Nonlinearity, localization and quantum chaos

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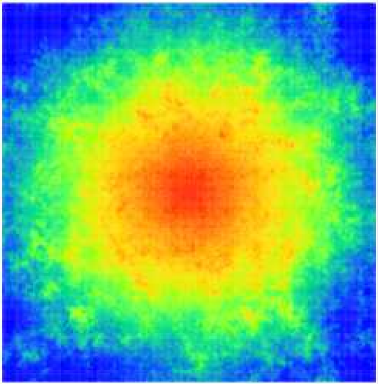

I.Garcia-Mata, DLS arXiv:0805.0539 (2008)

- Chirikov standard map and kicked NLSE PRA **44**, 3423 (1991)
- Delocalization in nonlinear kicked rotator model PRL **70**, 1787 (1993)

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- Delocalization in nonlinear Anderson model PRL **100**, 094101 (2008)
- Delocalization in nonlinear 2D Anderson model arXiv:0805.0539 (2008)
- Time revesal of Bose-Einstein condensates arXiv:0804.3514 (2008)

Chirikov standard map for soliton dynamics

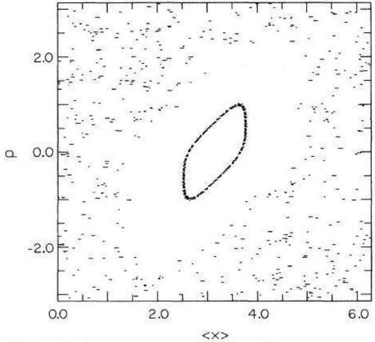


FIG. 1. Two phase-space trajectories with parameters $\beta=25$, $k=0.5$, and $T=2$ (classical K is 2), obtained by numerical in-

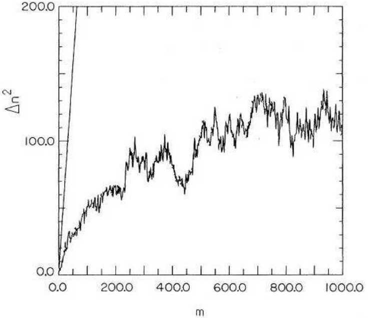


FIG. 4. Plot of the wave packet width in Fourier space Δn^2 vs number of periods m. Here $\beta=10$, $k=2.5$, $T=1$ and classical $K=5$; the initial soliton position and velocity are $x_0=0.2$ and

$$i\hbar \frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - g|\psi|^2 + k\cos x\ \delta_T(t)\right)\psi$$

$$\bar{p} = p + K\sin x\ ,\ \ \bar{x} = x + \bar{p}$$

PRA **44**, 3423 (1991)

Nonlinearity and Anderson localization: estimates

$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\psi_n|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1}) \, ; [-W/2 < E_n < W/2]$$

localization length $l \approx 96(V/W)^2$ (1D); $\ln l \sim (V/W)^2$ (2D)

Amplitudes C in the linear eigenbasis are described by the equation

$$i \frac{\partial C_m}{\partial t} = \epsilon_m C_m + \beta \sum_{m_1 m_2 m_3} V_{m m_1 m_2 m_3} C_{m_1} C^*_{m_2} C_{m_3}$$

the transition matrix elements are $V_{m m_1 m_2 m_3} = \sum_n Q^{-1}_{nm} Q_{n m_1} Q^*_{n m_2} Q_{n m_3} \sim 1/l^{3d/2}$. There are about l^{3d} random terms in the sum with $V \sim l^{-3d/2}$ so that we have $idC/dt \sim \beta C^3$. We assume that the probability is distributed over $\Delta n > l^d$ states of the lattice basis. Then from the normalization condition we have $C_m \sim 1/(\Delta n)^{1/2}$ and the transition rate to new non-populated states in the basis $\boldsymbol{m}$ is $\Gamma \sim \beta^2 |C|^6 \sim \beta^2/(\Delta n)^3$. Due to localization these transitions take place on a size l and hence the diffusion rate in the distance $\Delta R \sim (\Delta n)^{1/d}$ of $d-$ dimensional $\boldsymbol{m}-$ space is $d(\Delta R)^2/dt \sim l^2 \Gamma \sim \beta^2 l^2/(\Delta n)^3 \sim \beta^2 l^2/(\Delta R)^{3d}$. At large time scales $\Delta R \sim R$ and we obtain

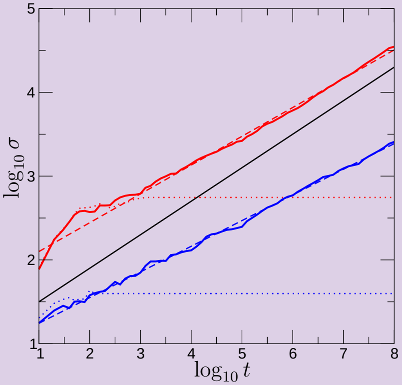
$$\Delta n \sim R^d \sim (\beta l)^{2d/(3d+2)} t^{d/(3d+2)}$$

Chaos criterion:

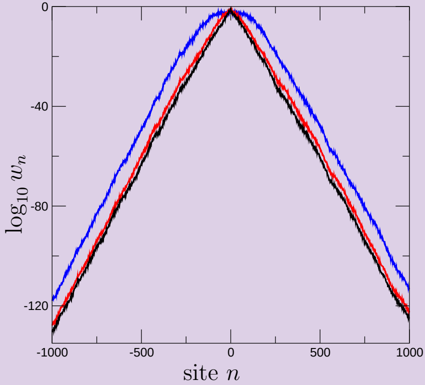
$$S = \delta\omega/\Delta\omega \sim \beta > \beta_c \sim 1$$

here $\delta\omega \sim \beta|\psi_n|^2 \sim \beta/\Delta n$ is nonlinear frequency shift
and $\Delta\omega \sim 1/\Delta n$ is spacing between exites eigenmodes
PRL **70**, 1787 (1993) ($d = 1$); arXiv:0805.0539 (2008) ($d \geq 1$)

Nonlinearity and Anderson localization (1D)



$W/V = 2, 4$, $\beta = 0, 1$; $\sigma = (\Delta n)^2 \propto t^{2/5}$

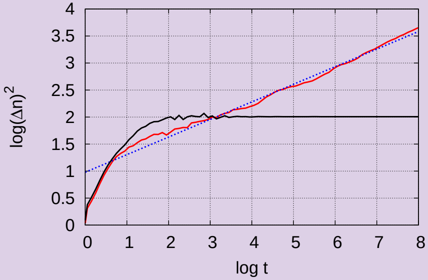


$W/V = 4$, $\beta = 1$, $t = 10^8$, $\beta = 0$

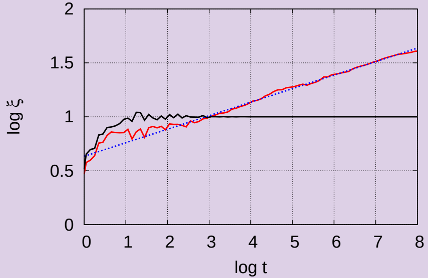
$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\psi_n|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1})\ ;[-W/2 < E_n < W/2]$$

PRL **100**, 094101 (2008)

Nonlinearity and Anderson localization (1D)



$W/V = 4$, $\beta = 1$, $\beta = 0$; $\alpha_1 = 0.325 \pm 0.003$ (theory 0.4)

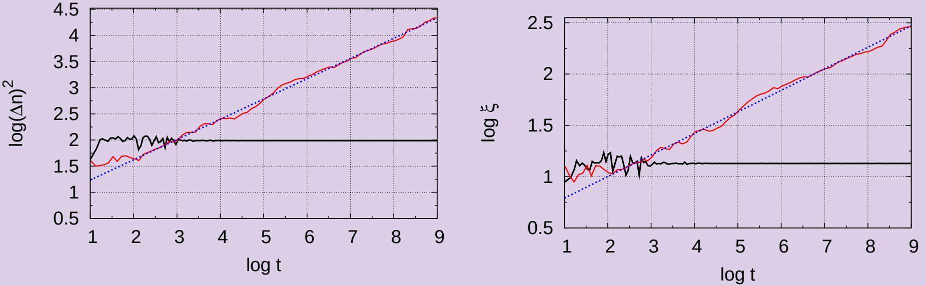


$\nu = 0.125 \pm 0.001$ (theory 0.2)

$$i\hbar\frac{\partial\psi_n}{\partial t} = E_n\psi_n + \beta|\ \psi_n\ |^2\psi_n + V(\psi_{n+1} + \psi_{n-1})$$

arXiv:0805.0539 (2008)

Kicked nonlinear rotator (1D)



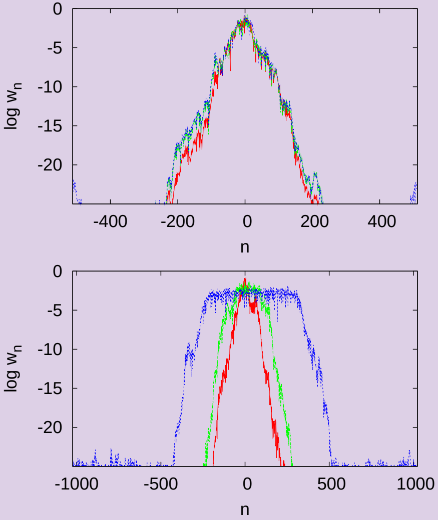
$\alpha_1 = 0.387 \pm 0.003$ (theory 0.4)

$\nu = 0.210 \pm 0.002$ (theory 0.2)

$$\psi_n(t+1) = e^{-iT\hat{n}^2/2 - i\beta|\psi_n|^2} e^{-ik\cos\hat{\theta}}\psi_n(t) \quad , (k = 3, T = 2, \beta = 0, 1)$$

arXiv:0805.0539 (2008)

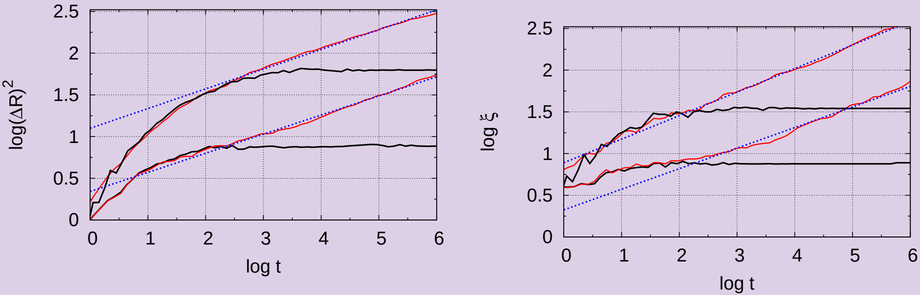
Kicked nonlinear rotator (1D)



$t = 10^3, 10^6, 10^9; \beta = 0; 1$

arXiv:0805.0539 (2008)

Nonlinearity and Anderson localization (2D)



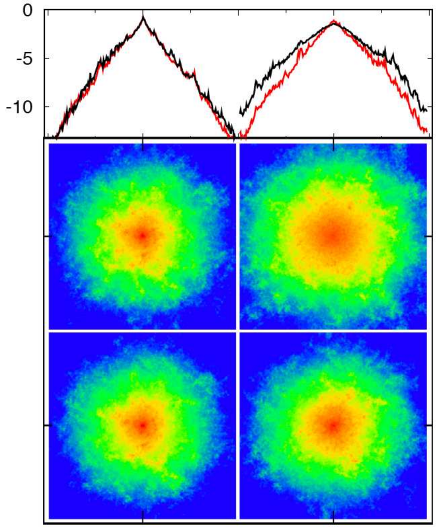
$W/V = 10, 15$, $\beta = 0, 1$; $\alpha_2 = 0.236.0.229 \pm 3$ (theory 0.25)

$\nu = 0.282, 0.247 \pm 0.005$ (theory 0.25)

$$i\hbar \frac{\partial \psi_n}{\partial t} = E_n \psi_n + \beta |\, \psi_n \,|^2 \psi_n + V(\psi_{n+1} + \psi_{n-1})$$

arXiv:0805.0539 (2008)

Nonlinearity and Anderson localization (2D)



$W = 10; \beta = 0, 1; t = 10^4, 10^6, 256 \times 256 lattice$

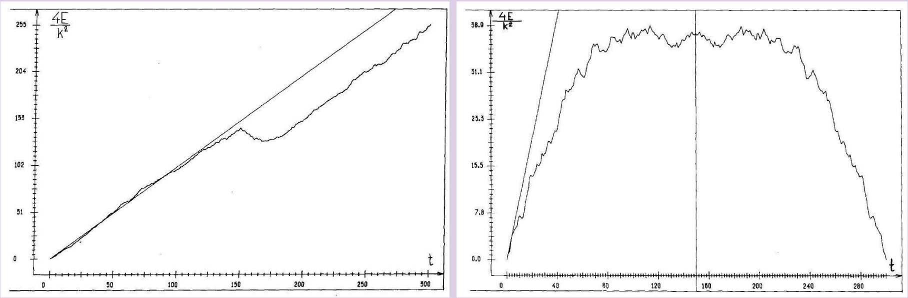
arXiv:0805.0539 (2008)

Boltzmann - Loschmidt dispute on time reversibility (1876)

* irreversible kinetic theory from reversible equations

Sitzungsberichte der Akademie der Wissebschaften, Wien,
II **73**, 128 (1876); **75**, 67 (1877)

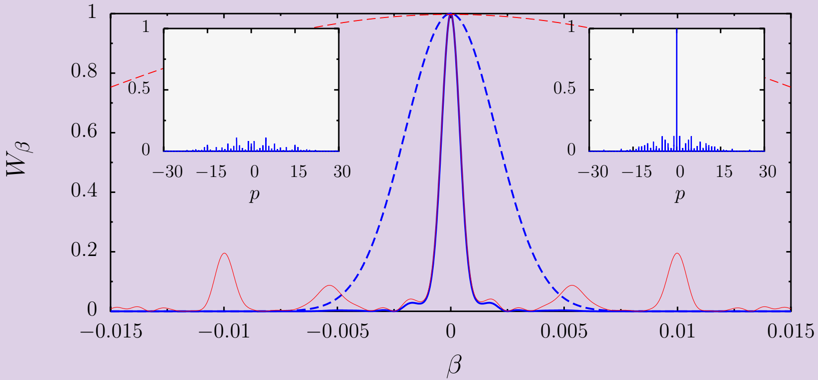
Time reversal for the Chirikov standard map



BESM-6 computation, rescaled energy or squared momentum vs. time t:
$K = 5$, $\hbar = 0$ (left), $\hbar = 1/4$ (right)
DLS, Physica D **8**, 208 (1983)

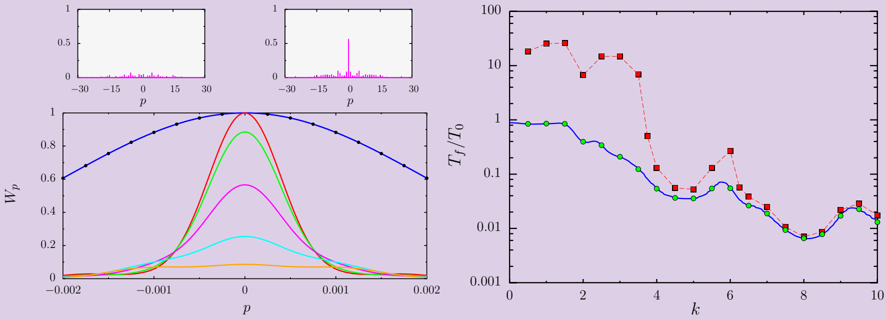
* Experimental realization of time reversal:
spin echo (E.L.Hahn (1950)); acoustic waves (M.Fink (1995));
electromagnetic waves (M.Fink (2004))

* Loschmidt cooling by time reversal of atomic matter waves



proposal of time reversal in kicked optical lattices:
$k = K/\hbar$, $\hbar = 4\pi + \epsilon$ (forward), $\hbar = 4\pi - \epsilon$ (back) and $k \to -k$;
Fig: $k = 4.5, \epsilon = 2, t_r = 10$, $k_B T_o/E_r = 2 \times 10^{-4}$ (red), $k_B T_o/E_r = 2 \times 10^{-6}$ (blue);
momentum β and energy E_r are give in recoil units
J.Martin, B.Georgeot, DLS, PRL **100**, 044106 (2008)

* Time reversal of Bose-Einstein condensates

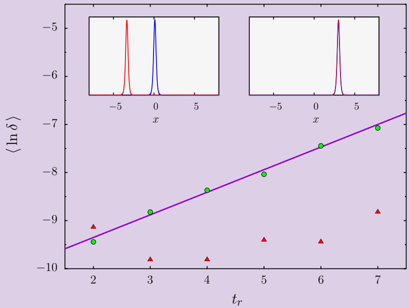


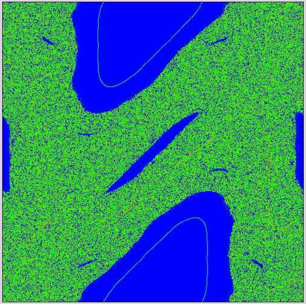
The Gross-Pitaevskii equation with kicks:

$$i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - g|\psi|^2 + k\cos x\ \delta_T(t)\right)\psi$$

Left: same as in previous Fig. for $g = 0, 5, 10(insets), 15, 20$ (top to bottom), ($t = 0$);
Right: cooling ratio T_f/T_0 for $g = 0$ (blue curve), $g = 0.5$ (green), $g = 10$ (red)
J.Martin, B.Georgeot, DLS, arXiv:0804.3514[cond-mat] (2008)

* Loschmidt paradox for Bose-Einstein condensates





Soliton initial condition (Zakharov, Shabat (1971)):

$$\psi(x,t) = \frac{\sqrt{g}}{2} \frac{\exp\left(ip_0(x-x_0-p_0t/2)+ig^2t/8\right)}{\cosh\left(\frac{g}{2}(x-x_0-p_0t)\right)}$$

Left: time resersal of soliton at $g = 10, k = 1, T = \hbar = 2, K = kT = 2, t_r = 40$ inside chaotic (left inset) and regular (right inset) domains; line shows divergence given by the Kolmogorov-Sinai entropy $h = 0.45$. Right: Poincaré section at $K = 2$

But the real BEC is quantum and should return back since the Ehrenfest time $t_E \sim |\ln \hbar_{eff}|/h \sim \ln N/2h \sim 13$ for BEC with $N = 10^5$ atoms

J.Martin, B.Georgeot, DLS, arXiv:0804.3514[cond-mat] (2008)

Possible experimental tests & applications

- BEC in disordered potential (Aspect, Inguscio)
- BEC time reversal (Phillips)
- nonlinear wave propagation in disordered media (Segev, Silberberg)
- lasing in random media (Cao)
- energy propagation in complex molecular chains (proteins, FPU)