Google matrix of world trade network

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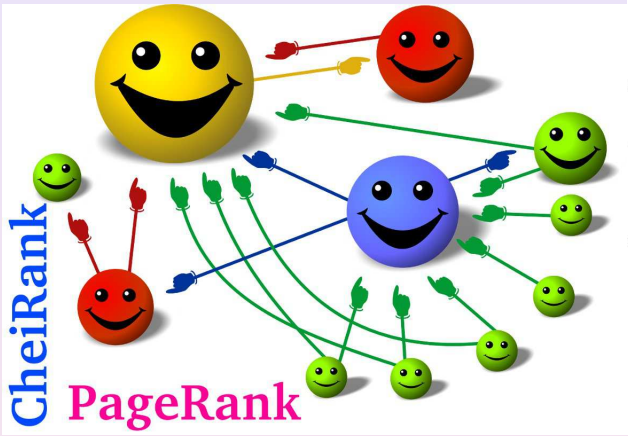
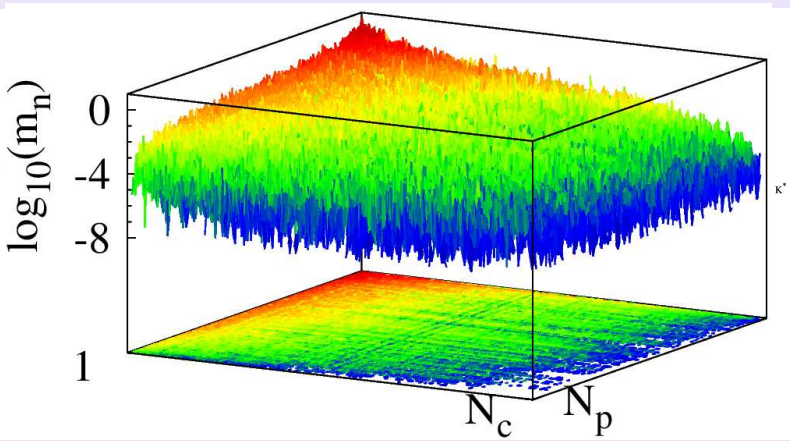
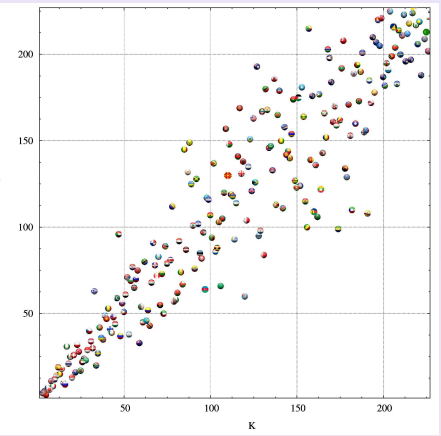
www.quantware.ups-tlse.fr/dima

EC FET Open NADINE project





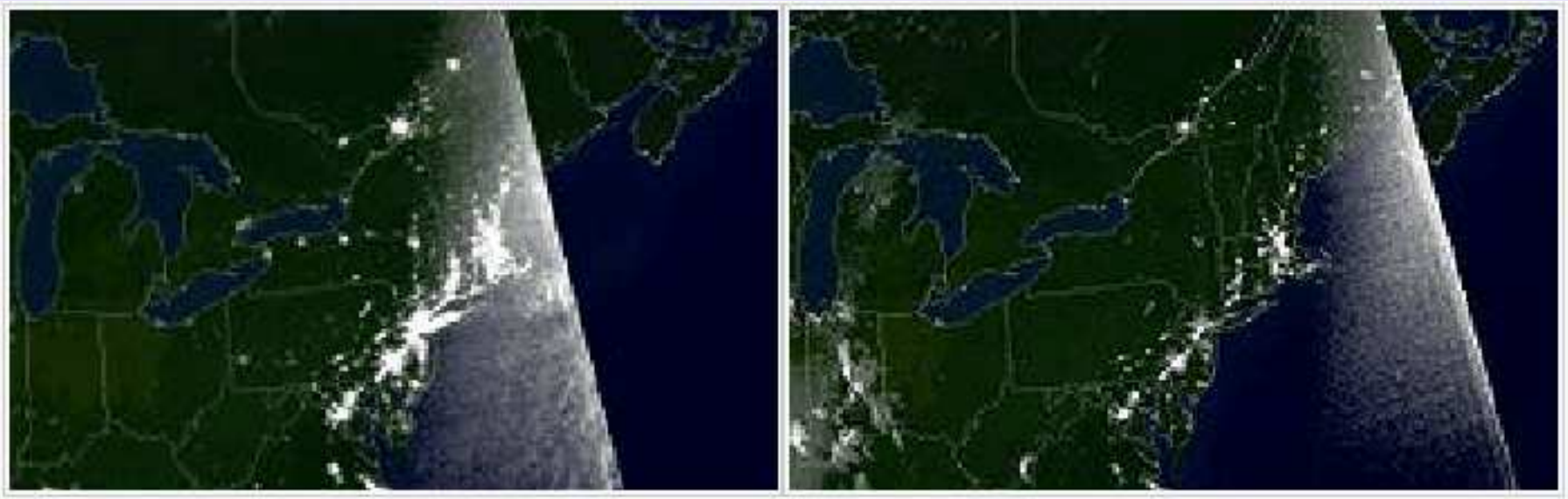


* Quantware group: classical/quantum chaos, dynamical systems, large matrices
* How Google search works, PageRank, CheiRank
* Examples of directed networks: Wikipedia, University networks, DvvaDi search; Ulam networks, Linux Kernel network, fractal Weyl law
* World trade from UN COMTRADE 1962 - 2009: arxiv:1103.5027
  => democratic treatment of all UN countries; ecology analysis arxiv:1201.3584
* Towards ranking of bank financial flows: WWW ==> WBW

S.Brin and L.Page, Comp. Networks ISDN Systems **30**, 107 (1998)

Monitoring of grids and networks

Any large network requires monitoring ...



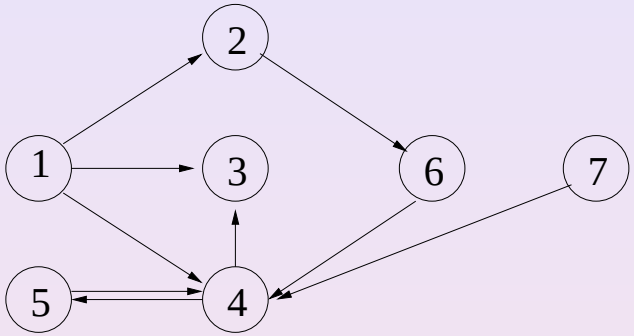
NOAA satellite imagery one day before and the night of the blackout.

Example of Northeast blackout of electical power grid, Aug 14, 2003.
Wikipedia article “Northeast blackout of 2003”

==> Analysis of network flows:
==> World Wide Web with $\sim 10^{11}$ *sites*
==> project launched at CERN by Tim Berners-Lee, 1991
==> World Bank Web exists (SWIFT ...)

How Google works

Markov chains (1906) and Directed networks



Weighted adjacency matrix

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

For a directed network with N nodes the adjacency matrix $\mathbf{A}$ is defined as $A_{ij} = 1$ if there is a link from node j to node i and $A_{ij} = 0$ otherwise. The weighted adjacency matrix is

$$S_{ij} = A_{ij} / \sum_k A_{kj}$$

In addition the elements of columns with only zeros elements are replaced by $1/N$.

How Google works

Google Matrix and Computation of PageRank

$\mathbf{P} = \mathbf{SP} \Rightarrow \mathbf{P}$= stationary vector of $\mathbf{S}$; can be computed by iteration of $\mathbf{S}$.

To remove convergence problems:

- Replace columns of 0 (dangling nodes) by $\frac{1}{N}$:

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{7} & 0 & 1 & 1 & 1 \\ 0 & 0 & \frac{1}{7} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & 0 \end{pmatrix}; \mathbf{S}^* = \begin{pmatrix} \frac{1}{7} & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & \frac{1}{7} \\ \frac{1}{7} & 0 & 0 & 0 & 0 & 1 & \frac{1}{7} \\ \frac{1}{7} & 0 & 0 & 0 & 0 & 0 & \frac{1}{7} \\ \frac{1}{7} & 0 & \frac{1}{2} & 0 & 1 & 0 & \frac{1}{7} \\ \frac{1}{7} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{7} \\ \frac{1}{7} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{7} \\ \frac{1}{7} & 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{7} \end{pmatrix}.$$

- To remove degeneracies of $\lambda = 1$, replace $\mathbf{S}$ by **Google matrix**
  $\mathbf{G} = \alpha\mathbf{S} + (1-\alpha)\frac{\mathbf{E}}{N}$; $GP = \lambda P$ => Perron-Frobenius operator
- α models a random surfer with a random jump after approximately 6 clicks (usually $\alpha = 0.85$); PageRank vector => P at $\lambda = 1$ ($\sum_j P_j = 1$).
- CheiRank vector P^*: $G^* = \alpha\mathbf{S}^* + (1-\alpha)\frac{\mathbf{E}}{N}$, $G^*P^* = P^*$
  ($\mathbf{S}^*$ with inverted link directions)
  Fogaras (2003) ... Chepelianskii arXiv:1003.5455 (2010) ...

Real directed networks

Real networks are characterized by:

- **small world property**: average distance between 2 nodes $\sim \log N$
- **scale-free property**: distribution of the number of ingoing or outgoing links $\rho(k) \sim k^{-\nu}$

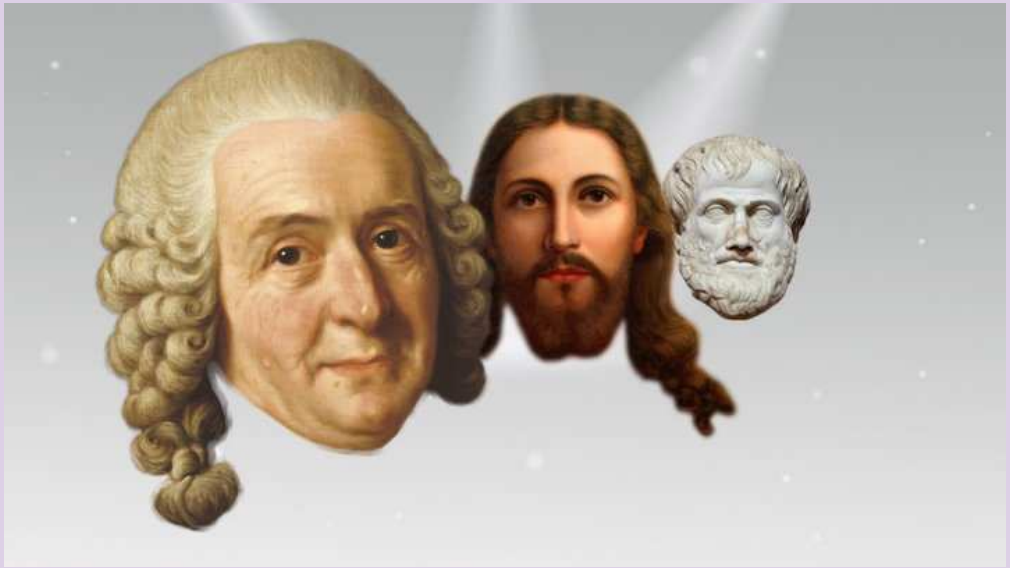
PageRank vector for large WWW:

- $P(K) \sim 1/K^{\beta}$, where K is the ordered rank index
- number of nodes N_n with PageRank P scales as $N_n \sim 1/P^{\nu}$ with numerical values $\nu = 1 + 1/\beta \approx 2.1$ and $\beta \approx 0.9$.
- PageRank $P(K)$ on average is proportional to the number of ingoing links
- CheiRank $P^*(K^*) \sim 1/K^{*\beta}$ on average is proportional to the number of outgoing links ($\nu \approx 2.7; \beta = 1/(\nu - 1) \approx 0.6$)
- WWW at present: $\sim 10^{11}$ web pages

Donato *et al.* EPJB **38**, 239 (2004)

Top historical figures of 24 Wikipedia editions

Top global PageRank historical figures: Carl Linnaeus, Jesus, Aristotle ...

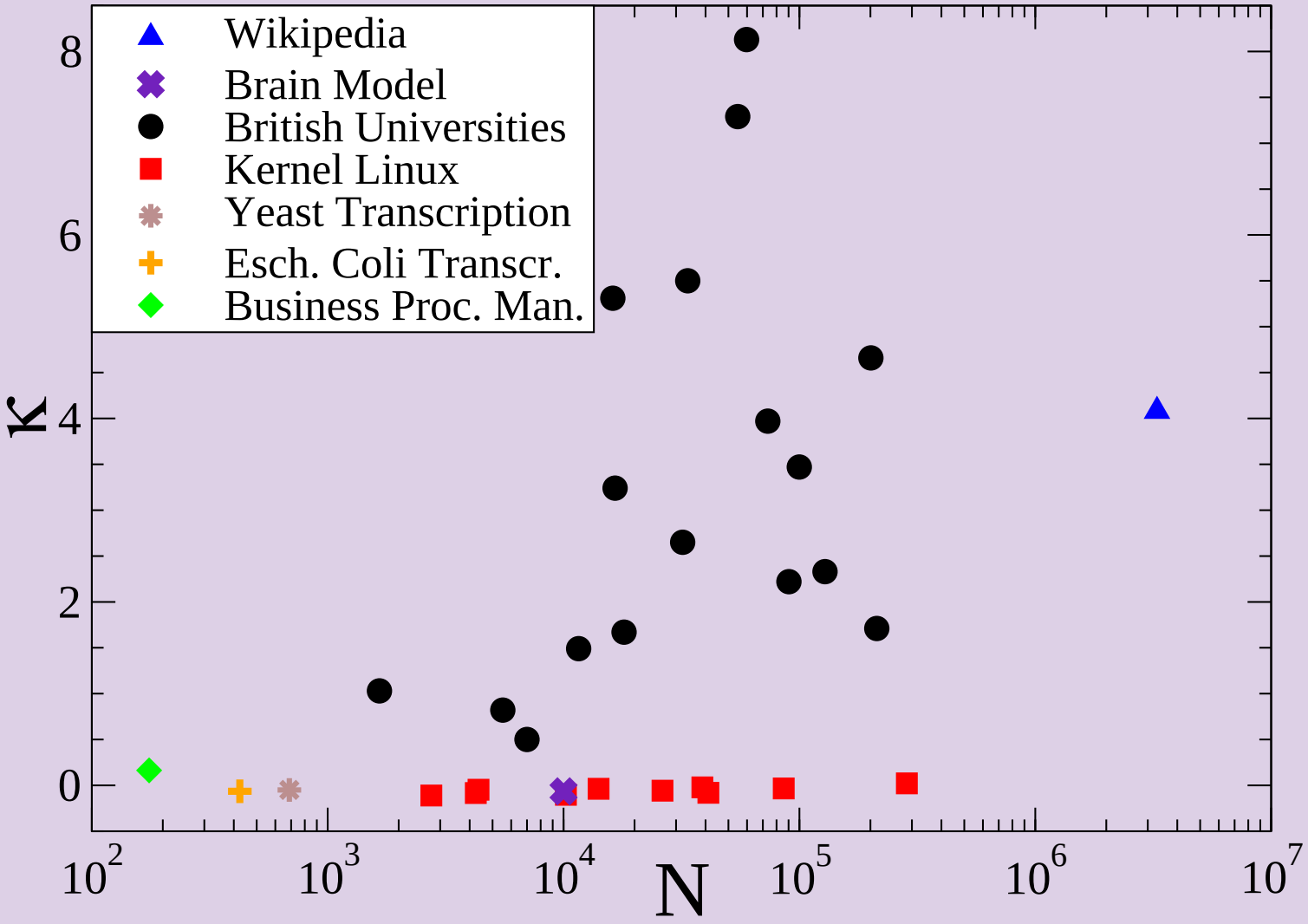


Y.-H.Eom,P.Aragon, D.Laniado, A.Kaltenbrunner, S.Vigna, DLS arXiv:1405.7183[cs.SI] (2014) Media highlights: The Guardian, The Independent, The Washington Post, France24, Uppsala Universitet: “Carl Linnaeus ranked most influential person of all time” ...

Competitors: MIT Pantheon project; Stony-Brook NY (2013)

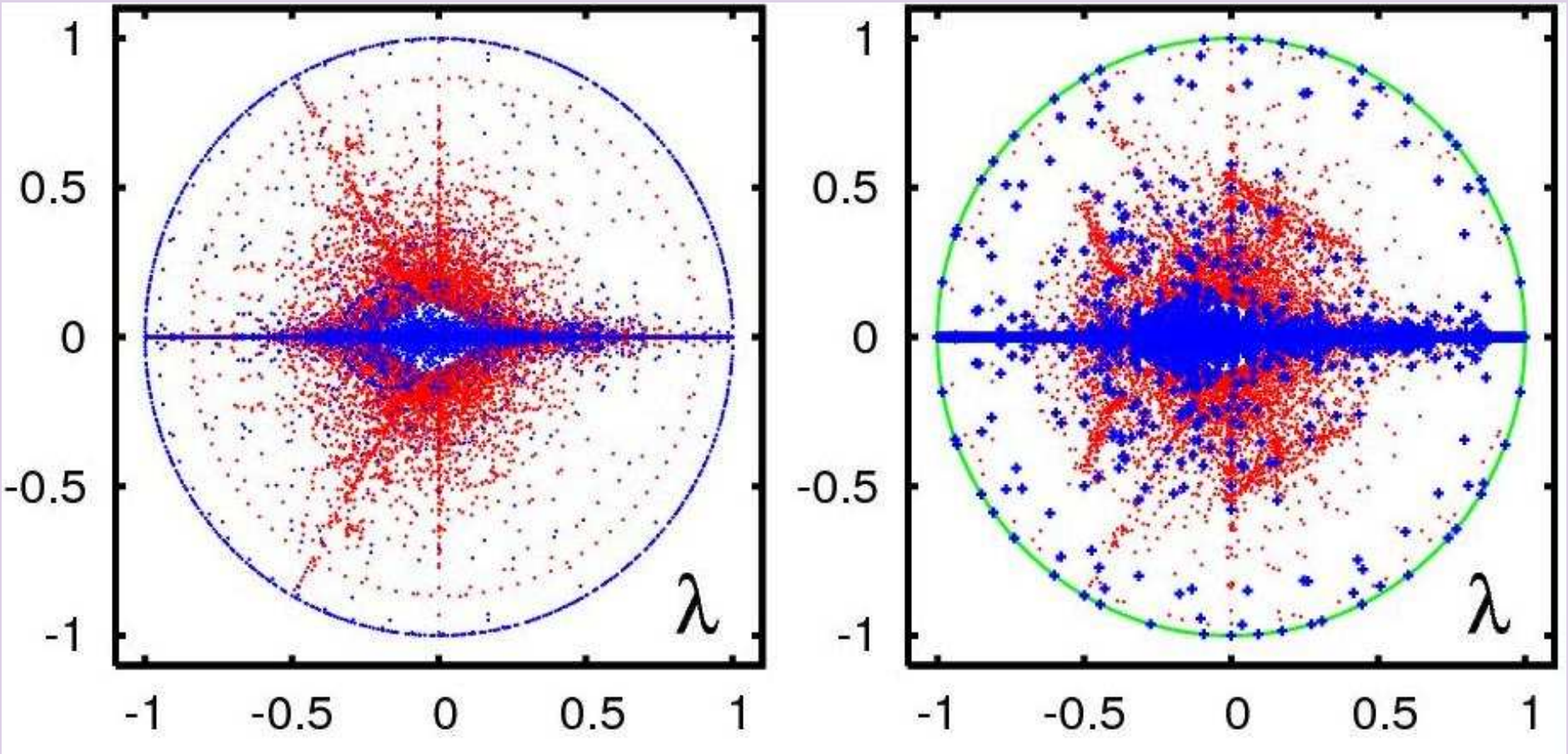
PageRank 10 WIKI RU: Peter the Great, Carl Linnaeus, Napoleon, Joseph Stalin, Vladimir Lenin, Catherine the Great, Jesus, Aristotle, Vladimir Putin

Correlator of PageRank and CheiRank



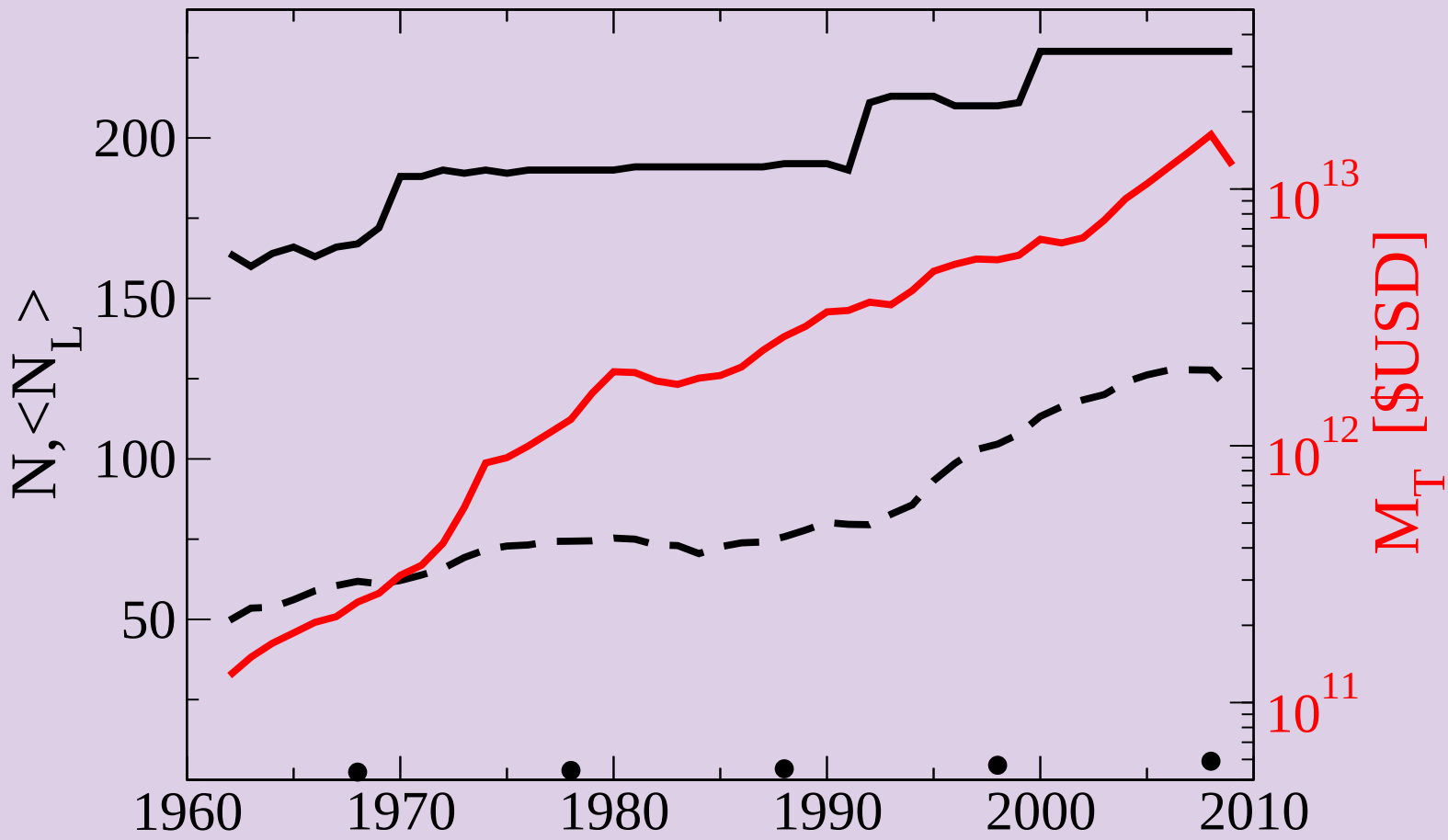
$$\kappa = N \sum_i P(K(i)) P^*(K^*(i)) - 1$$

Spectrum of UK University networks



Arnoldi method: Spectrum of Google matrix for Univ. of Cambridge (left) and Oxford (right) in 2006 ($N \approx 200000$, $\alpha = 1$). [Frahm, Georgeot, DS arxiv:1105.1062 (2011)]

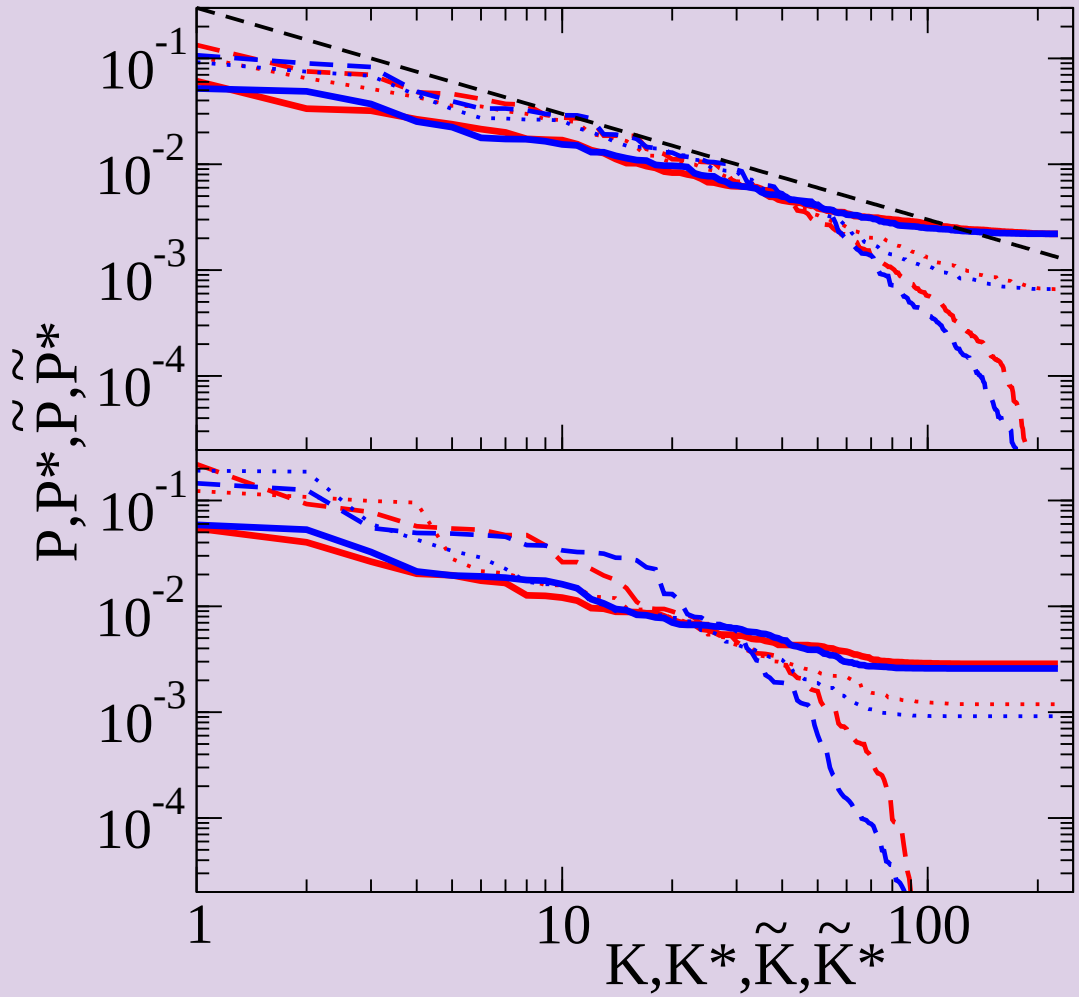
World trade network (WTN) of United Nations COMTRADE 1962-2009



Number of countries (black), links (dashed/points) and mass volume in USD (red)

Leonardo Ermann, DS arxiv:1103.5027 (2011)

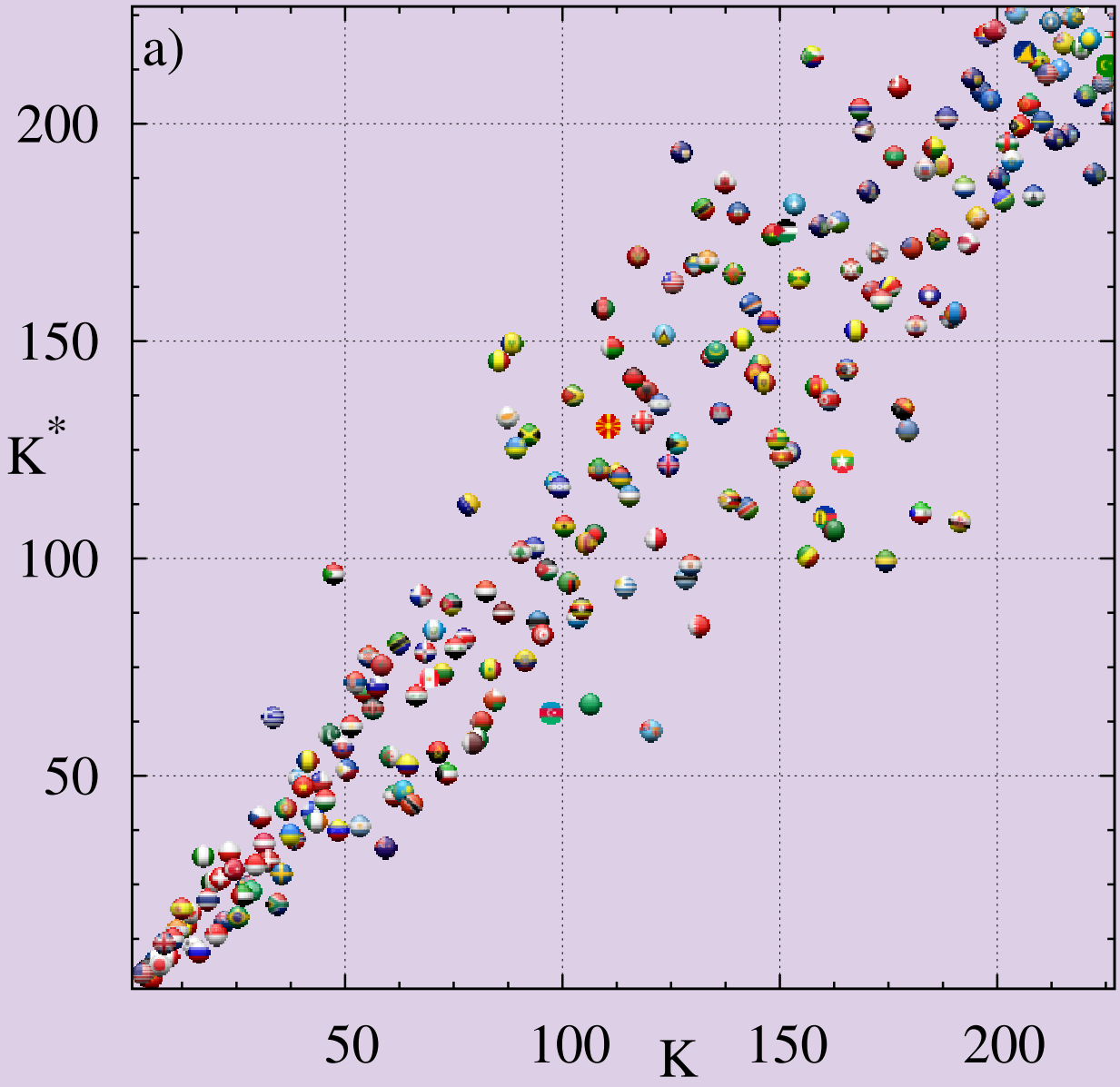
PageRank, CheiRank of World Trade



Year 2008: Probabilities of PageRank $P(K)$ (red), CheiRank $P^*(K^*)$ (blue) for all commodities (top) and crude petroleum (bottom), $\alpha = \mathbf{0.5}; 0.85$ (full/dotted); (dashed curves are for ImportRank, ExportRank); dashed line Zipf law $P \sim 1/K$; 227 countries

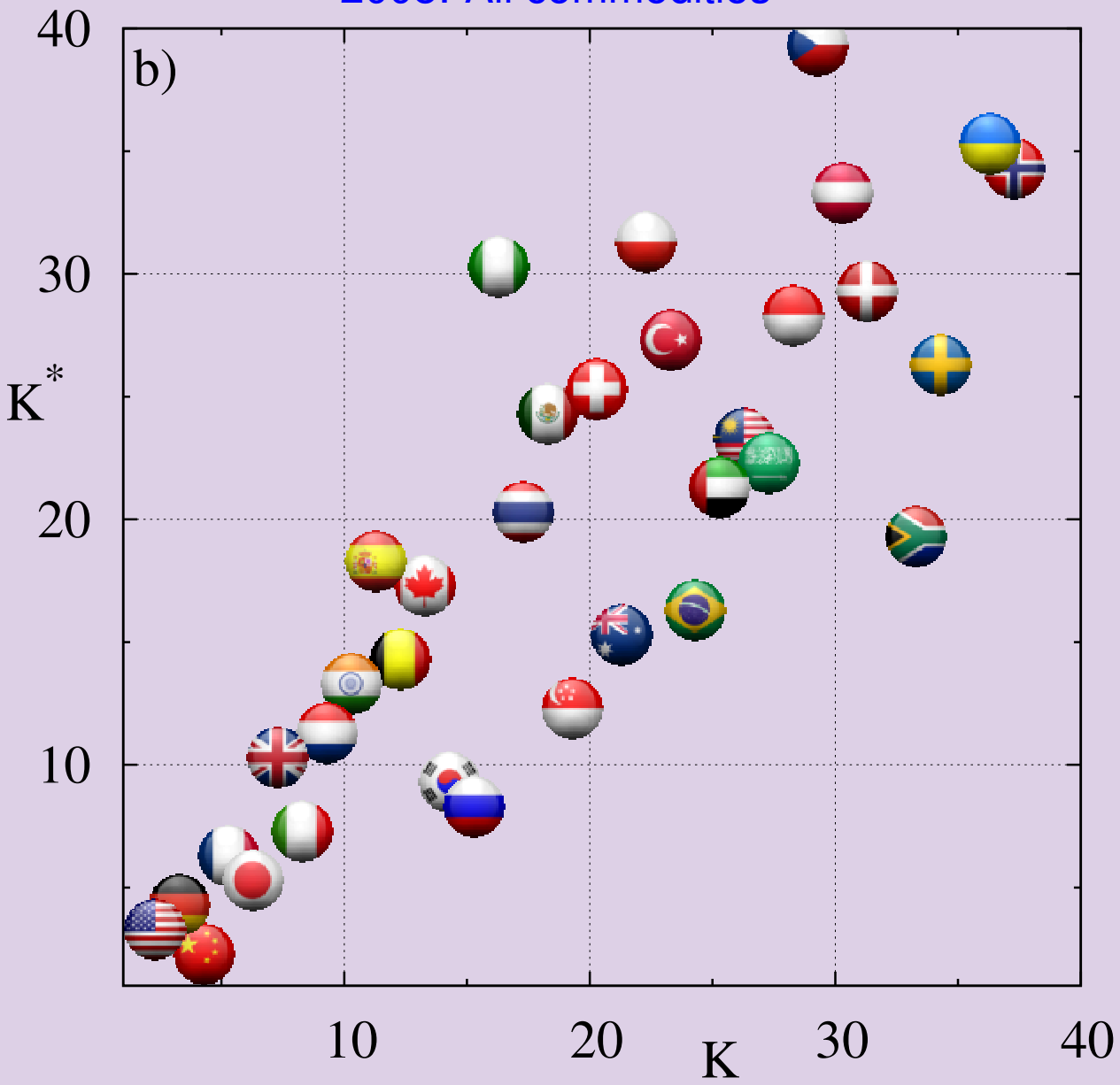
Ranking of World Trade

2008: All commodities



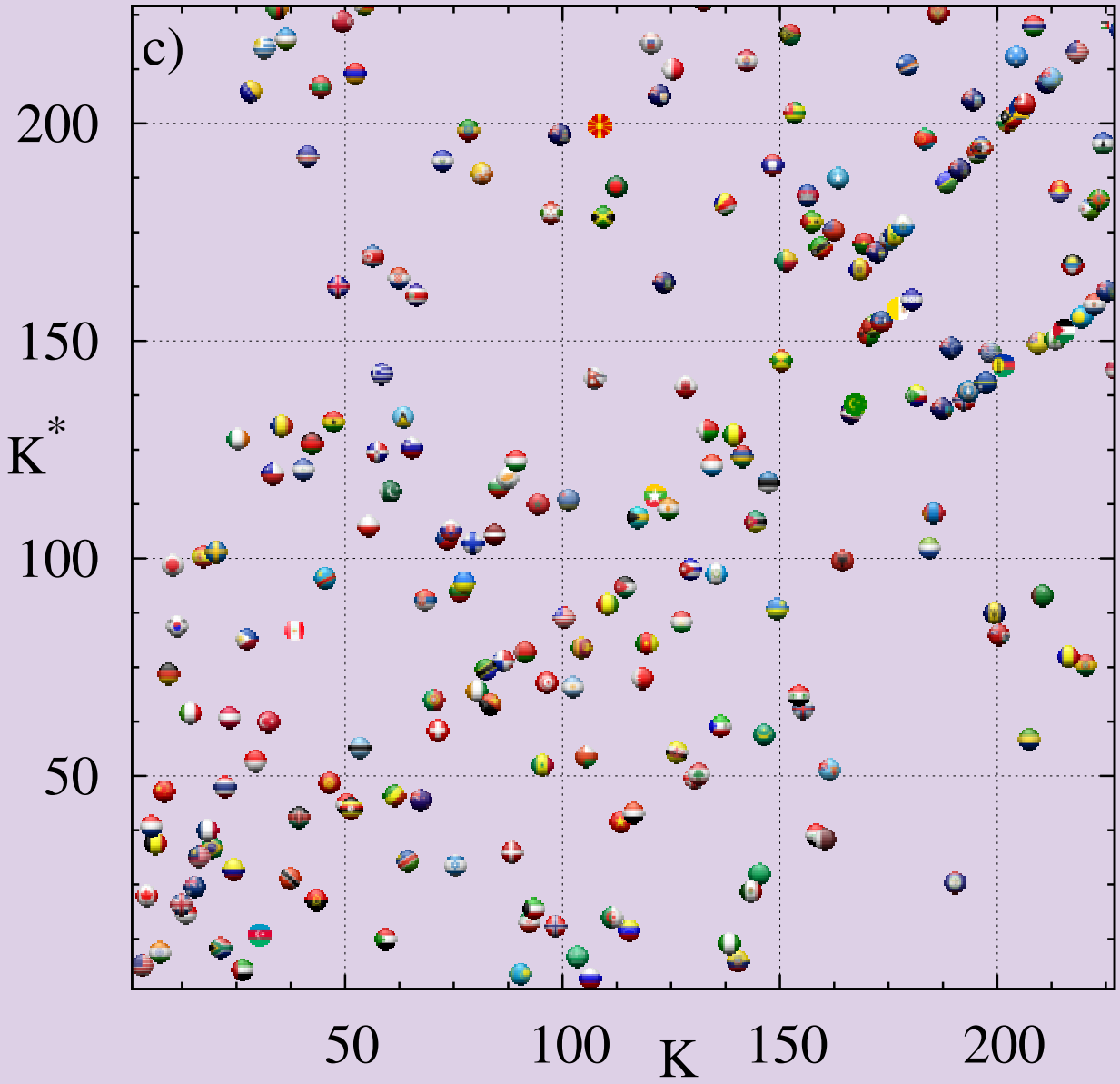
Ranking of World Trade

2008: All commodities



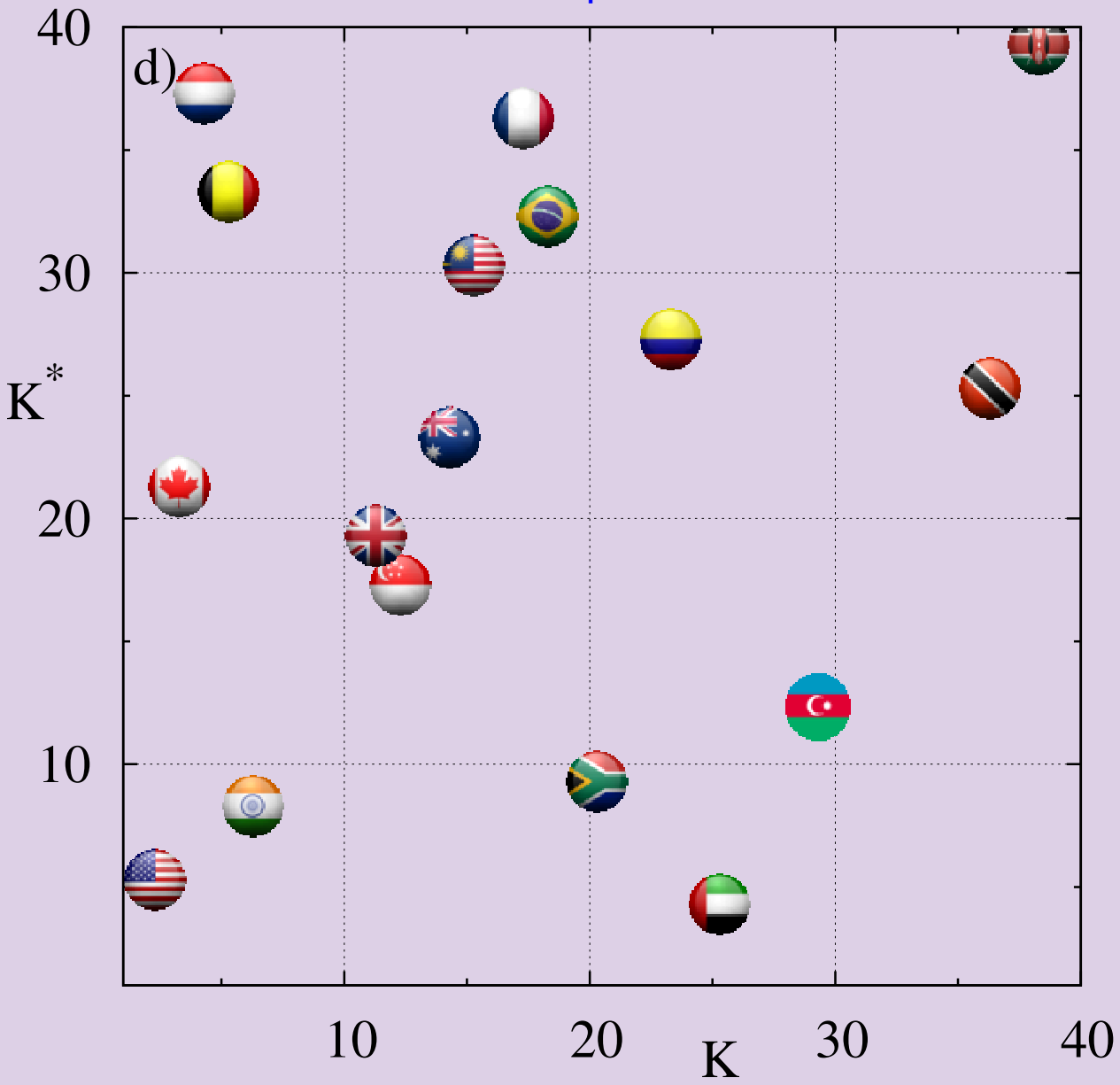
Ranking of World Trade

2008: Crude petroleum



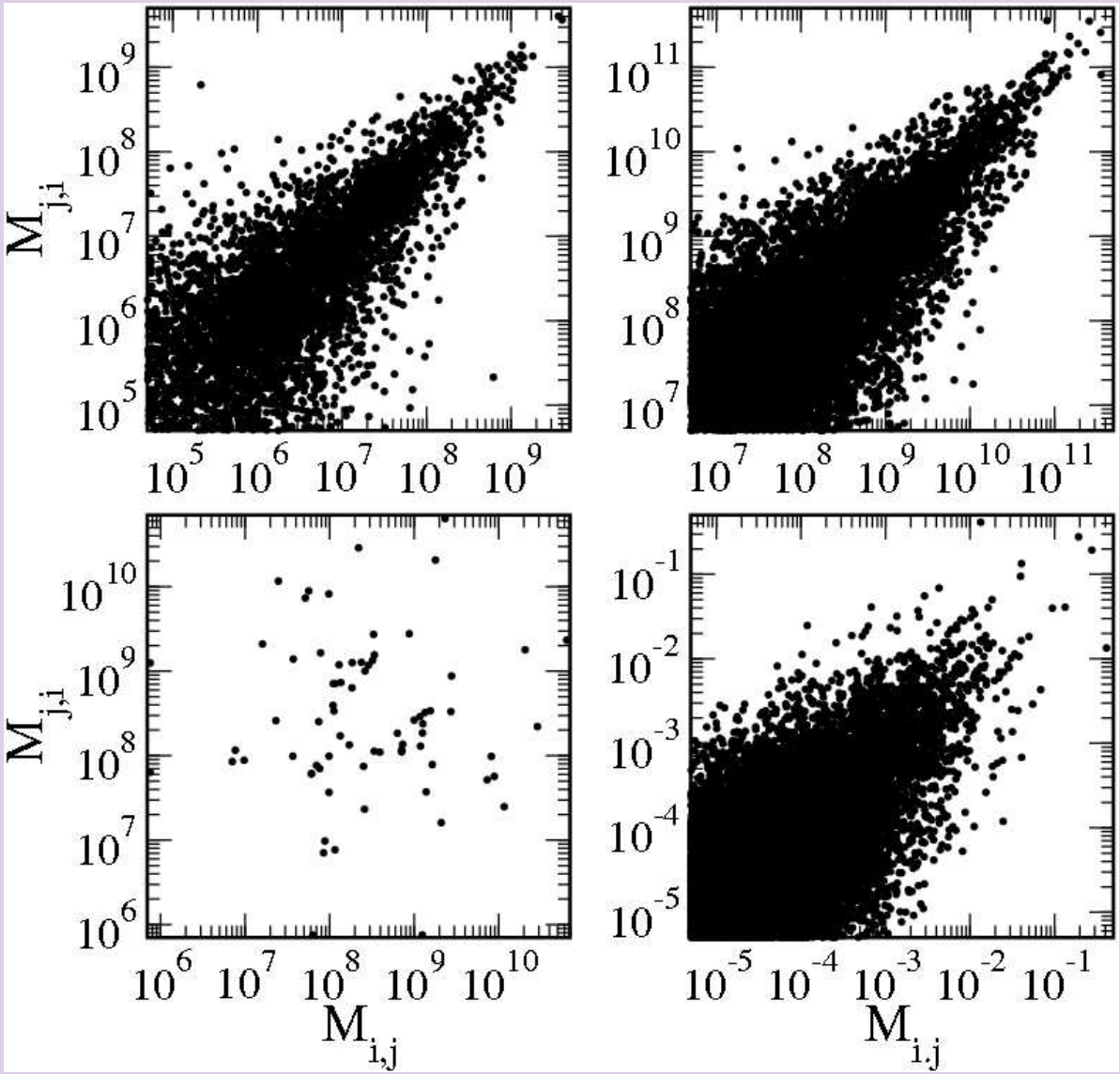
Ranking of World Trade

2008: Crude petroleum



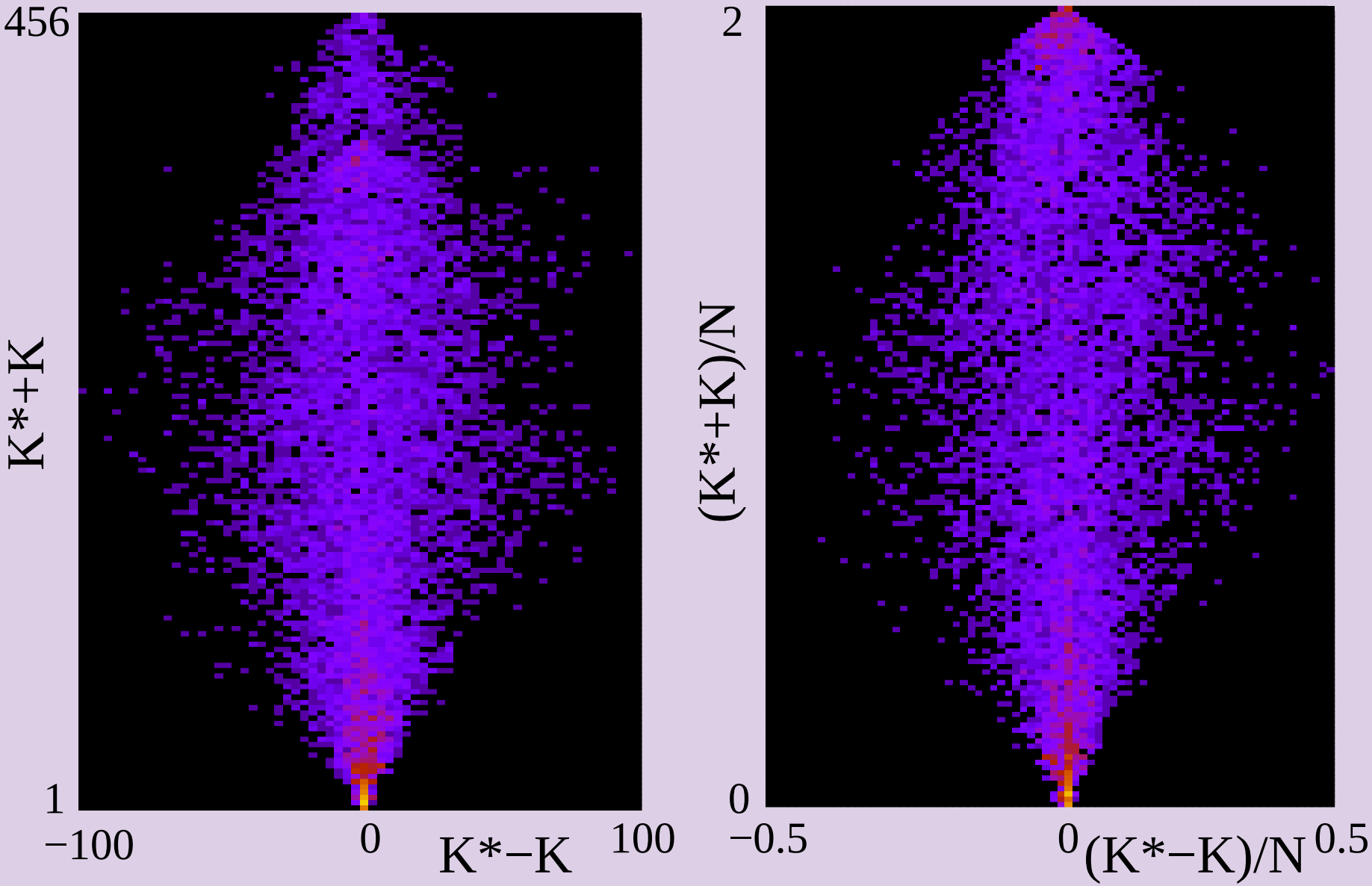
Mass flow on World Trade Network (WTN)

RMT model $M_{ij} = \epsilon_i \epsilon_j / ij$ (all commod. 1962/2008 left/right top; petroleum left bottom; model right bottom)



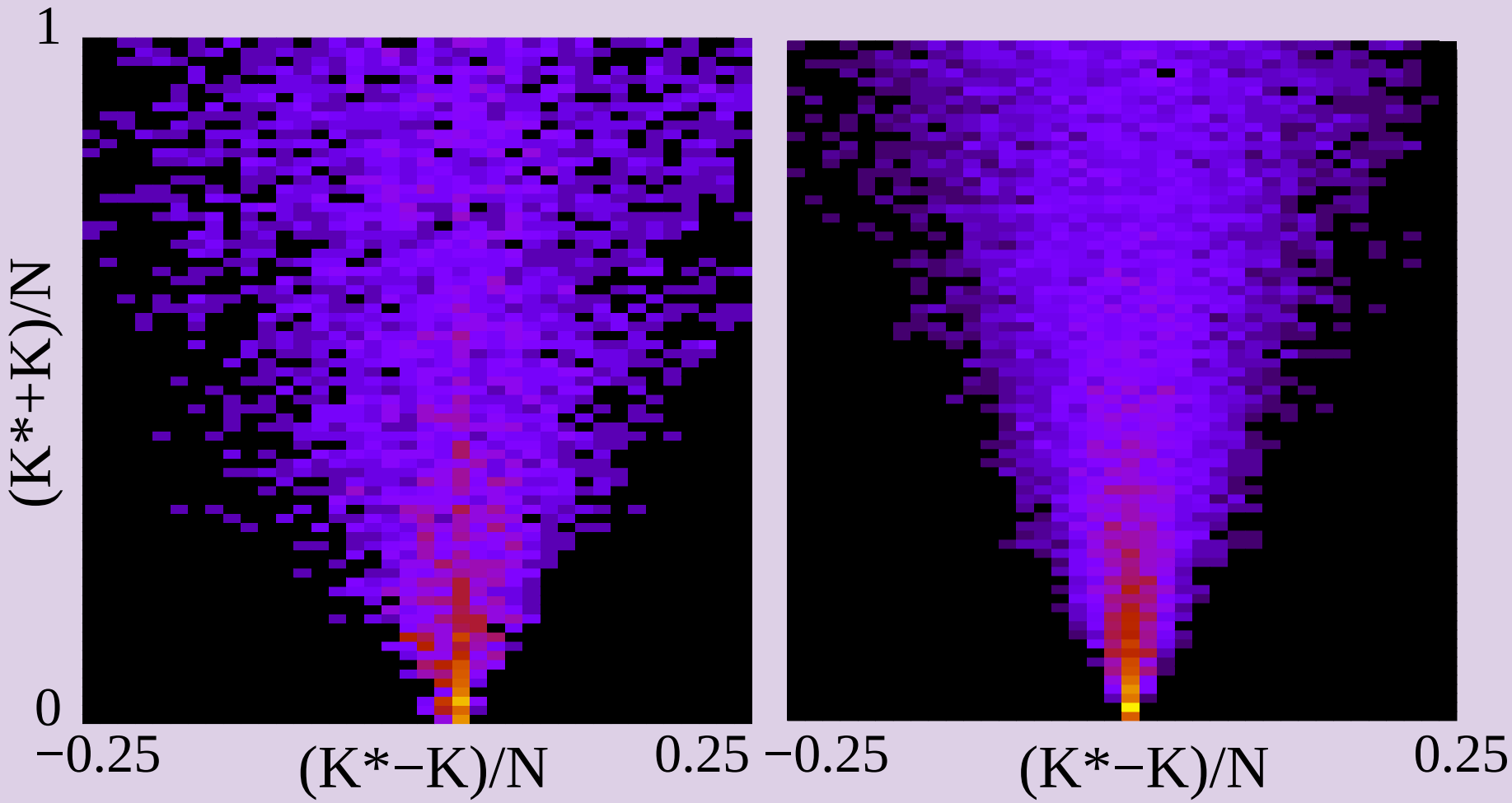
Global distribution for WTN

All commodities 1962-2009



Global distribution for WTN

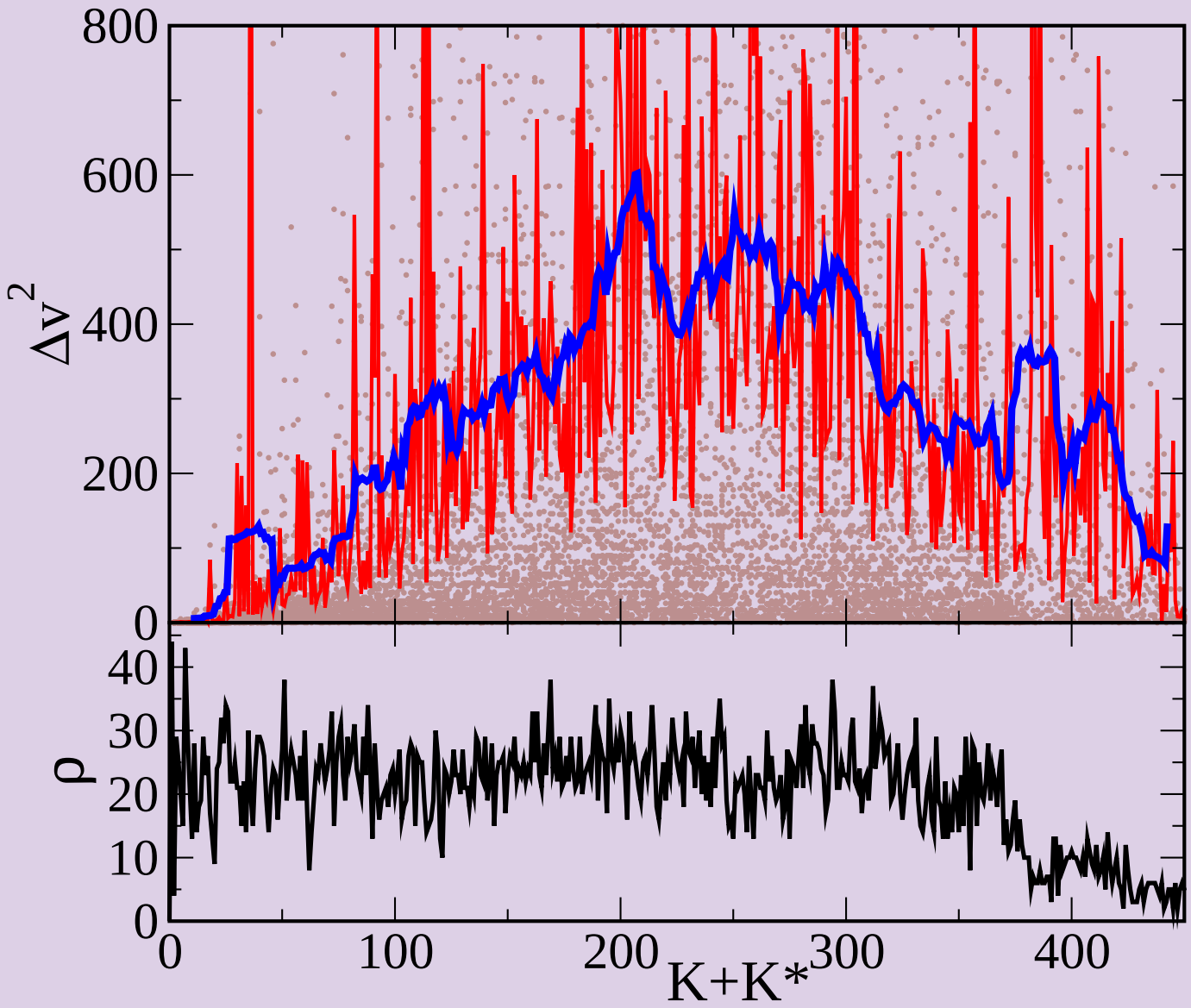
All commodities 1962-2009: left - zoom, right - RMT model



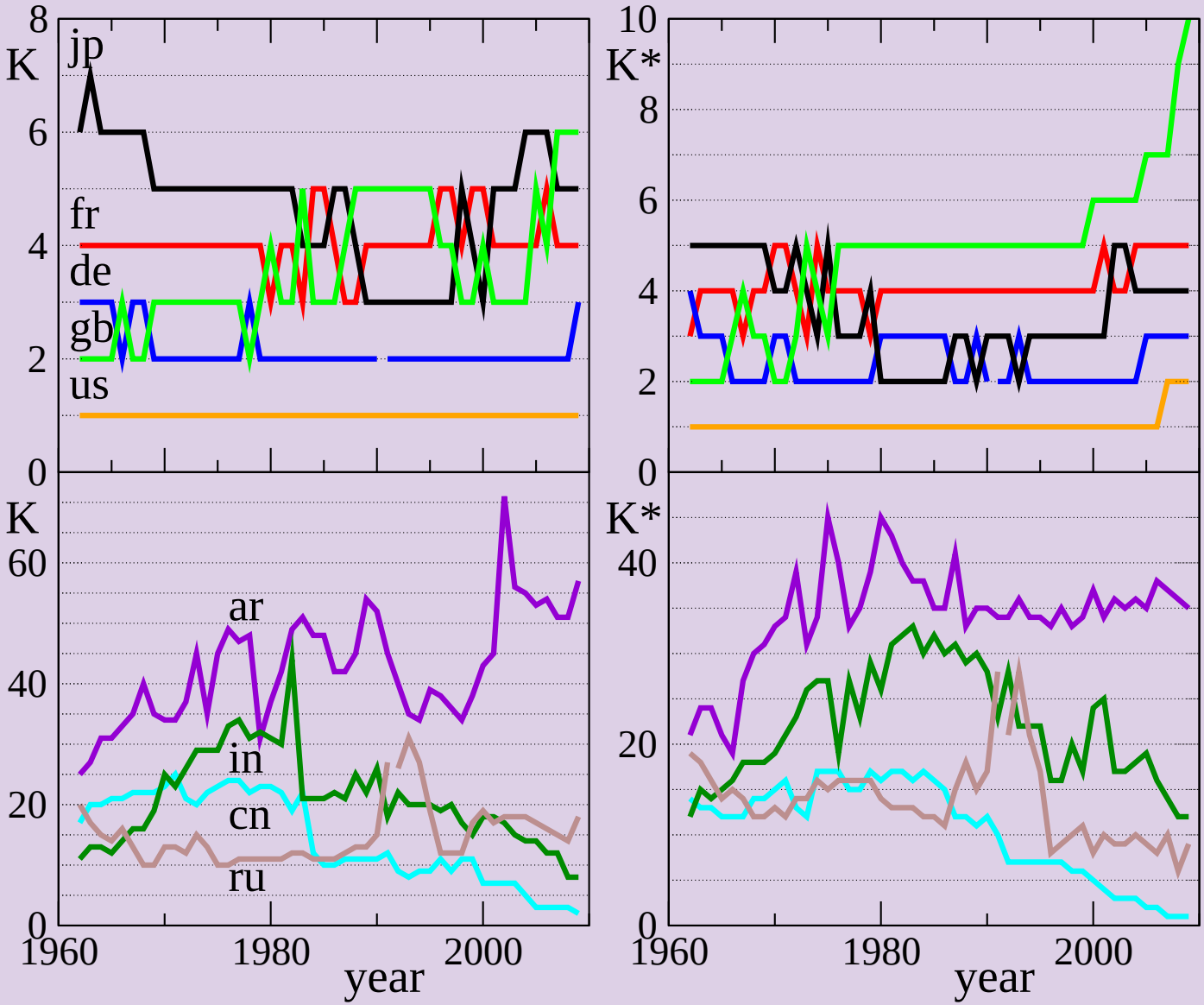
The poor stay poor and the rich stay rich

Velocity fluctuations for WTN

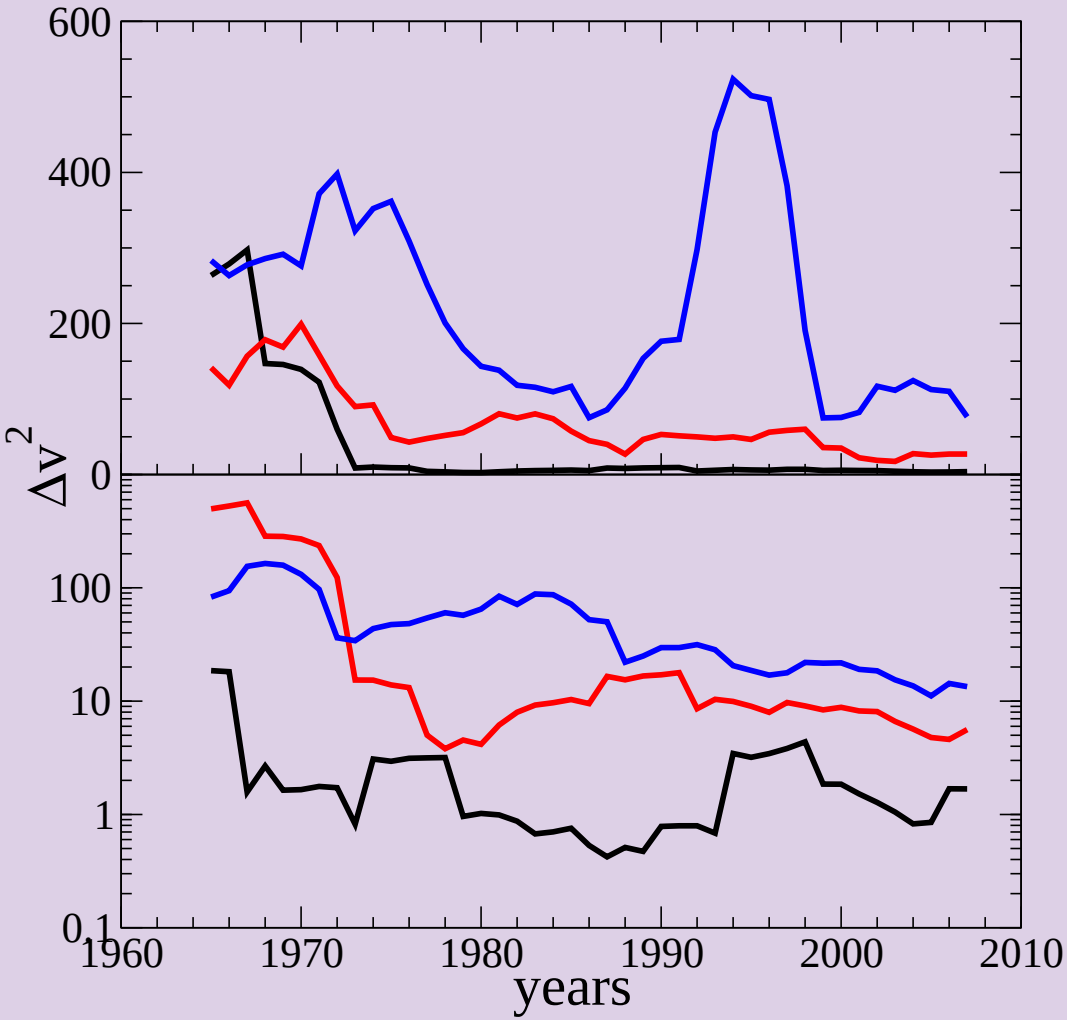
1962-2009: Rank velocity fluctuations $(\Delta v)^2 = (\Delta K)^2 + (\Delta K^*)^2$



Rank evolution in time



Rank evolution in time



Top: $1 \leq K + K^* \leq 40$; $41 \leq K + K^* \leq 80$; $81 \leq K + K^* \leq 120$;
Bottom: $1 \leq K + K^* \leq 20$; $21 \leq K + K^* \leq 40$; $41 \leq K + K^* \leq 60$

Rank table 2008 (74% of countries of G20)

Table 1. Top 20 ranking for *all commodities* – 2008.

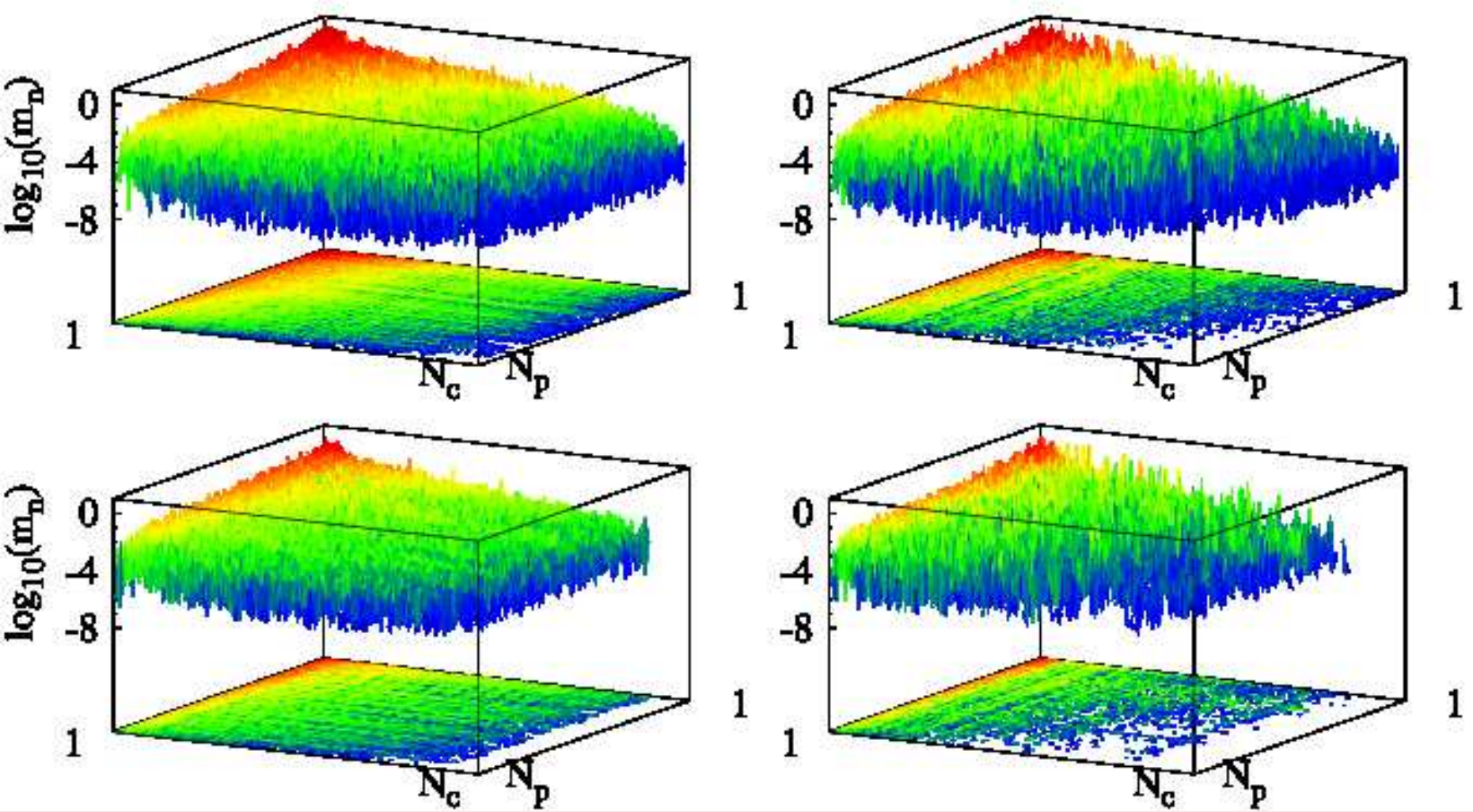
Ran	K	K^*	K_2	$\tilde{K}$	$\tilde{K}^*$
1	USA	China	USA	USA	China
2	Germany	USA	China	Germany	Germany
3	China	Germany	Germany	China	USA
4	France	Japan	Japan	France	Japan
5	Japan	France	France	Japan	France
6	UK	Italy	Italy	UK	Netherlands
7	Italy	Russian Fed.	UK	Netherlands	Italy
8	Netherlands	Rep. of Korea	Netherlands	Italy	Russian Fed.
9	India	UK	India	Belgium	UK
10	Spain	Netherlands	Rep. of Korea	Canada	Belgium
11	Belgium	Singapore	Belgium	Spain	Canada
12	Canada	India	Russian Fed.	Rep. of Korea	Rep. of Korea
13	Rep. of Korea	Belgium	Canada	Russian Fed.	Mexico
14	Russian Fed.	Australia	Spain	Mexico	Saudi Arabia
15	Nigeria	Brazil	Singapore	Singapore	Singapore
16	Thailand	Canada	Thailand	India	Spain
17	Mexico	Spain	Australia	Poland	Malaysia
18	Singapore	South Africa	Brazil	Switzerland	Brazil
19	Switzerland	Thailand	Mexico	Turkey	India
20	Australia	U. Arab Emir.	U. Arab Emir.	Brazil	Switzerland

Rank table 2008

Table 2. Top 20 ranking for *crude petroleum* – 2008.

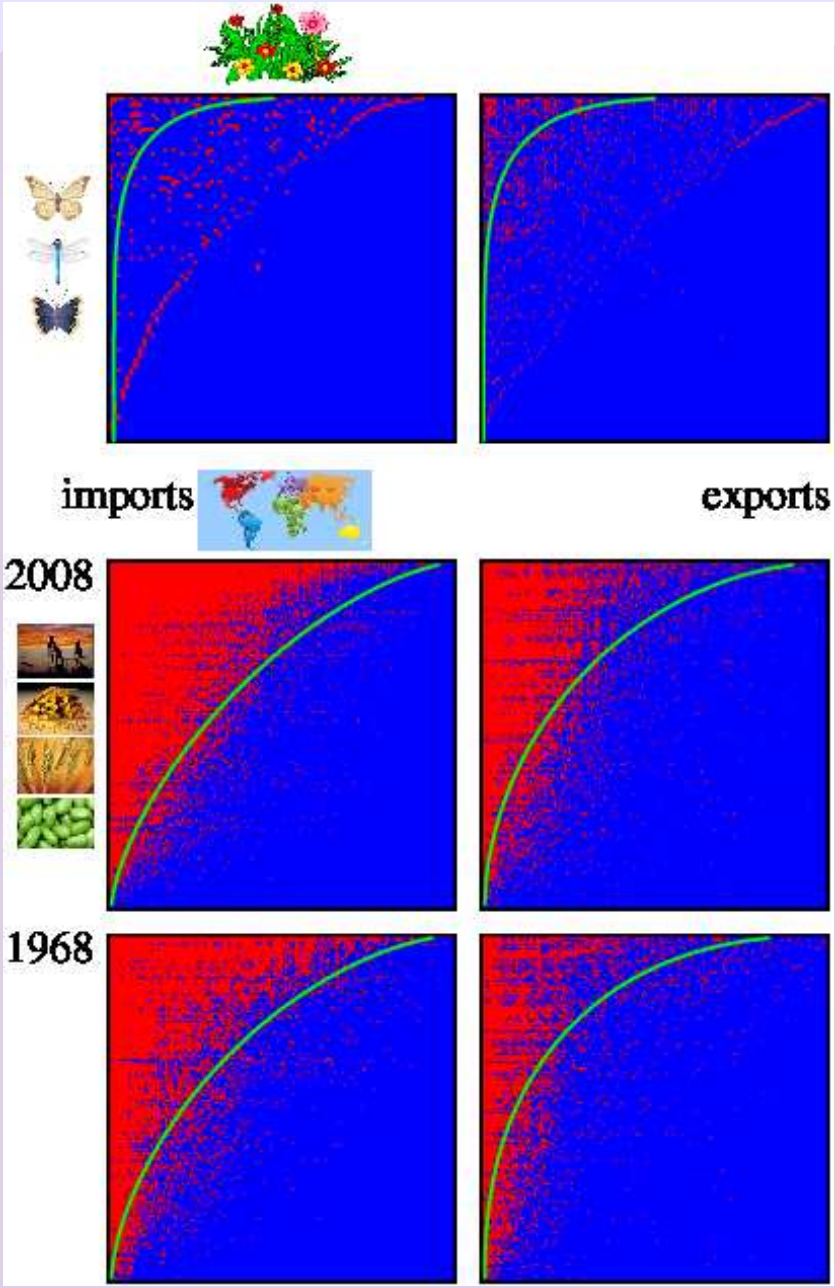
Ran	K	K^*	K_2	$\tilde{K}$	$\tilde{K}^*$
1	USA	Russian Fed.	USA	USA	Saudi Arabia
2	Canada	Kazakhstan	India	Japan	Russian Fed.
3	Netherlands	U. Arab Emir.	Singapore	China	U. Arab Emir.
4	Belgium	USA	UK	Italy	Nigeria
5	India	Ecuador	South Africa	Rep. of Korea	Iran
6	China	Saudi Arabia	Canada	India	Venezuela
7	Germany	India	Australia	Germany	Norway
8	Japan	South Africa	U. Arab Emir.	Netherlands	Canada
9	Rep. of Korea	Nigeria	Colombia	France	Angola
10	UK	Sudan	Azerbaijan	UK	Iraq
11	Singapore	Azerbaijan	Malaysia	Spain	Libya
12	Italy	Venezuela	Brazil	Singapore	Kazakhstan
13	Australia	Norway	Belgium	Canada	Kuwait
14	Malaysia	Iran	Trinidad and Tobago	Thailand	Azerbaijan
15	Spain	Algeria	France	Belgium	Algeria
16	France	Singapore	Netherlands	Brazil	Mexico
17	Brazil	Kuwait	Kenya	Turkey	UK
18	Sweden	UK	Angola	South Africa	Qatar
19	South Africa	Angola	China	Poland	Oman
20	Thailand	Canada	Thailand	Australia	Netherlands

Ecological analysis of world trade



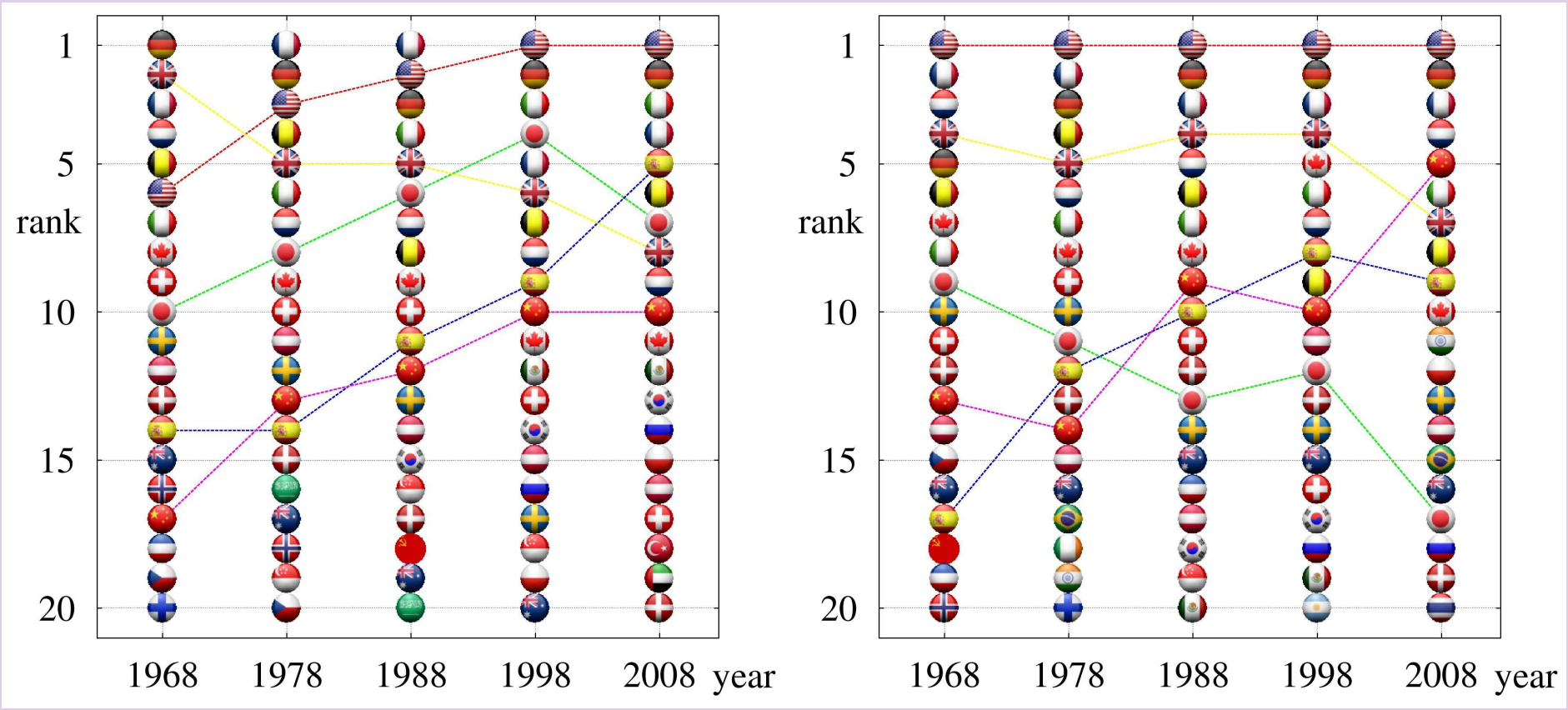
Normalized monetary trade volume: import (left), export (right), 1968 (bottom) and 2008 (top); arxiv:1201.3584; countries/products $N_c = 164, 227 / N_p = 182$; import/export $M^{(i)}_{p,c} = \sum_{c'=1}^{N_c} M^{p}_{c,c'}$ / $M^{(e)}_{p,c} = \sum_{c'=1}^{N_c} M^{p}_{c',c}$; $M_{p,c} > / < \mu => 1/0$

Plants-animals => Countries-products



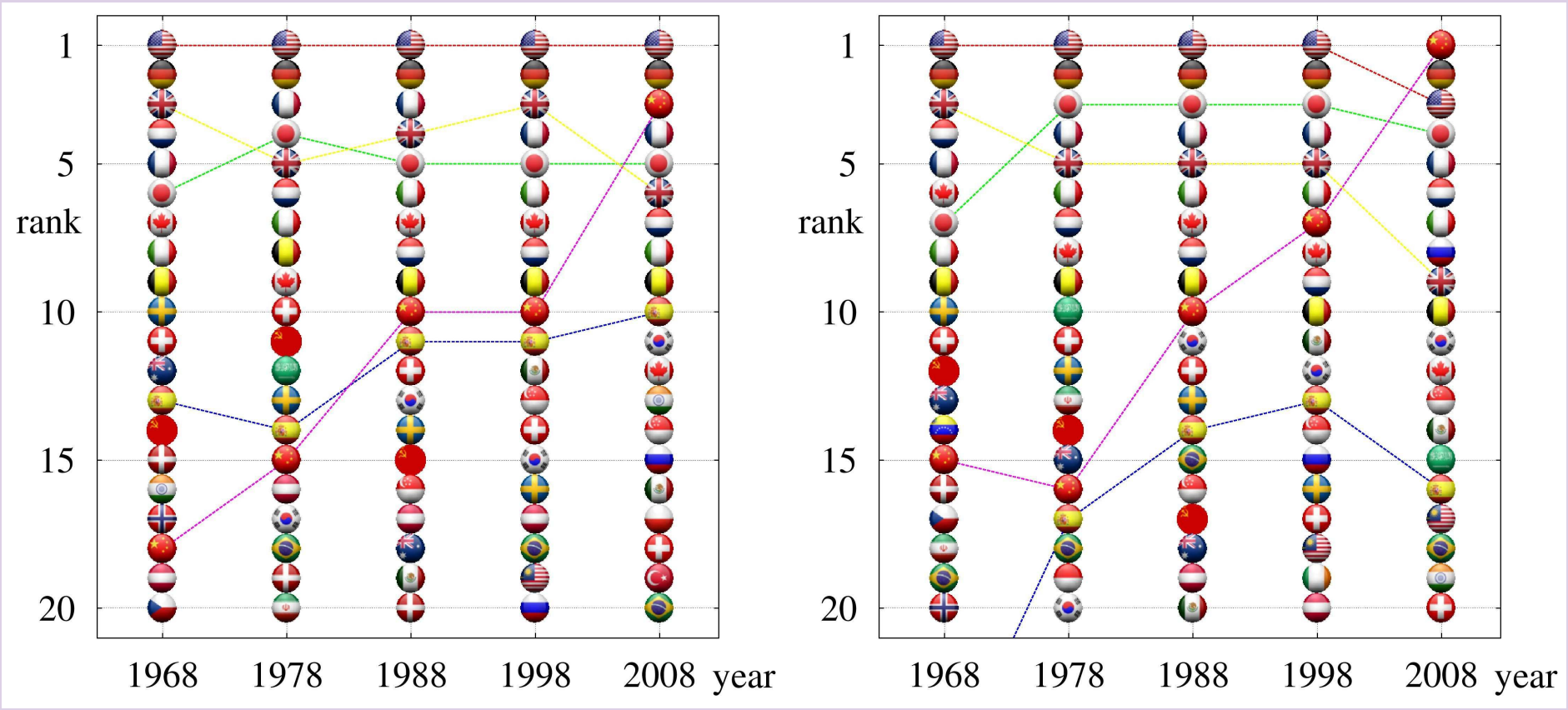
Mutualistic nestedness matrix:
Top: two ecological systems
from J.Bascompte et al.
"The architecture of mutualistic networks minimizes competition and increases biodiversity“
Nature 458, 1018 (2009);
Middle-bottom: WTN data
Nestedness ordering algorithm

Ecological ranking of world trade (countries)



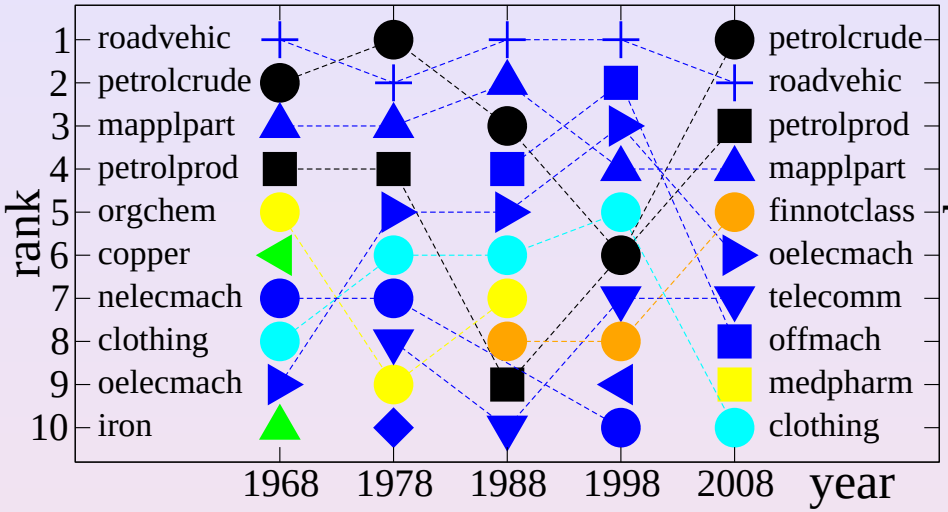
Left: import; Right: export

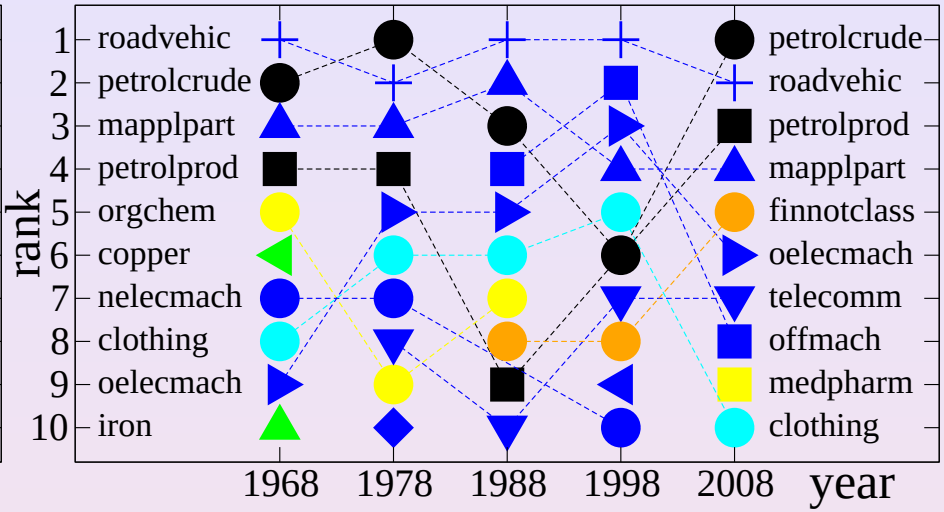
Trade volume ranking of world trade (countries)

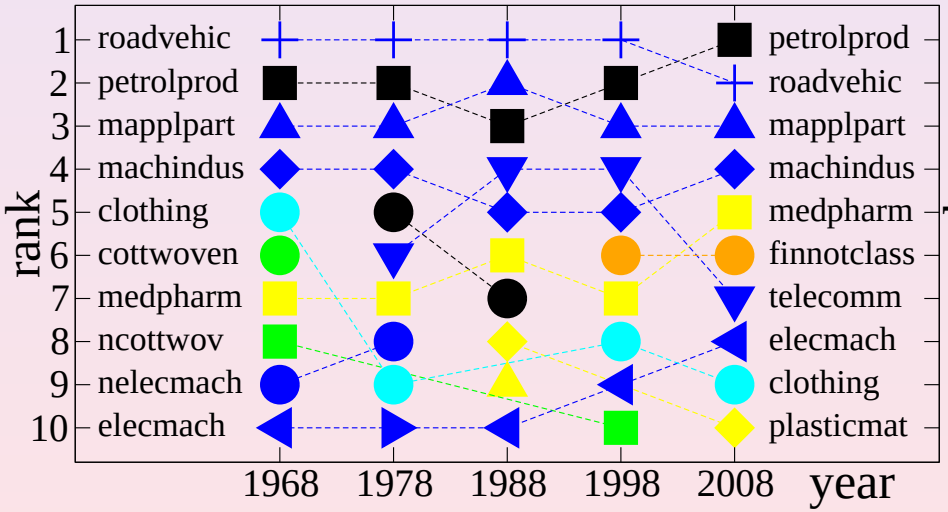


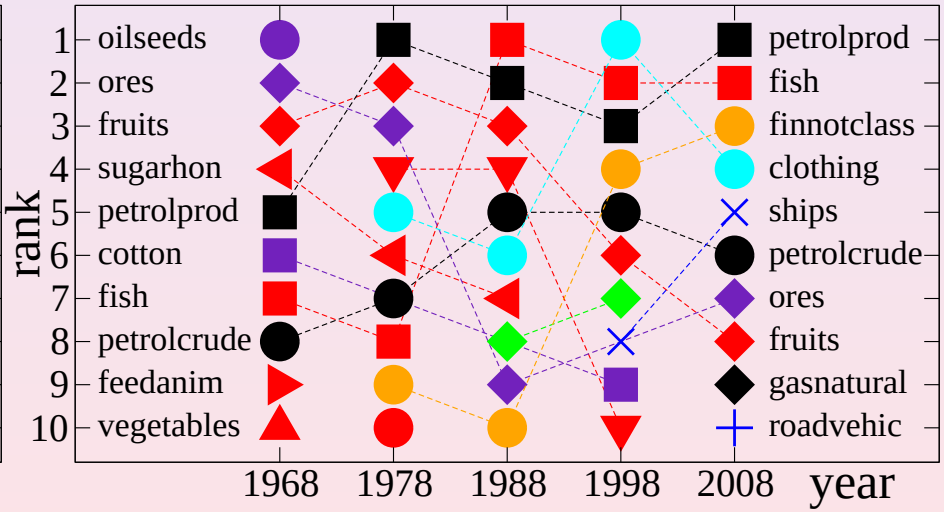
Left: import; Right: export

Ecological ranking of world trade (products)





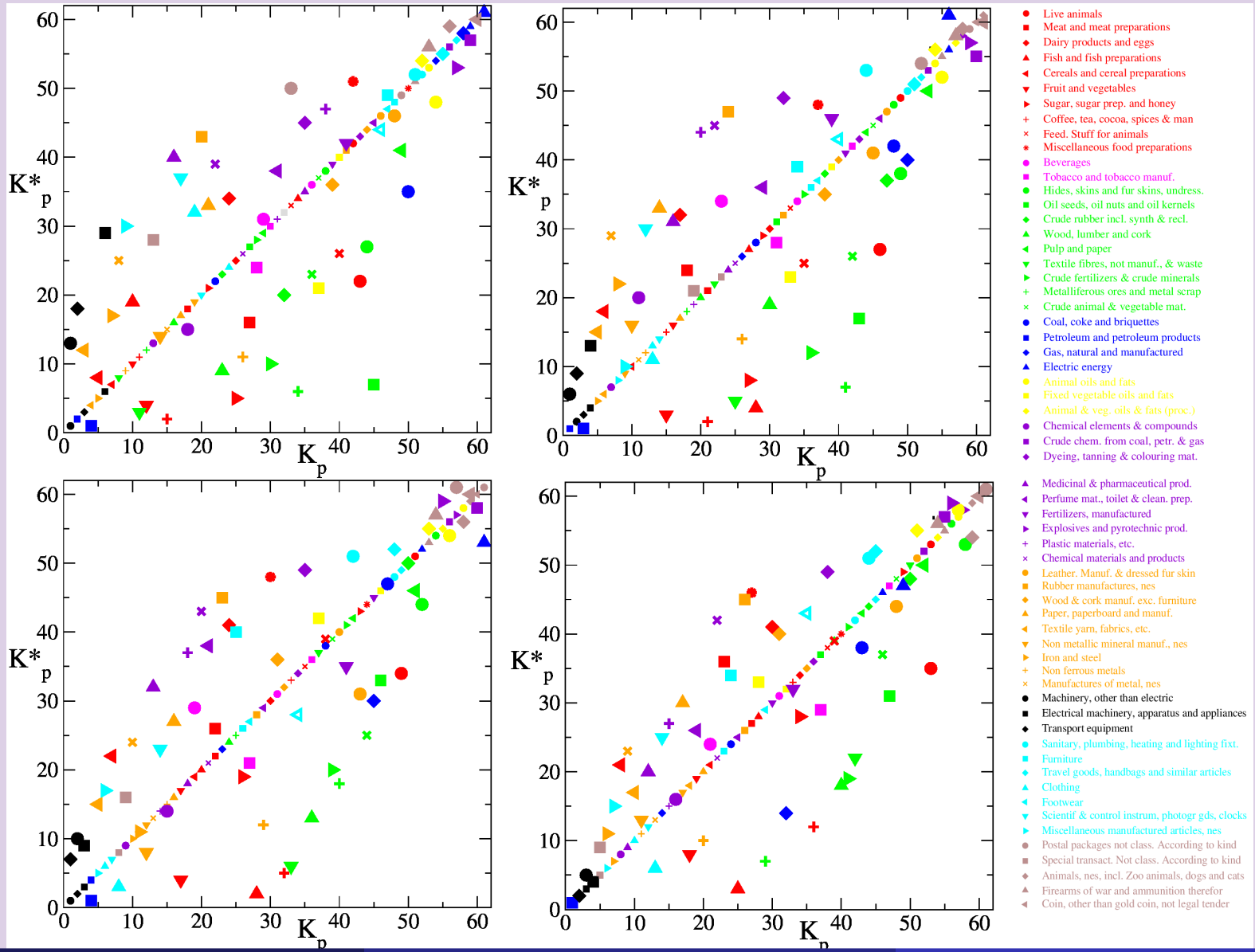




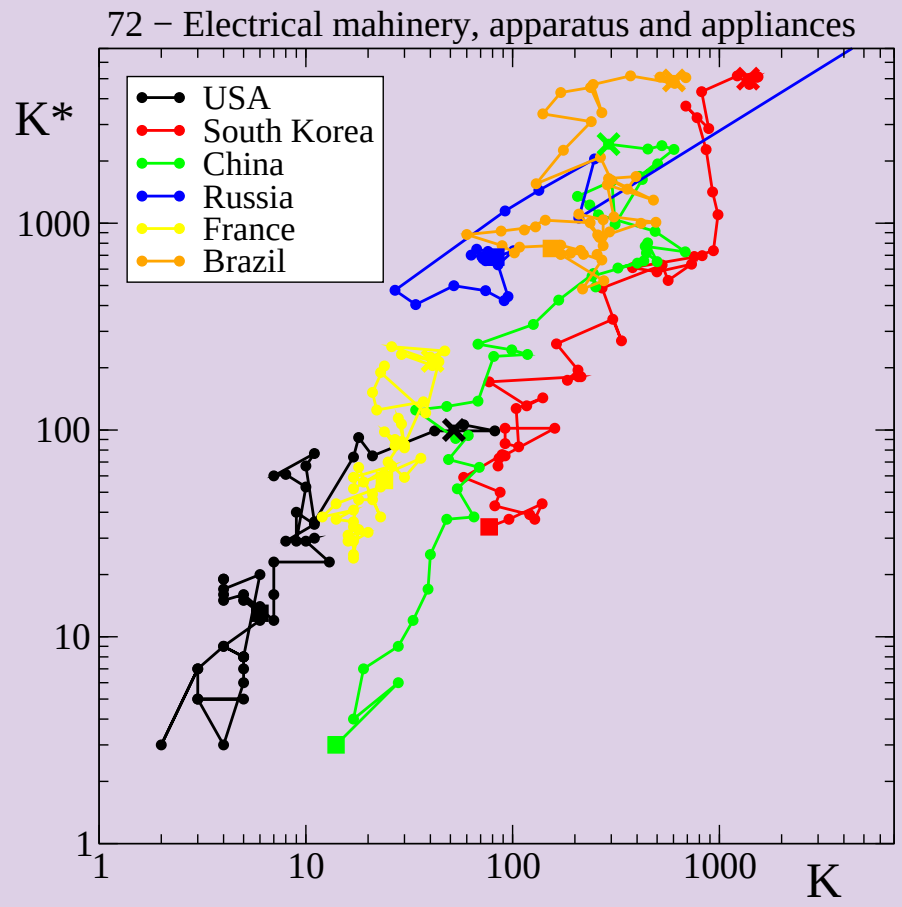
Top: trade volume; Left bottom: EcoloRank imp; Right bottom: EcoloRank exp

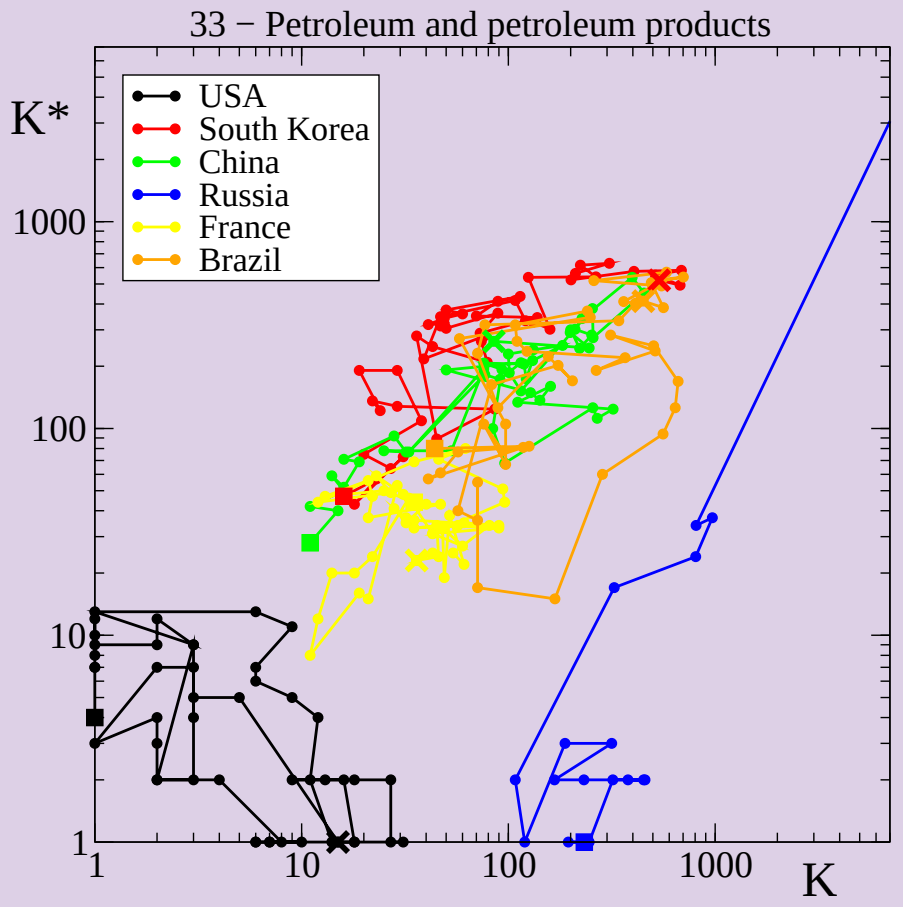
Google matrix of multiproduct trade

$N_p = 61$ products, $N_c = 227$ countries, $N = 13847$, $G_{ij} = \alpha S_{ij} + (1-\alpha) v_i e_j$



Time evolution of ranking for 72, 33





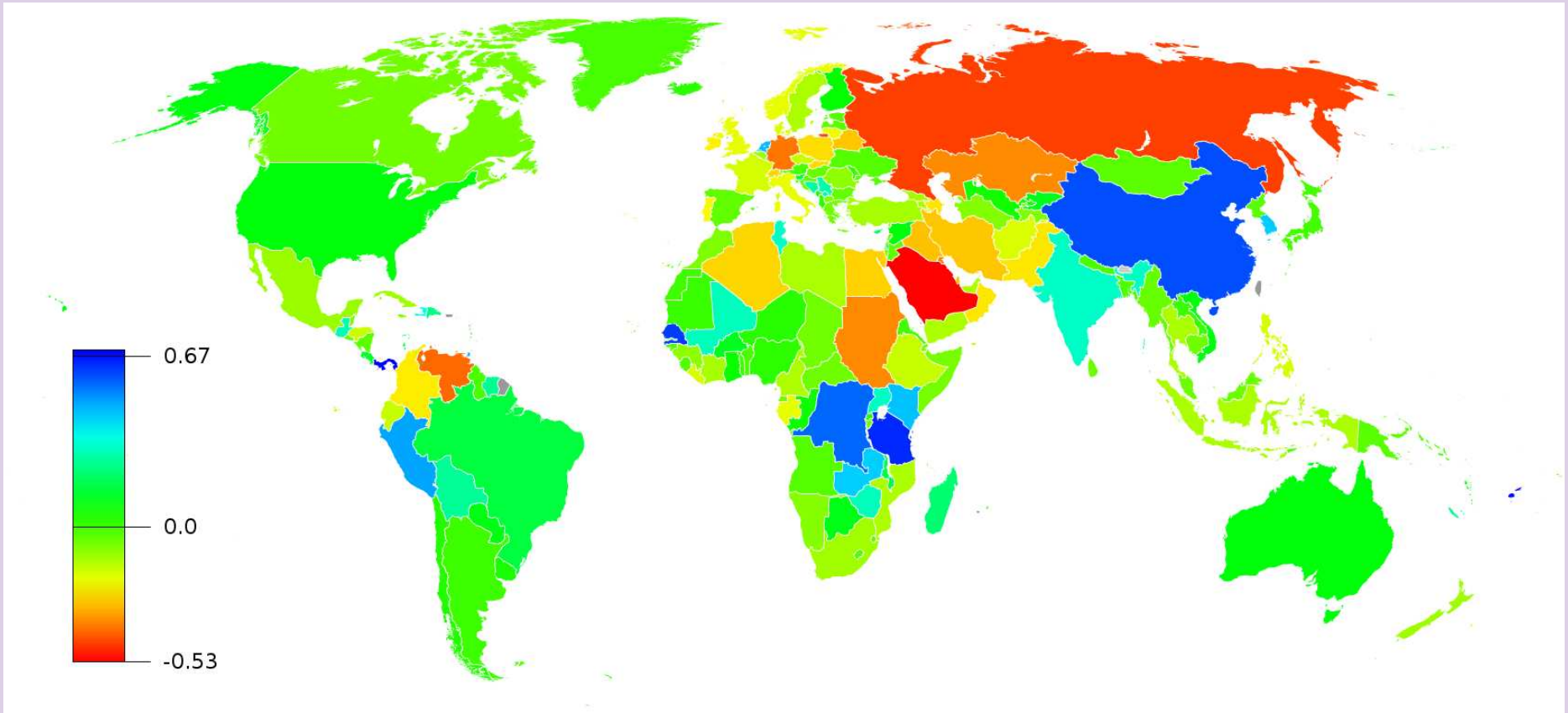
Top ranking of countries and products

Table 3. Top 20 ranks for global PageRank K, CheiRank K^*, 2dRank K_2, ImportRank $\hat{K}$ and ExportRank $\hat{K}^*$ for given country and product code for year 2008.

#	K country & code		K^* country & code		K_2 country & code		$\hat{K}$ country & code		$\hat{K}^*$ country & code	
1	USA	33	Russia	33	Germany	73	USA	33	China	72
2	USA	73	China	84	USA	73	USA	71	Russia	33
3	USA	71	Germany	73	USA	33	USA	72	China	71
4	USA	93	Japan	73	USA	71	USA	73	Germany	73
5	Germany	73	USA	73	India	33	Japan	33	Germany	71
6	USA	72	China	72	Singapore	33	China	72	Saudi Arabia	33
7	France	73	USA	33	Germany	71	China	33	USA	71
8	Germany	71	India	33	USA	72	Germany	71	Japan	73
9	Singapore	33	USA	71	France	73	Germany	73	USA	73
10	India	33	China	71	Netherlands	33	Netherlands	33	Japan	71
11	China	33	Singapore	33	USA	93	Germany	72	USA	72
12	Netherlands	33	Saudi Arabia	33	Nigeria	33	China	71	China	89
13	France	33	Germany	71	Germany	72	USA	89	Germany	72
14	UK	71	USA	72	China	72	Italy	33	China	84
15	UK	73	France	73	China	71	Germany	33	Japan	72
16	Germany	72	Thailand	3	UK	33	South Korea	33	South Korea	72
17	USA	89	Kazakhstan	33	Germany	93	France	73	France	73
18	South Korea	33	U. Arab Emir.	33	China	33	China	28	Italy	71
19	France	71	USA	28	South Korea	33	Germany	93	U. Arab Emir.	33
20	Sudan	73	Netherlands	33	Australia	33	India	33	Germany	93

Sensitivity to pertoleum price variation

derivative of balance $b_c = [\sum_p (P^*_{cp} - P_{cp})]/[\sum_p (P_{cp} + P^*_{cp})]$



WBW: Towards bank financial network ranking

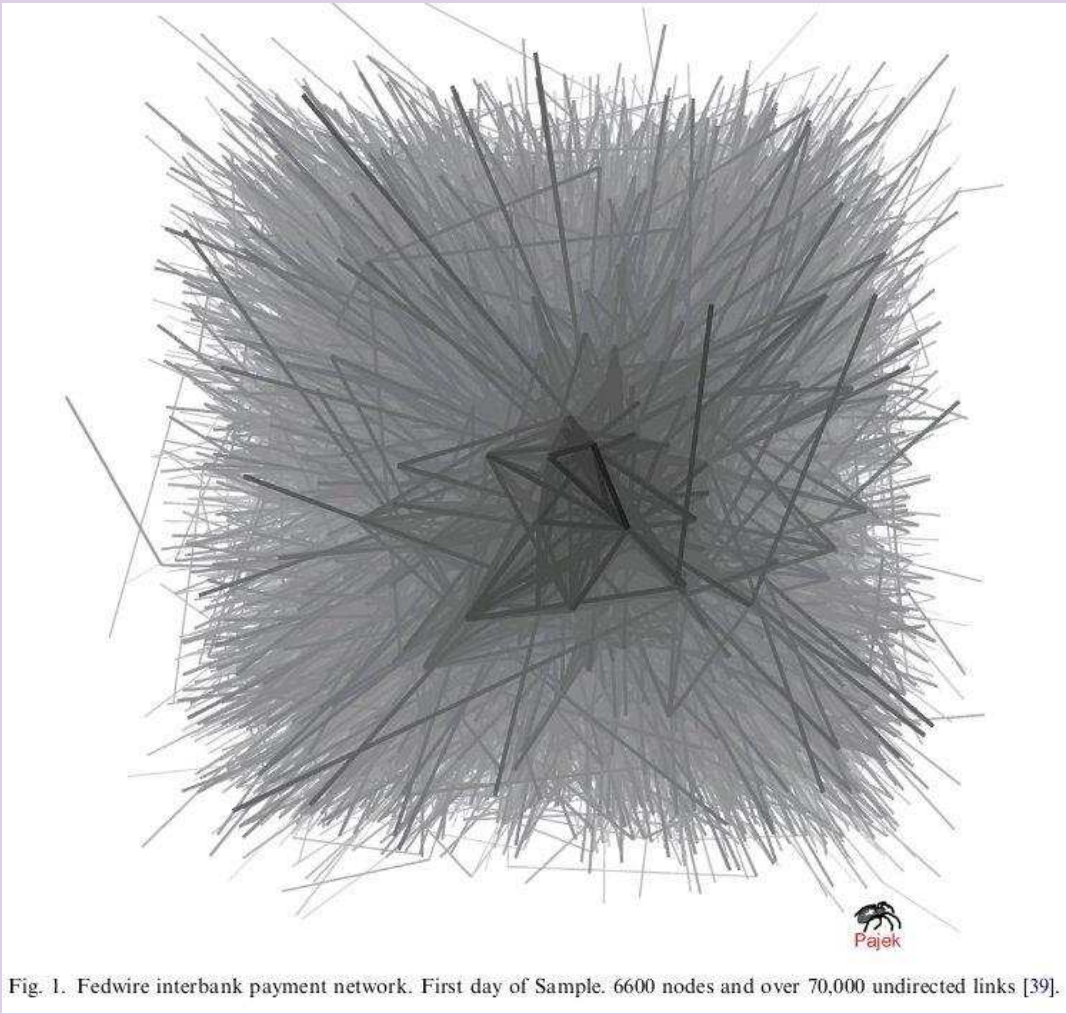


Fig. 1. Fedwire interbank payment network. First day of Sample. 6600 nodes and over 70,000 undirected links [39].

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Google Matrix Applications

practically to everything ....
http://www.quantware.ups-tlse.fr/ecoledeluchon/

more data at Refs. below and
http://www.quantware.ups-tlse.fr/QWLIB/2drankwikipedia/ .../tradecheirank/
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