & 1 & 0 \\ 0 & 1/3 & 1/2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- To remove degeneracies of $\lambda = 1$, replace **S** by **Google matrix**

  $\mathbf{G} = \alpha\mathbf{S} + (1-\alpha)\frac{\mathbf{E}}{N}$; $\quad GP = \lambda P$ => Perron-Frobenius operator

  Import

- α models a random surfer with a random jump after approximately 6 clicks (usually $\alpha = 0.85$); PageRank vector => P at $\lambda = 1$ ($\sum_j P_j = 1$).

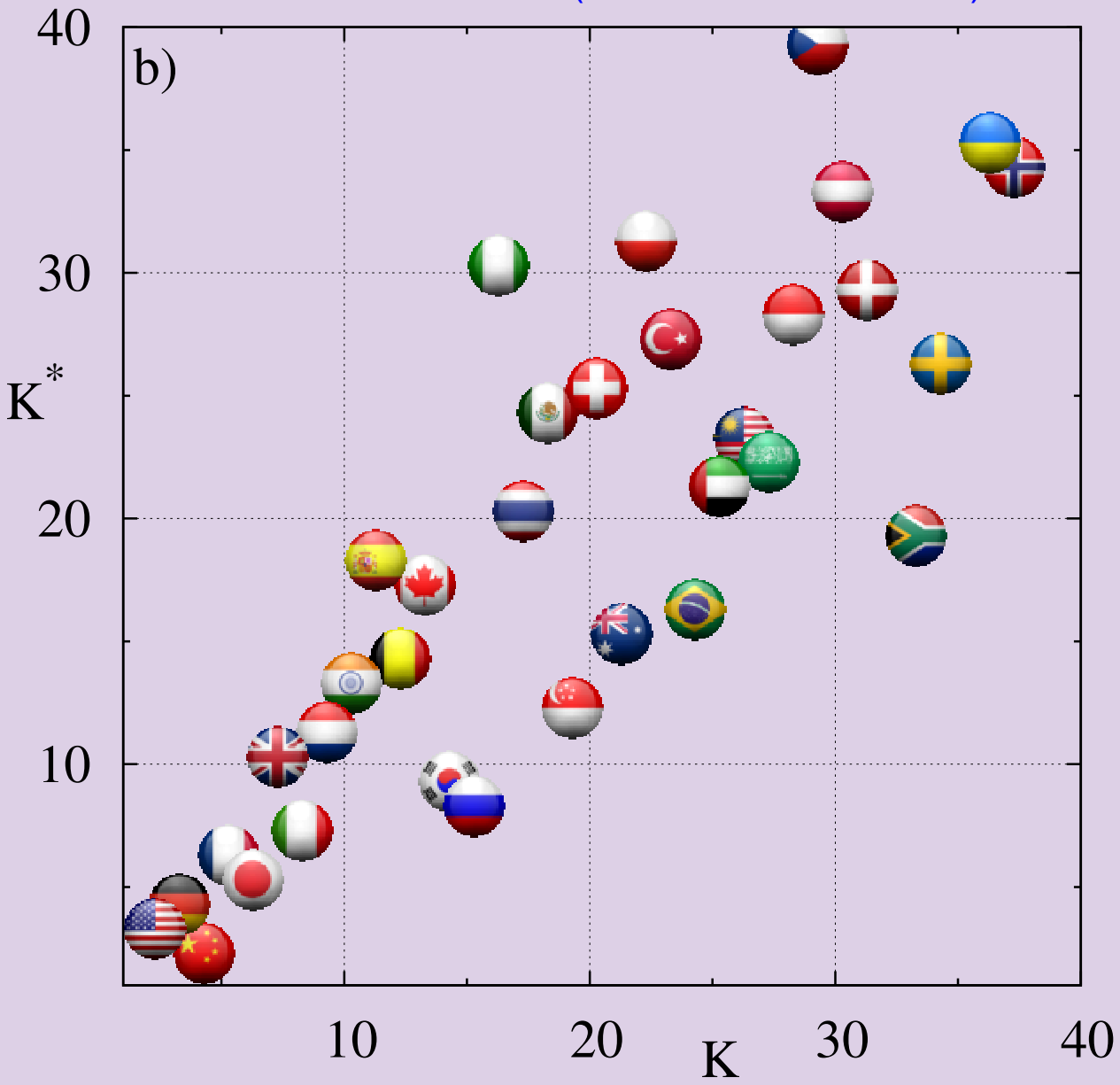
- CheiRank vector P^*: $G^* = \alpha\mathbf{S}^* + (1-\alpha)\frac{\mathbf{E}}{N}$, $G^*P^* = P^*$ Export

  ($\mathbf{S}^*$ with inverted link directions)

  Fogaras (2003) ... Chepelianskii arXiv:1003.5455 (2010) ...

Ranking of World Trade

2008: All commodities (about 200 countries)



Reduced Google matrix (REGOMAX)

A selected network of interest with $N_r < N$ nodes called reduced network.
Block structure of G matrix:

$$G = \begin{pmatrix} G_{rr} & G_{rs} \\ G_{sr} & G_{ss} \end{pmatrix}$$

with s index for scattering network $N_s = N - N_r$.
Reduced G_R matrix

$$G_{\rm R} P_r = P_r \quad , \quad G_{\rm R} = G_{rr} + G_{rs}(\mathbf{1} - G_{ss})^{-1} G_{sr} = G_{pr} + G_{rr} + G_{qr}$$

Useful expansion

$$(\mathbf{1} - G_{ss})^{-1} \quad = \quad \mathcal{P}_c \frac{1}{1-\lambda_c} + \mathcal{Q}_c \sum_{l=0}^{\infty} \bar{G}_{ss}^{l}$$

with projector $\mathcal{P}_c = \psi_R \psi_L^T$ on eigenstate of maximal eigenvalue λ_c of G_{ss}, the complementary projector $\mathcal{Q}_c = \mathbf{1} - \mathcal{P}_c$ and $\bar{G}_{ss} = \mathcal{Q}_c G_{ss} \mathcal{Q}_c$.
K.Frahm, DS arxiv:1602.02394 (2016);
K.Frahm, K.Jaffrres-Runser, DS EPJB **89**, 269 (2016)

Protein-protein interactions for cancer networks

PLOS ONE | Reduced Google matrix for directed biological networks

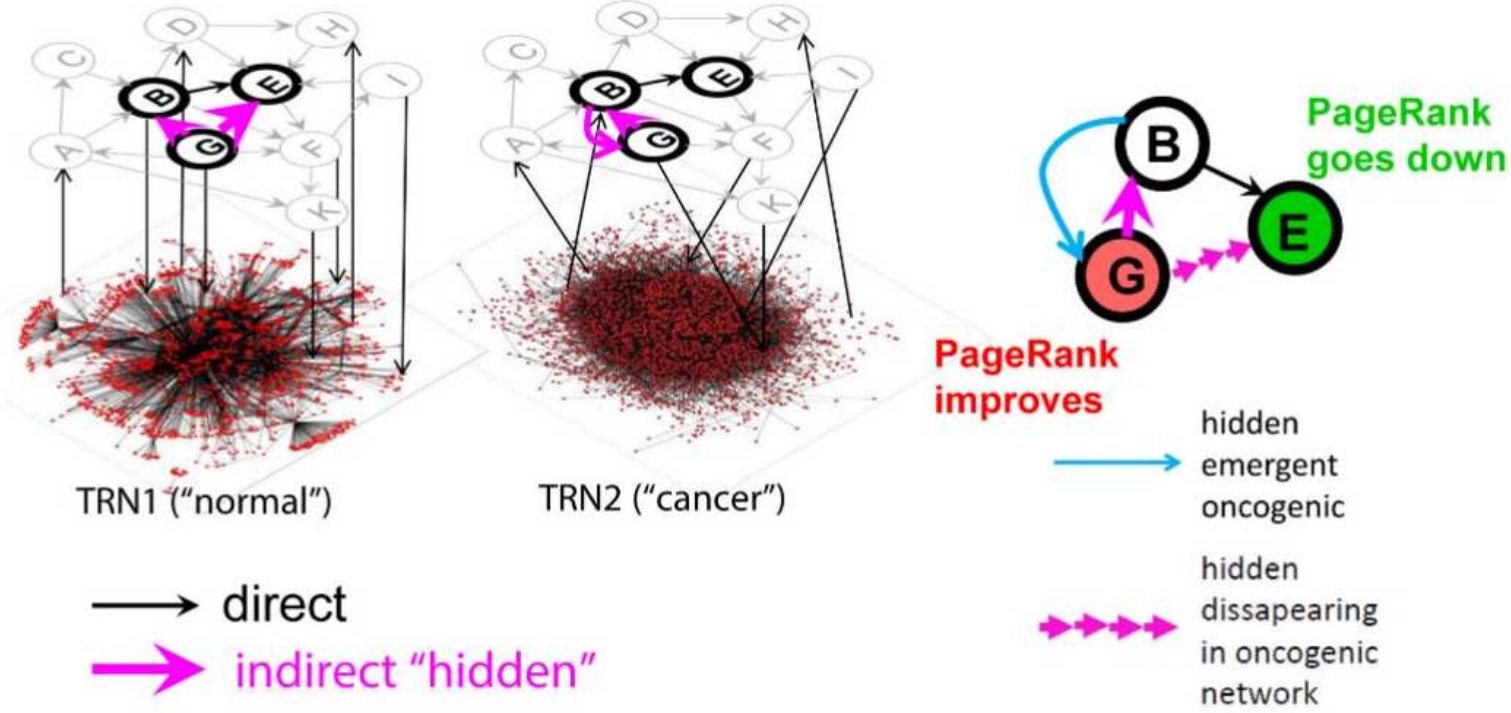


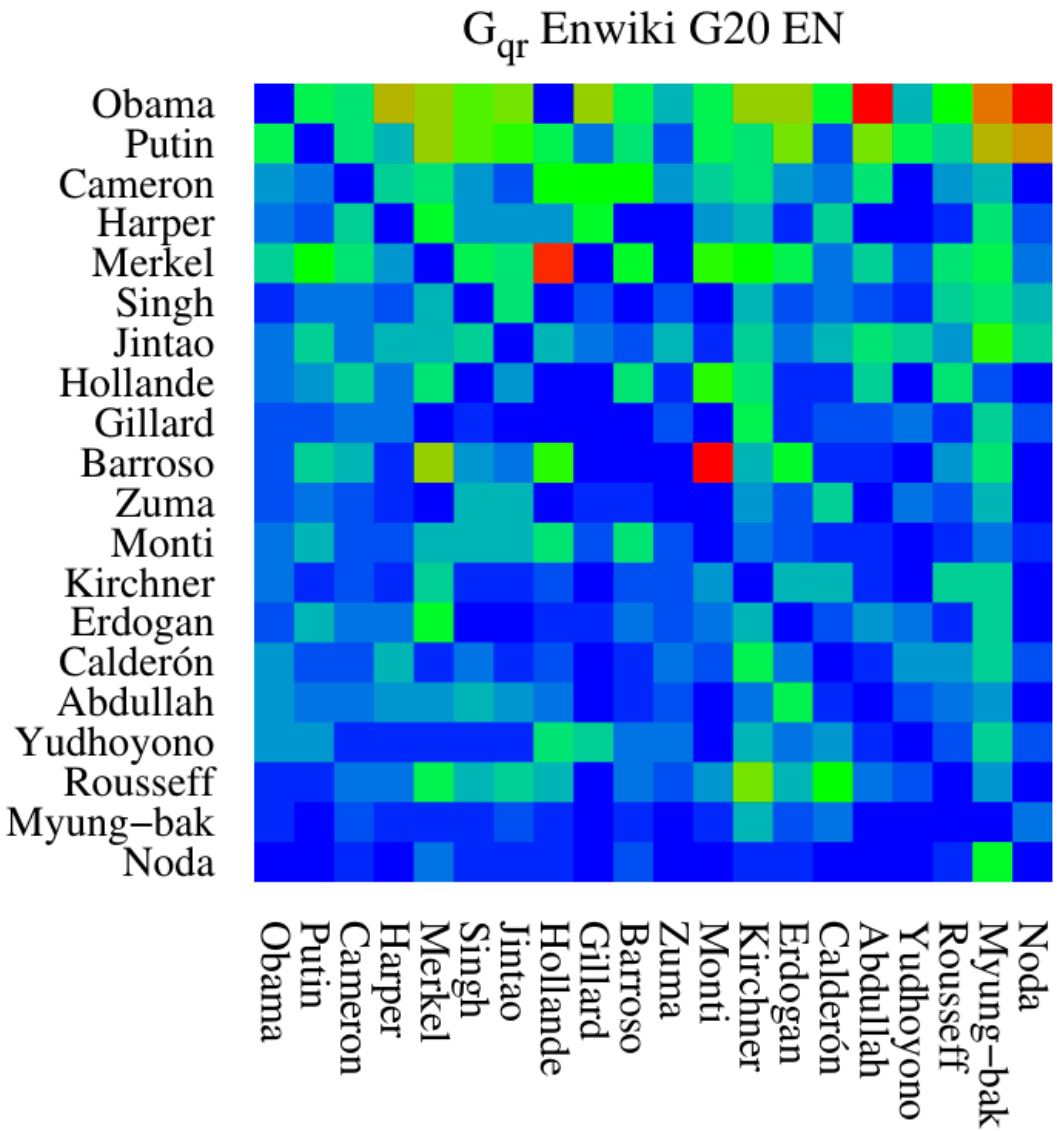
Fig 1. Using reduced Google matrix approach for inferring hidden causal relations in signaling pathways. Here the structure of the context-dependent global regulatory network is symbolically shown as consisting of two layers: the upper (nodes A-K) is the global signaling network whose structure does not depend on the context and the lower is a symbolic view of the contextual transcriptional regulatory network (TRN) whose structure can change between a "normal" and a "cancer" cell. Thick node borders denote a pathway embedded into the global signaling network. Black arrows denote direct physical interactions. Pink arrows denote inferred hidden directed regulations through the global regulatory network (both layers). In the final representation of the pathway (on the right), one can show those hidden regulations which emerge or disappear due to the changes in the TRN structure. Also, the color of the pathway nodes can show the direction of PageRank change: green corresponds to the PageRank decreased in the cancer network while red corresponds to the opposite.

https://doi.org/10.1371/journal.pone.0190812.g001

Lages, DS, Zinovyev (PLOS ONE 2018) [SIGNOR database]

G-reduced: G20 political leaders 2012-ENWIKI2013

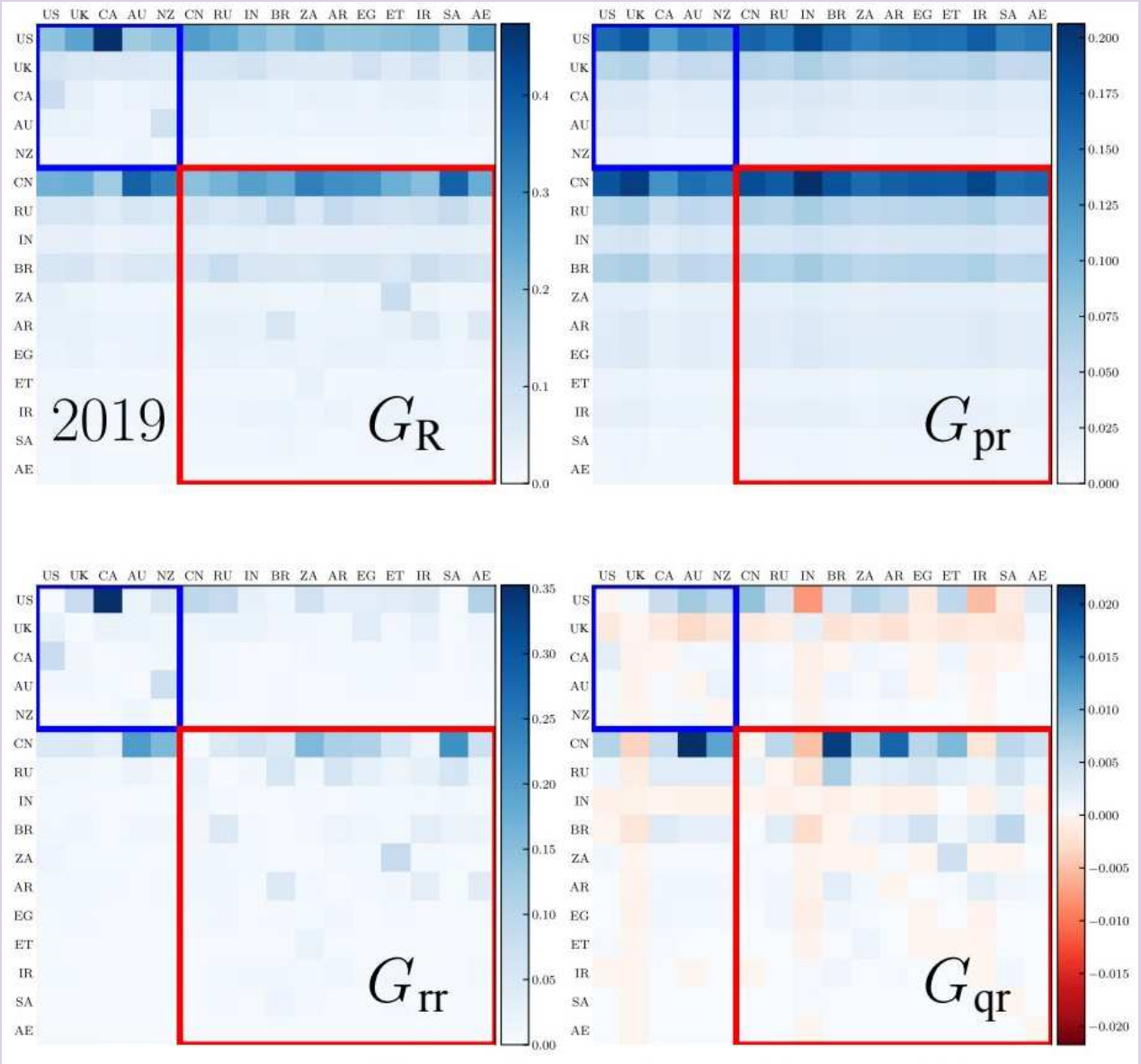
G_R example: G20 political leaders 2012 indirect links of G_{qr} (non-diagterms)



G20	EN	Enwiki
Name	Friends	Followers
Obama	Putin	Noda
	Merkel	Abdullah
	Calderón	Myung-bak
Putin	Merkel	Noda
	Obama	Myung-bak
	Barroso	Merkel
Cameron	Putin	Gillard
	Obama	Barroso
	Merkel	Hollande
Harper	Obama	Merkel
	Cameron	Gillard
	Putin	Myung-bak
Merkel	Barroso	Hollande
	Putin	Monti
	Obama	Kirchner

G-reduced for WTN ANGL-BRICS+

REGOMAX for 5+11 countries of 194 ones of WTN in 2019



Overview

* Mathematical Ising-type network model of opinion formation in WTN
* BRICS+ and China currencies become world dominant around 2016
* Reduced Google matrix (REGOMAX) algorithm allows to determine direct and hidden interactions between a group of selected nodes embedded in a huge network
* Applications to various types of directed networks (WTN, Wikipedia, protein-protein interactions, ...)