LECTURE 6

[65] D.L.Shepelyansky, "Hydrogen in Monochromatic Field: Stabilization and Channeling vs. Chaos", Proc. XIII Int. Conf. on Atomic Physics, Munich, 1992 in: Atomic Physics 13, Edited by H.Walther, T.W.Hansch, and B.Neizert, AIP 1993, p.425 [Rydberg stabilization].

[66] F.Benvenuto, G.Casati, D.L.Shepelyansky, "Classical Stabilization of the Hydrogen Atom in a Monochromatic Field", Phys. Rev. A, v.47, N2 (1993) p. R786-R789 [Rydberg stabilization].

[67] D.L.Shepelyansky, "Kramers Map Approach for stabilization of Hydrogen Atom in a Monochromatic Field", Phys. Rev. A. v.50 (1994) p.575-583 [Rydberg stabilization].

[68] F.Benvenuto, G.Casati, D.L.Shepelyansky, "Rydberg Stabilization of Hydrogen Atom in Strong Fields: The "Magic Mountain" in the Chaotic Sea", Z. Phys. B v.94 (1994) p.481-486 [Rydberg stabilization].

[69] F.Benvenuto, G.Casati, D.L.Shepelyansky, "Chaotic Autoionization of Molecular Rydberg States", Phys. Rev. Lett. v.72 (1994) p. 1818.

[70] B.V.Chirikov and V.V.Vecheslavov, Astron. Astrophys. **221** (1989) 146 [Halley's comet].

(47)

Conditions of Kepler map picture

* $\Delta E \sim k\omega \sim \frac{2.6\,\varepsilon}{\omega^{4/3}} \gg \frac{1}{2}\left(\frac{\varepsilon}{\omega}\right)^2$

$$\varepsilon \ll \varepsilon_{ATI} \approx 5\,\omega^{4/3}$$

* $l < \left(\frac{3}{\omega}\right)^{1/3}$

Stabilization

(F 33, 34)

$m = \text{const}$

Strong field limit

$m = 0 \rightarrow$ unavoidable collision with nucleus

$$\frac{2\varepsilon}{\omega^2} \gtrsim \frac{l^2}{2} \Longrightarrow \varepsilon > \frac{\omega^2 l^2}{4} \qquad \left(l > \left(\frac{3}{\omega}\right)^{1/3}\right)$$

$m \neq 0 \Longrightarrow$ charged thread potential

$$H = \frac{\tilde{p}_z^2}{2} + \frac{p_\rho^2}{2} + \frac{m^2}{2\rho^2} - \frac{1}{(\rho^2+z^2)^{1/2}} + \varepsilon z \sin\omega t$$

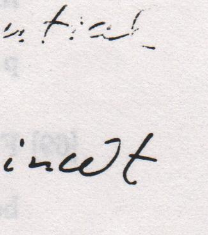
KH transformation

$$H = \frac{p_z^2}{2} + \frac{p_\rho^2}{2} + \frac{m^2}{2\rho^2} - \frac{1}{\left[\rho^2 + \left(z - \frac{\varepsilon}{\omega^2}\sin\omega t\right)^2\right]^{1/2}}$$

averaged potential

$$H_{av} = \frac{p_z^2}{2} + \frac{p_\rho^2}{2} + \frac{m^2}{2\rho^2} + 2\sigma(z)\ln\left(\frac{\rho\omega^2}{\varepsilon}\right)$$

$$\sigma(z) = \frac{\omega^2}{\pi\varepsilon}\,\frac{1}{\left(1 - \left(\frac{z\omega^2}{\varepsilon}\right)^2\right)^{1/2}}$$



$\rho_{min} \sim \frac{m\sqrt{\varepsilon}}{\omega}$, $\quad \Omega \approx \frac{\omega^2}{\varepsilon m}$

adiabatic motion: $\Omega \ll \omega$

Stabilization

$\Omega \sim \frac{\omega^2}{\varepsilon m} << \omega$ — adiabatic motion

* Stabilization border

$$\delta = \frac{\omega}{\Omega} \gg 1 \Longrightarrow \varepsilon > \varepsilon_{stab} = \beta \frac{\omega}{m}$$

($\beta = const \approx 12$)

Collision estimate:

$$\Delta p \sim \frac{\Delta t}{\rho_{min}^2} \sim \frac{\omega}{\varepsilon \rho_{min}}, \quad \rho_{min} \sim \frac{m\sqrt{\varepsilon}}{\omega}, \quad \Delta t \sim \frac{\rho_{min}}{v} \sim \frac{\omega \rho_{min}}{\varepsilon}$$

$$\Delta E \sim (\Delta p)^2 < \frac{\omega^2}{\varepsilon} \rightarrow \quad \varepsilon > \varepsilon_{stab}$$

* Destabilization border

$$\rho_{min} \sim \frac{\omega\sqrt{\varepsilon}}{m} > 2n_0^2$$

$$\varepsilon < \varepsilon_{destab} \approx \frac{16}{\pi} \frac{\omega^2 n_0^2}{m^2} L$$

$$\left(L = \frac{1}{2} \ln\left(\frac{2\varepsilon}{e\pi\omega^2 m^2}\right)\right)$$

Size of the atom

$$\Delta r \sim \begin{cases} 2n_0^2 & (\gg \alpha = \frac{\varepsilon}{\omega^2}) \\ \frac{\varepsilon}{\omega^2} & (\gg 2n_0^2) \end{cases}$$

Stabilization for $m << \left(\frac{3}{\omega}\right)^{1/3}$

For $m >> \left(\frac{3}{\omega}\right)^{1/3}$ atom is stable up to ε_{destab}

F35

$W_{stab} = 1 - W_{ion}$

$t_{int} = 500\ \omega_0$ field periods
500 orbital periods

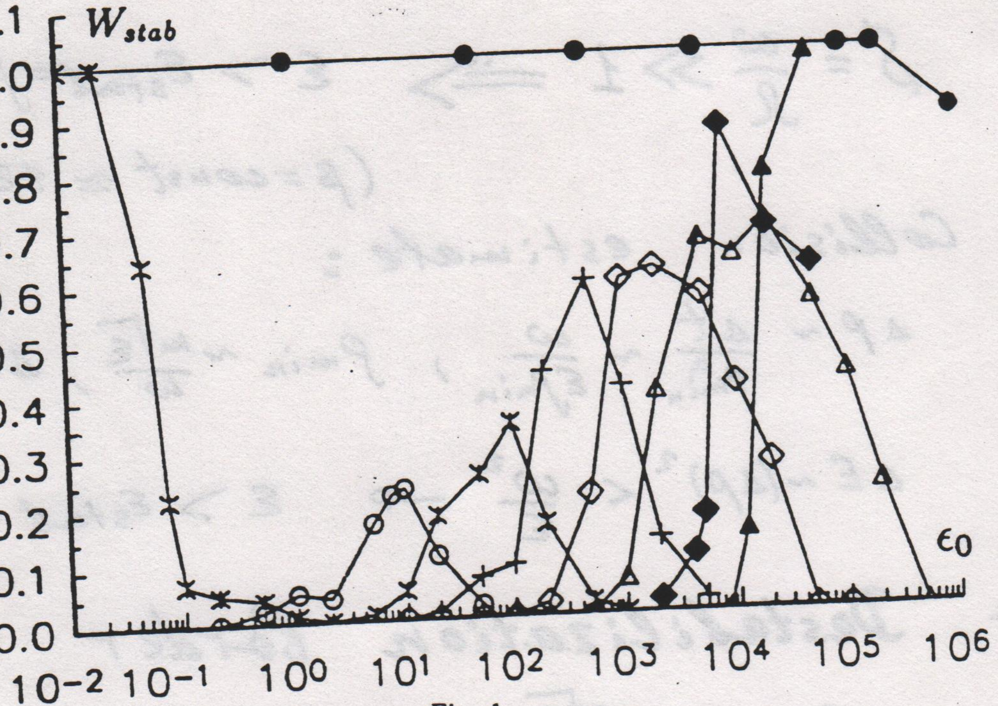


Fig. 1

ω_0
0.3 (O)
1. (*)
3. (+)
10. (◇)
30. (△)
100. (◆)
300. (▲)
1000. (●)

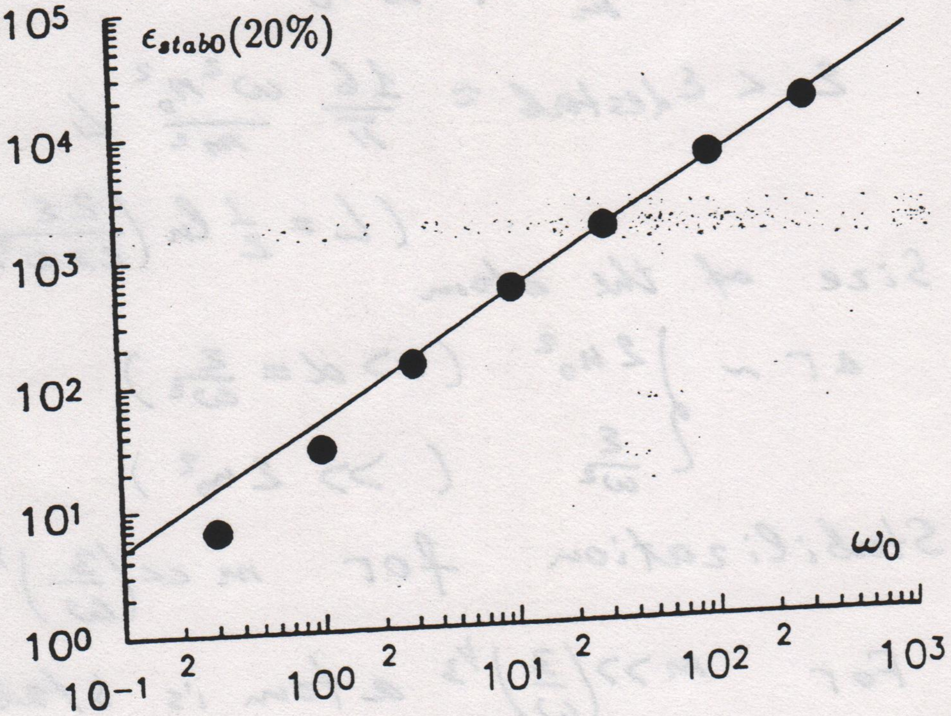


Fig. 2

$\frac{m}{n_0} = 0.25$

$\frac{\ell}{n_0} = 0.3$

(D.S. 1992.)

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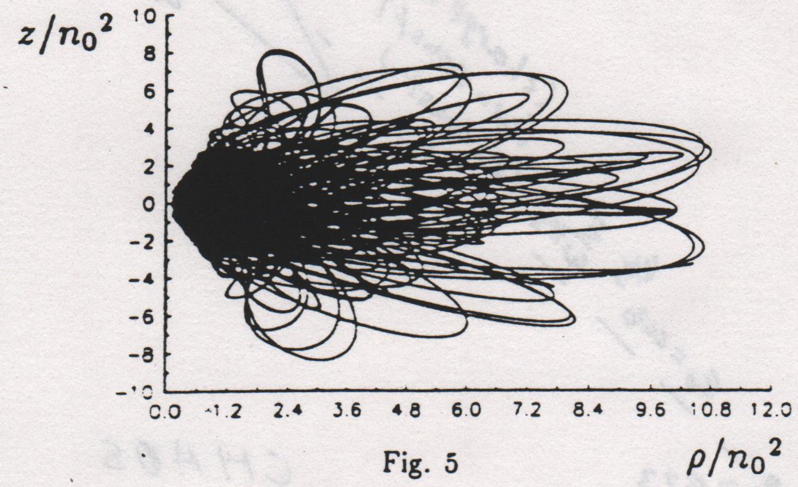


Fig. 5

$\omega_0 = 100$

$\mathcal{E}_0 = 8000$

$\frac{m}{n_0} = 0.25$

$\frac{\ell}{n_0} = 0.3$

$t_{int} = 10^5$

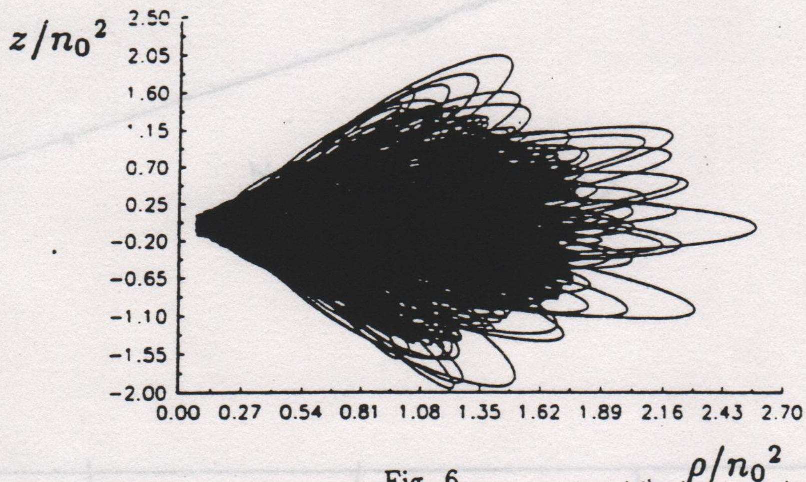


Fig. 6

$\omega_0 = 300$

$\mathcal{E}_0 = 20000$

$\frac{m}{n_0} = 0.25$

$\frac{\ell}{n_0} = 0.3$

$t_{int} = 10^5$ field period.

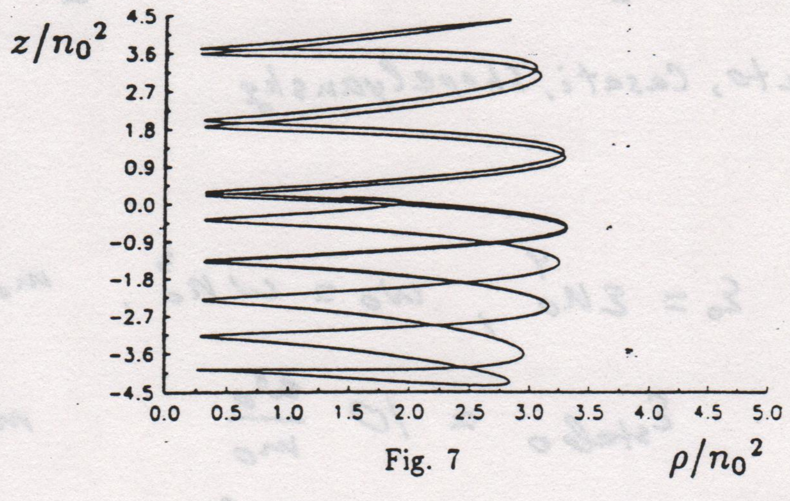


Fig. 7

$\omega_0 = 1000$

$\mathcal{E}_0 = 5 \cdot 10^6$

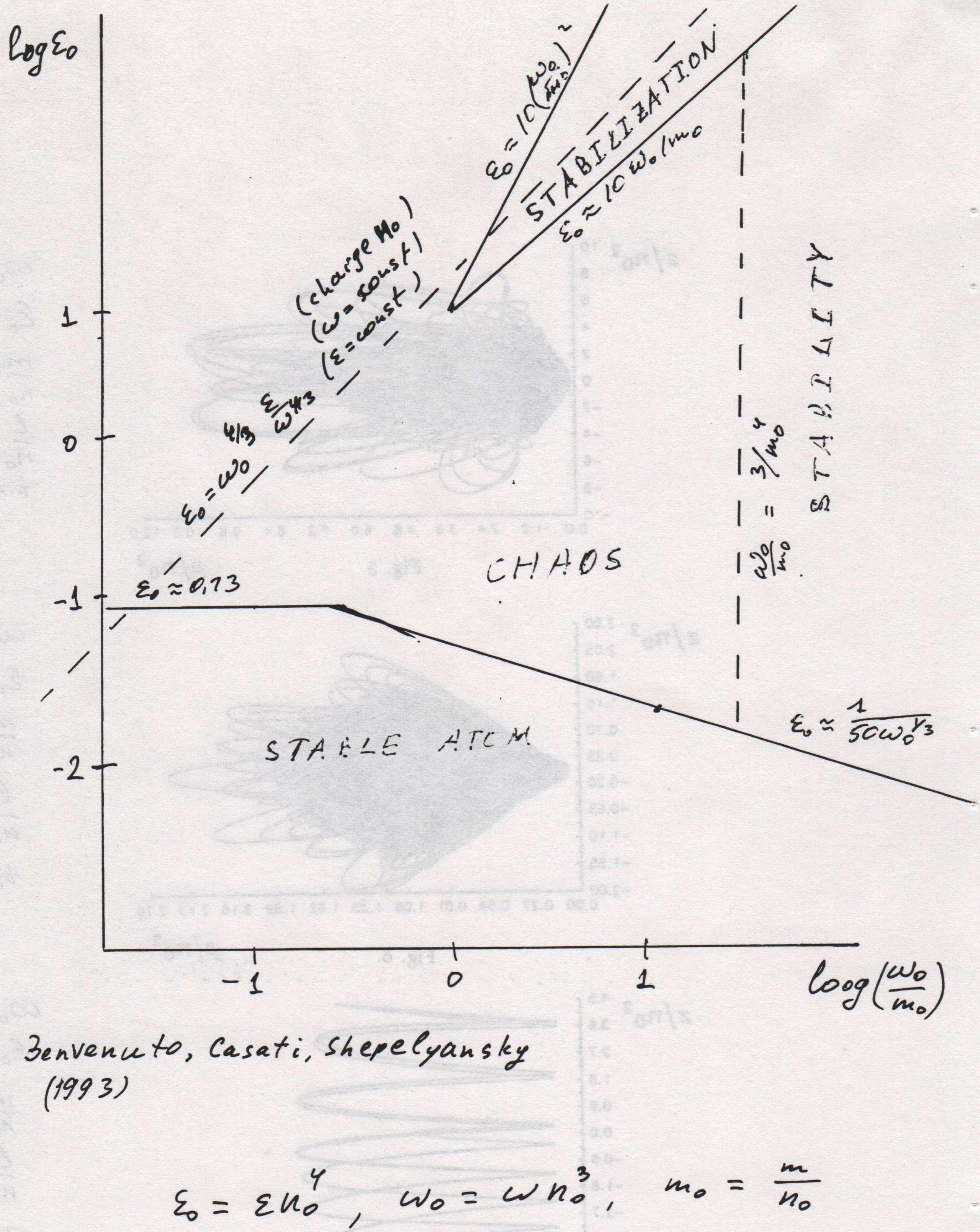
$\frac{m}{n_0} = 0.25$

$\frac{\ell}{n_0} = 0.3$

$t_{int} = 5 \cdot 10^4$ field period.

(D. S. 1992)

F34



Benvenuto, Casati, Shepelyansky (1993)

$$\varepsilon_0 = \varepsilon n_0^4, \quad \omega_0 = \omega n_0^3, \quad m_0 = \frac{m}{n_0}$$

$$\varepsilon_{stab\,0} \simeq 10\,\frac{\omega_0}{m_0} \qquad m_0^3 < \frac{3}{\omega_0}$$

$$\varepsilon_{destab\,0} \simeq \frac{164}{\pi}\,\frac{\omega_0^2}{m_0^2}.$$

F35

(49) Kramers-map approach for stabilization

1-d Kramers model (ρ-direction)

$$H = \frac{p_\rho^2}{2} + \frac{m^2}{2\rho^2} - \frac{1}{\left[\rho^2 + \frac{\varepsilon^2}{\omega^4}(\sin\gamma + \sin\omega t)^2\right]^{1/2}}$$

$$Z = -\frac{\varepsilon}{\omega^2}\sin\gamma = \text{const}$$

(F36 a,б)

$$\alpha = \frac{\varepsilon}{\omega^2} \ll n_0^2$$

Kramers map

$$\bar{E} = E + J\, h(\phi)\,; \quad \bar{\phi} = \phi + 2\pi\omega(-2\bar{E})^{-3/2}$$

$$S = \frac{\varepsilon m}{\omega} \gg 1 \qquad \text{stabilization parameter}$$

$$K = 6\pi\omega J n^5 > 1$$

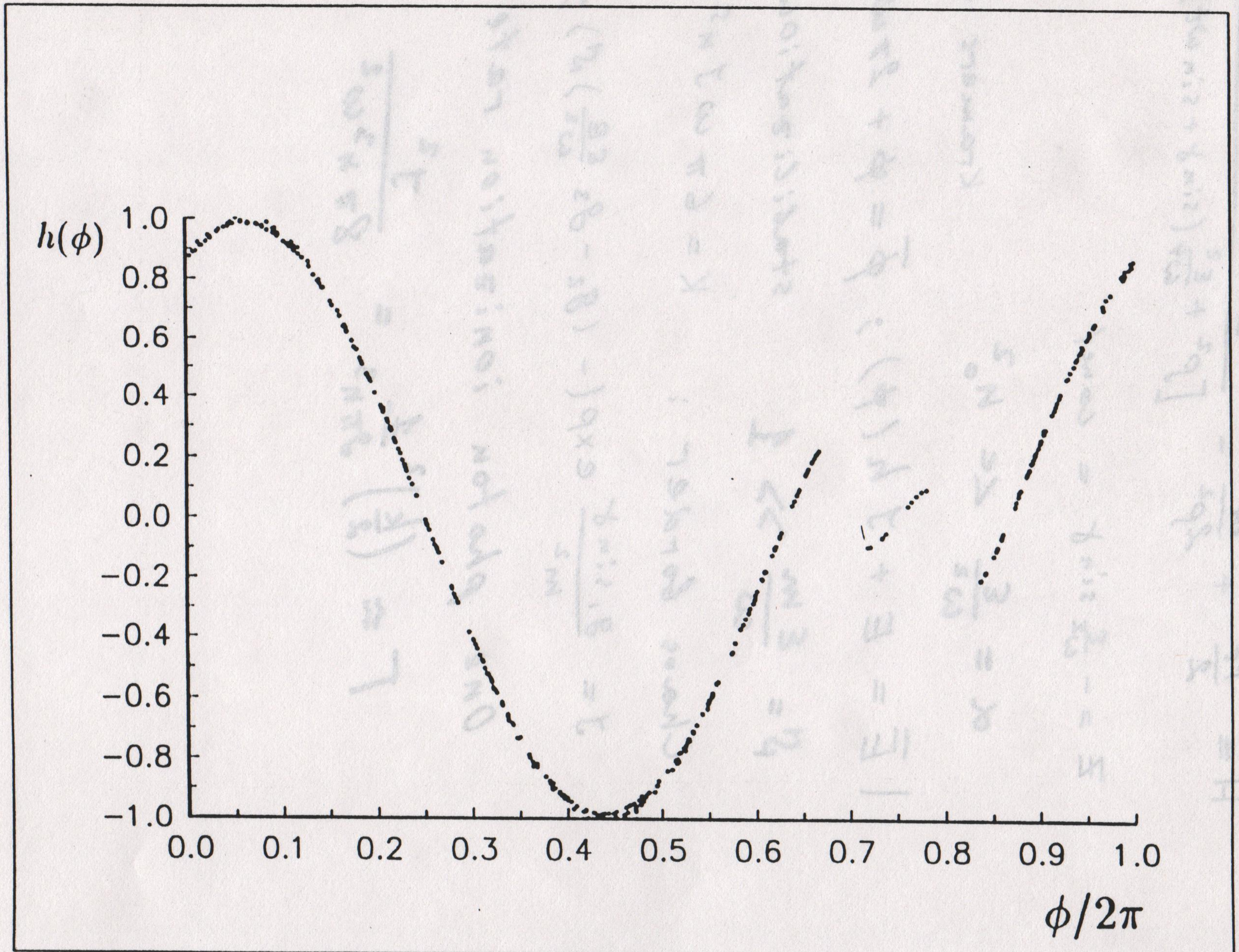
Chaos border:

$$J = \frac{g_1 \sin\gamma}{m^2} \exp\left(-\left(g_2 - g_3 \frac{\varepsilon E}{\omega^2}\right) S\right); \qquad g_1 \approx 0.1,\ g_2 \approx 0.2,\ g_3 \approx 0.1$$

One photon ionization rate:

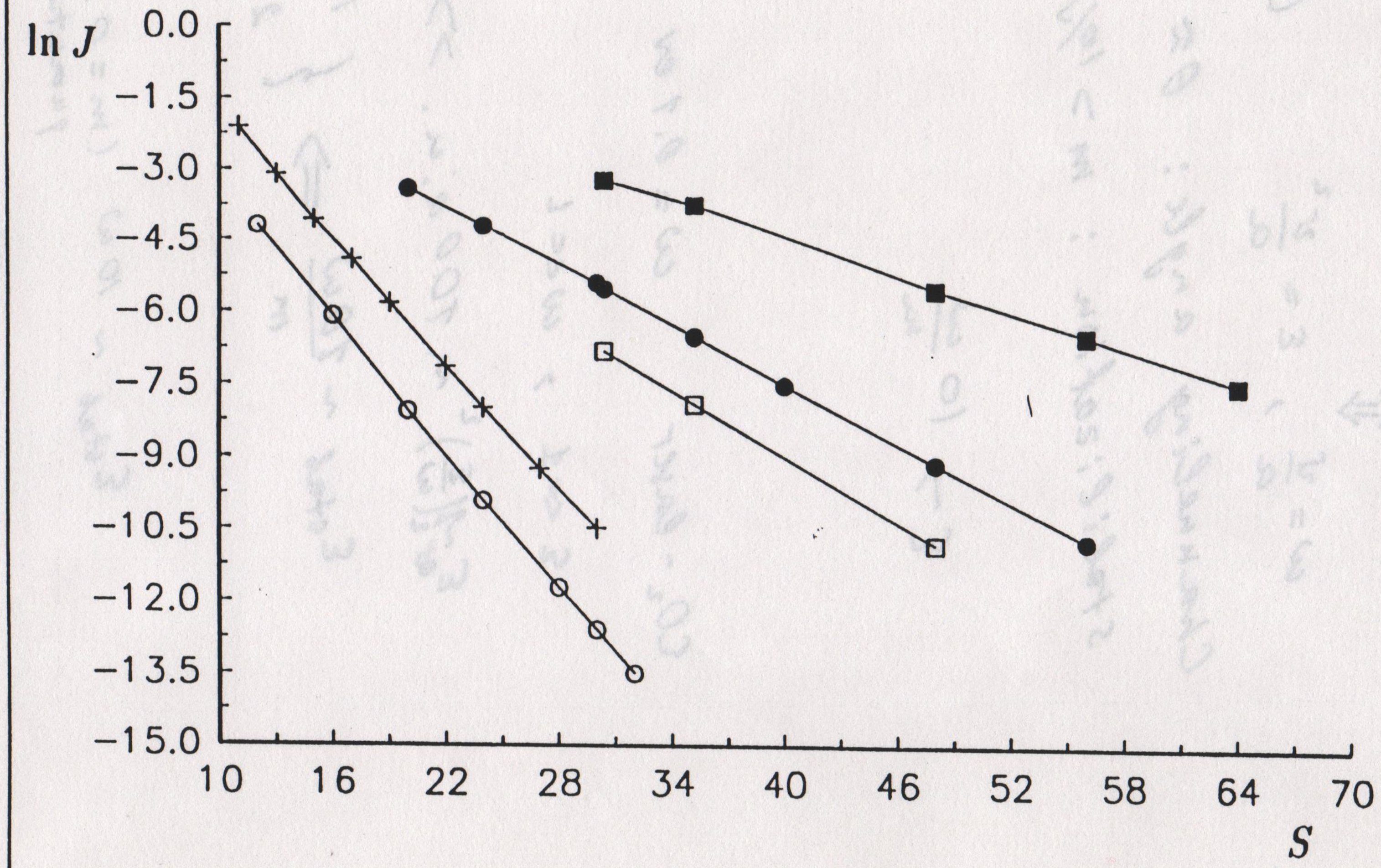
$$\Gamma \approx \left(\frac{k}{2}\right)^2 \frac{1}{2\pi n^3} = \frac{J^2}{8\pi n^3 \omega^2}$$

F36 (a)



$h(\phi)$
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\phi/2\pi$

F36(6)



ln J
0.0
−1.5
−3.0
−4.5
−6.0
−7.5
−9.0
−10.5
−12.0
−13.5
−15.0
10
16
22
28
34
40
46
52
58
64
70
S

Channeling Analogy

$$a = \frac{\varepsilon}{\omega^2}, \quad v = \frac{\varepsilon}{\omega} \qquad \left(\frac{\varepsilon}{\omega^2} \gtrsim n_0^2\right)$$

$$\Updownarrow$$

$$\omega = \frac{v}{a}, \quad \varepsilon = \frac{v^2}{a} \qquad (v \sim c \sim 137)$$

Channeling angle: $\theta \approx \frac{v_\perp}{v} \sim \frac{1}{v\sqrt{a}} \sim \left(\frac{\omega^{4/3}}{\varepsilon}\right)^{3/2} \ll 1$

stabilization : $m > 10/v$ (always)

$$\varepsilon > 10\,\frac{\omega}{m}$$

CO_2 - laser $\qquad \omega = 0.1$ ev $\left(\frac{1}{300},\ \omega_0 \approx 100 \text{ for } n_0 = 30\right)$

$\varepsilon \ll 1, \quad \omega \ll 1$

$E_{at} \sim \frac{1}{2}\left(\frac{\varepsilon}{\omega}\right)^2 \sim 100$ a.u. $\gg 13$ ev

$\varepsilon_{stab} \sim \frac{10\omega}{m} \Rightarrow \begin{cases} 10^7 \text{ V/cm} & (m \sim 15) \\ 2\cdot 10^8 \text{ V/cm} & (m = 0, 1) \end{cases}$

quantum case 0 as 1

$m \to m+1$

$\varepsilon_{stab} \sim 10\omega$ $(m = 0)$ quantum

Physical scales

Chaotic autoionization of molecular Rydberg states (Benvenuto, Casati, Shepelyansky (1994))

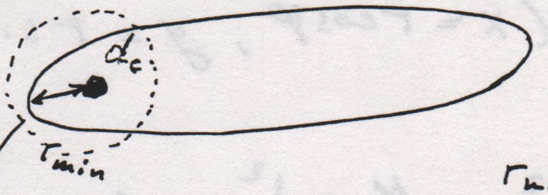
$e = \hbar = m = 1$ (a.u.)

$E_0 = -\frac{1}{2n^2} \sim \frac{1}{r} \rightarrow r \sim n^2$

$\Delta E \sim \omega \sim \frac{1}{n^3}$

$H_{int} = \varepsilon z \sim H_0 = E_0 \rightarrow \varepsilon \sim 1/n^4$

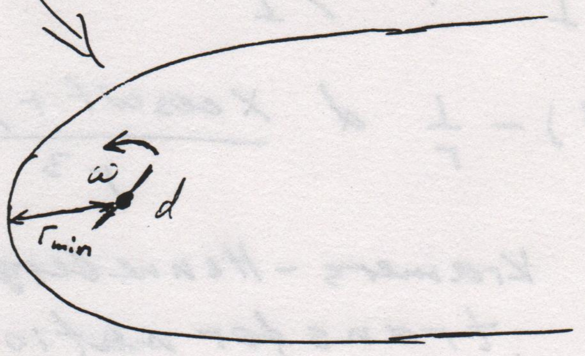
Opposite to T. Seligman, M. Lombardi, P. Labastie



$r_{min} \gg d_c \sim d$

$r_{min} = n^2(1-e) \approx \; ; \; e = \sqrt{1 - \frac{l^2}{n^2}}$

$\approx l^2/2$



$\omega \sim \frac{1}{n_0^3}$

Born-Oppenheimer

⇓

HET

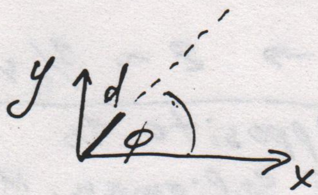
$n_0 \approx 70$; $\omega \approx \frac{1}{n_0^3} \Rightarrow 20\,GHz$

$\varepsilon \sim \frac{0.1}{n_0^4} \Rightarrow 20\ V/cm$

Interaction between rotating dipole (core) and Rydberg electron

$m = l$ (plane motion)

$$H = \frac{1}{2}(\hat{p}_x^2 + \hat{p}_y^2) + \frac{L^2}{2I} - \frac{1}{r} + d\,\frac{x\cos\phi + y\sin\phi}{r^3}$$



$$J = L + m = const$$

$(x = r\cos\varphi,\ y = r\sin\varphi)$

2 freedoms

$$H = \frac{p_r^2}{2} + \frac{m^2}{2r^2} + \frac{(J-m)^2}{2I} - \frac{1}{r} + \frac{d\cos\psi}{r^2}$$

$\psi = \phi - \varphi \qquad (m, \psi)\ (p_r, r)$

$$\dot{\phi} = \omega = L/I = (J-m)/I$$

$$H = \frac{1}{2}(p_x^2 + p_y^2) - \frac{1}{r}\ d\ \frac{x\cos\omega t + y\sin\omega t}{r^3}$$

$\Downarrow$

$d \ll r_{min}$

Kramers-Henneberger transformation (exact)

$$H = \frac{1}{2}(p_x^2 + p_y^2) - \frac{1}{((x + d\cos\omega t)^2 + (y + d\sin\omega t)^2)^{1/2}}$$

$$H = \frac{\tilde{p}_x^2 + \tilde{p}_y^2}{2} - \frac{1}{r} + \mathcal{E}(x\cos\omega t + y\sin\omega t)$$

$\mathcal{E} = d\omega^2$

microwave problems

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Hydrogen in circular polarized monochromatic field

$$\begin{cases} \bar{N} = N + k \sin \Phi \\ \bar{\Phi} = \Phi + 2\pi \omega (-2\omega \bar{N})^{3/2} \end{cases}$$ Kepler map

$N = E/\omega$; $E = -1/2n^2$; $\omega_0 = \omega n_0^3 > 1$

F37 F38 F39 $k = 2.6\, d\, \omega^{1/3} \left(1 + \frac{l^2}{2n^2} + 1.09 \omega^{1/3} l\right)$

F40 $m = l < \left(\frac{3}{\omega}\right)^{1/3} \longrightarrow d \ll r_{min} \approx \frac{l^2}{2} < \frac{1}{\omega^{2/3}}$

Chaos border

Chirikov st. map

$\bar{N} = N + k \sin \Phi$

$\bar{\Phi} = \Phi + T \bar{N}$

$T = 6\pi \omega^2 n_0^5$

Diffusion $K = kT > 1$ $D = k^2/2$

$t_D = N_I^2 / D$; $N_I = \frac{1}{2 n_0^2 \omega}$

In rotatin frame

$$H = \frac{p_r^2}{2} + \frac{m^2}{2r^2} - \frac{1}{r} - \omega m + d \omega^2 \cos \psi$$

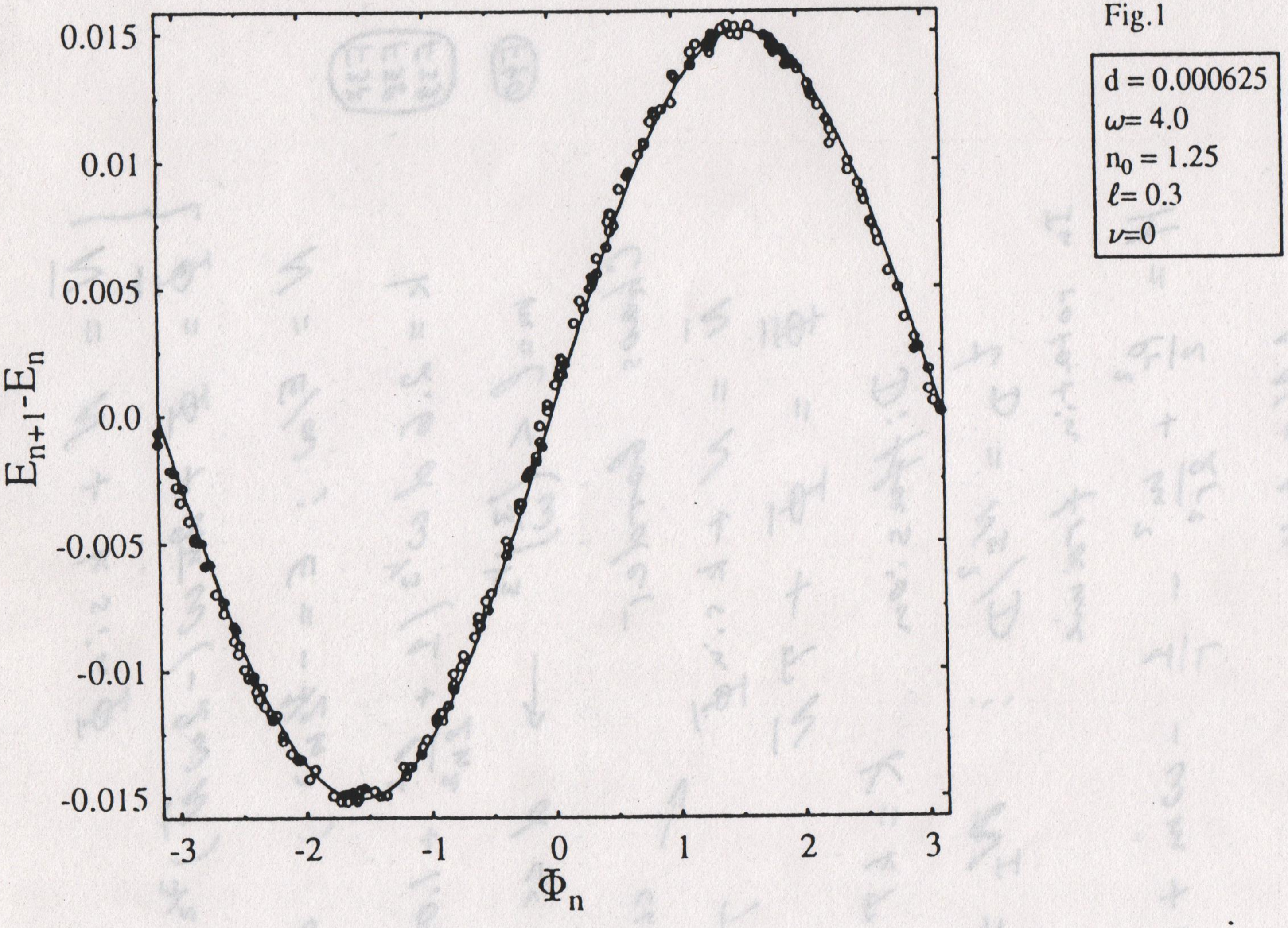
$\Delta N = \Delta m$

$\Delta E = k\omega \ll E_{mol} = \frac{L^2}{2I} \approx \omega L$

$1 < k \ll L$

Halley comet

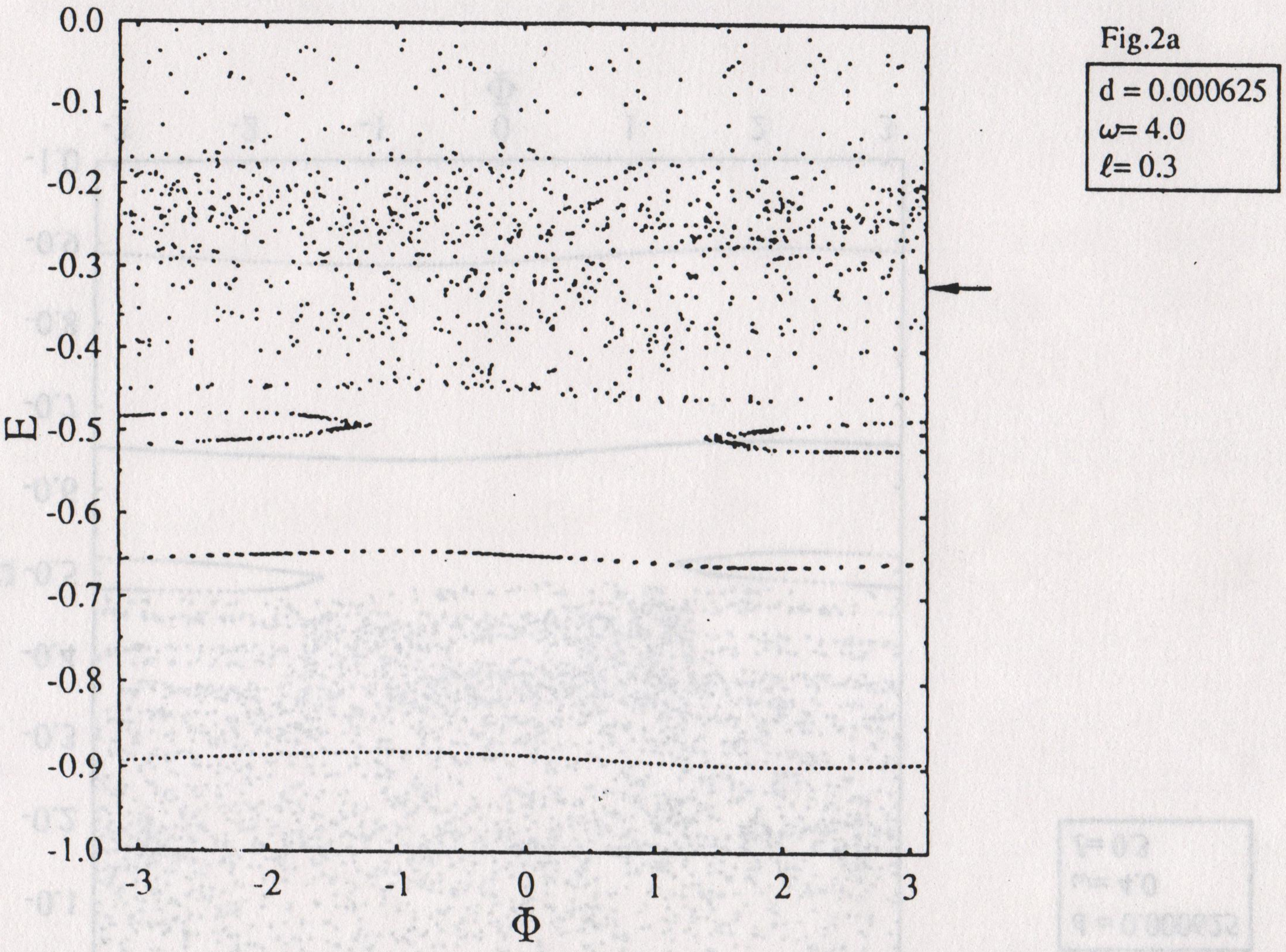
Fig.1


E_{n+1}-E_n
0.015
0.01
0.005
0.0
-0.005
-0.01
-0.015
-3
-2
-1
0
1
2
3
Φ_n
d = 0.000625
ω= 4.0
n_0 = 1.25
ℓ= 0.3
ν=0

F38

111

Fig.2a



F 39

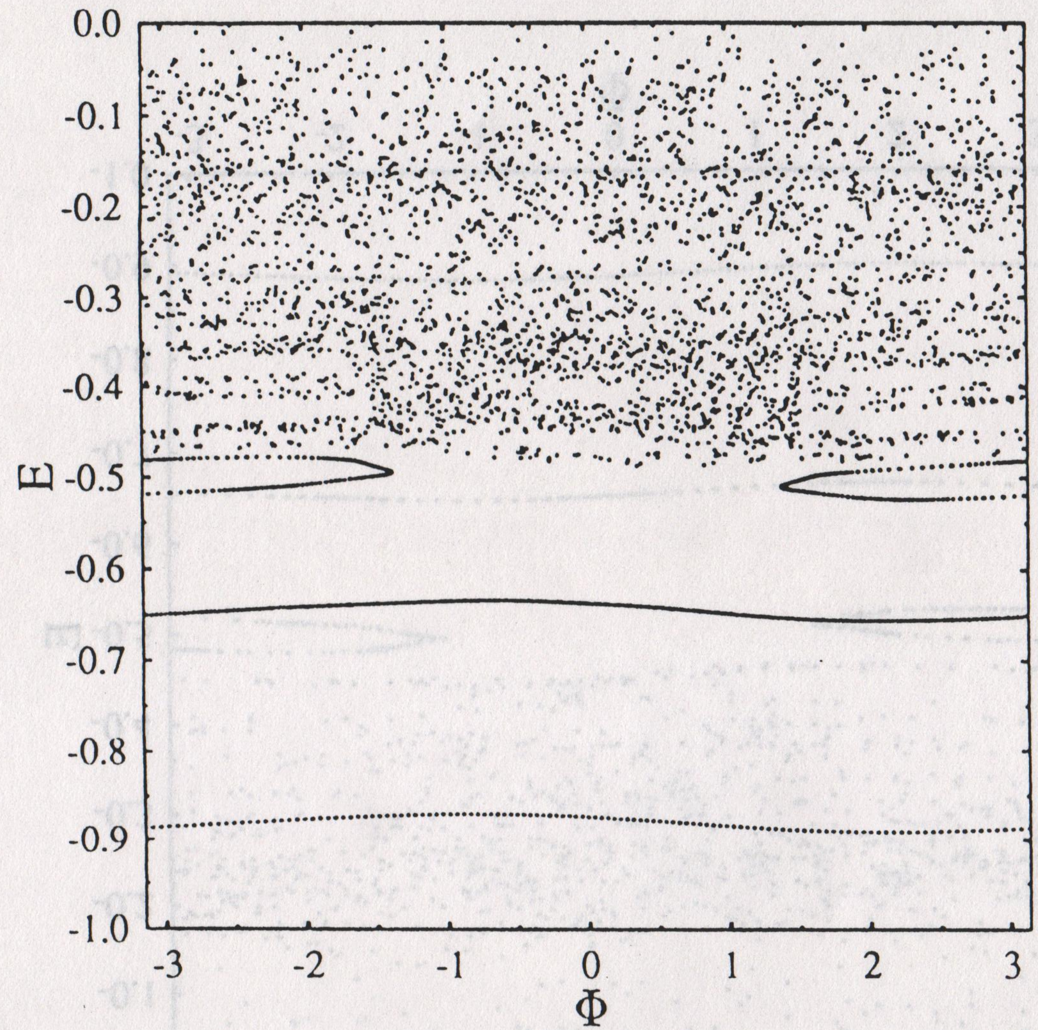


Fig.2b (Kepler Map)

$d = 0.000625$
$\omega = 4.0$
$\ell = 0.3$

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* Quantum stability border

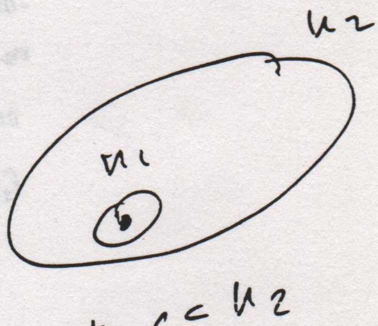
$k > 1 \rightarrow d \gtrsim \frac{1}{5\omega^{4/3}}$ Shuryak border (1976)

for $n_0 = 40$, $\omega_0 = 4 \rightarrow d > 5$

* Localization of chaos

$1 \ll l_\phi \approx \frac{k^2}{2} < N_I$

$d < 1/(\sqrt{6}\, n_0 \omega^{5/6})$



* doubly excited electrons

$\omega \sim \frac{1}{n_1^3} \frac{\partial \mu_1}{\partial l_1} \sim \frac{1}{n_2^3}$ $n_1 \ll n_2$

$d \sim n_1 b_1$ (b_1 - Runge-Lenz vector)

$n_1^2 \ll l^2$ ($d \ll r_{min}$)

$l < \left(\frac{3}{\omega}\right)^{1/3} \rightarrow \omega n_1^3 \ll 1$

* quadrupole $Q \sim d^2$

$\Delta E \sim k\omega \sim d^2 \omega^2$

Chaotic dynamics of comet Halley

B. V. Chirikov, V. V. Vecheslavov

Astr. Astr. 1989

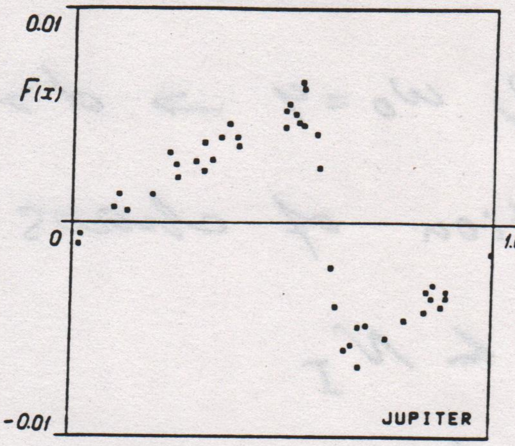
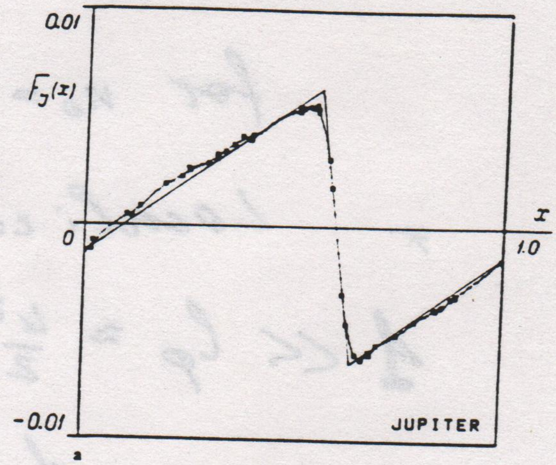


Fig. 1. The full perturbation of comet Halley vs. Jupiter's phase



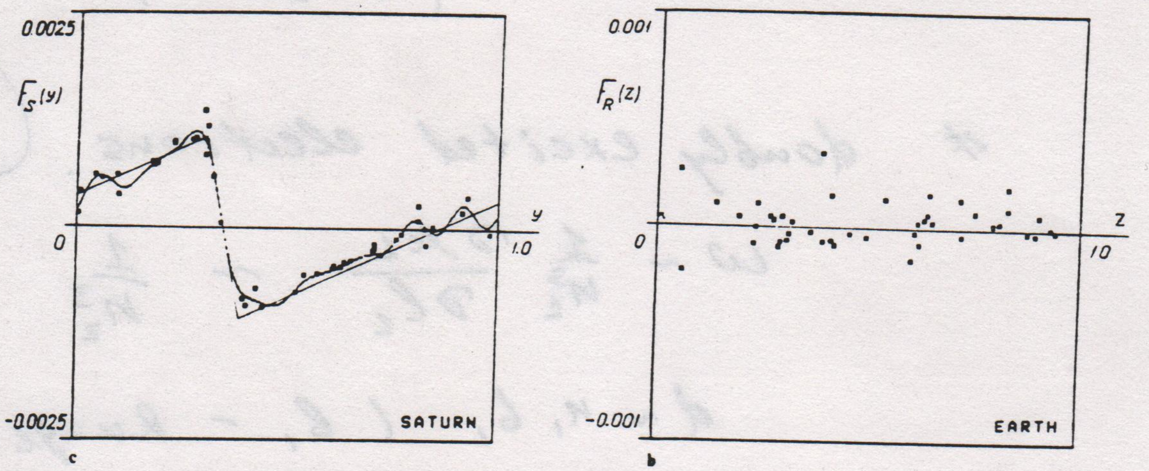


Fig. 2a–c. Comet Halley's perturbation by Jupiter (a), by Saturn (b), and residual perturbation (c). Curves are Fourier approximation (FA), straight lines are "saw-tooth" approximation (STA)

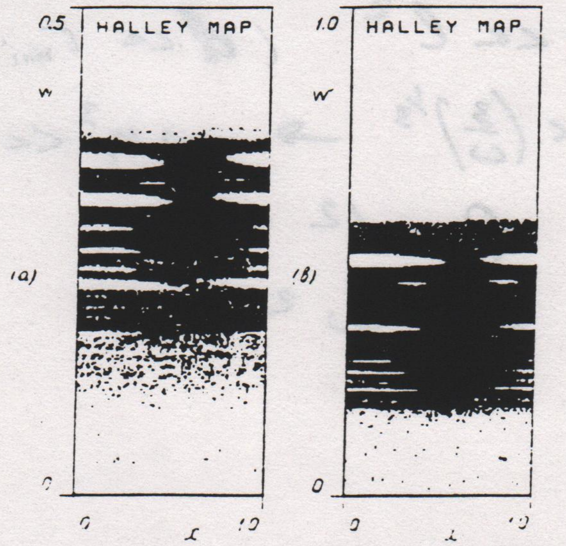


Fig. 5a and b. Two examples of comet Halley's global dynamics in model (3), $N_p \approx 4.1 \cdot 10^4$, $t_p \approx 4.5 \cdot 10^6$ yr (a); with a variable nongravitational acceleration (20), $F \sim 3 \cdot 10$ ..., N ... 10 ..., $\bar{N}_{pd} \approx 3.1 \cdot 10$ ..., $\bar{t}_{pd} \approx 2.1 \cdot 10$ ... yr (b)

$$W_{n+1} = W_n + F(X_n)$$

$$X_{n+1} = X_n + W_{n+1}^{-3/2}$$

$$W_n \approx -2E_n$$

Diffusive ionization

$$\bar{t}_D \approx 4 \cdot 10^6 \text{ years}$$

(46)