Dynamical thermalization in finite interacting systems

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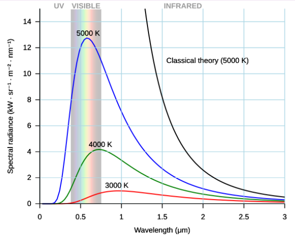
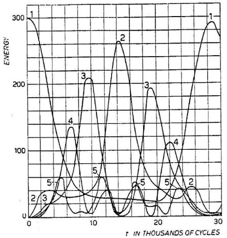
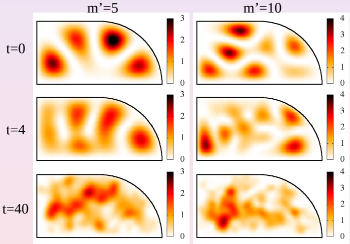
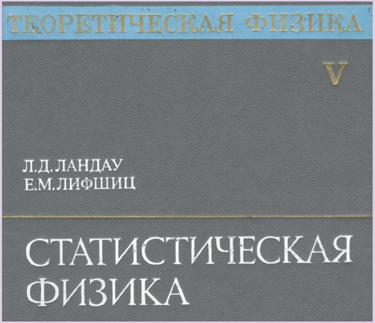






Planck's law (1900); Fermi-Pasta-Ulam problem (1955);
Bose-Einstein condensate (BEC) in Bunimovich stadium (2015) (left to right)

Energy equipartition over degrees of freedom



§ 44] ЗАКОН РАВНОРАСПРЕДЕЛЕНИЯ 151

Соответственно теплоемкость $c_p = c_v + 1$ равна

$$c_p = \frac{l+2}{2}. \qquad (44,2)$$

Таким образом, чисто классический идеальный газ должен обладать постоянной теплоемкостью. Формула (44,1) позволяет при этом высказать следующее правило: на каждую переменную в энергии $\varepsilon(p, q)$ молекулы приходится по равной доле 1/2 в теплоемкости c_v газа ($k/2$ в обычных единицах), или, что то же, по равной доле $T/2$ в его энергии. Это правило называют *законом равнораспределения*.

Имея в виду, что от поступательных и вращательных степеней свободы в энергию $\varepsilon(p, q)$ входят только соответствующие им импульсы, мы можем сказать, что каждая из этих степеней свободы вносит в теплоемкость вклад, равный 1/2. От каждой же колебательной степени свободы в энергию $\varepsilon(p, q)$ входит по две переменных (координата и импульс), и ее вклад в теплоемкость равен 1.

Nauka Moscow (1976)

Dynamical thermalization in finite isolated systems?

Chirikov criterion for onset of chaos (1959)

Izrailev, Chirikov Dokl. Akad. Nauk SSSR 166: 57 (1966)

Integrability of nonlinear Schrödinger equation

Zakharov, Shabat, Zh. Eksp. Teor. Fiz. 61: 118 (1971)

BEC in Bunimovich stadium

Gross-Pitaevskii equation (GPE or NSE)

Model description. – The model is described by the GPE for BEC in the de-symmetrized Bunimovich stadium billiard with Dirichlet boundary conditions:

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi(\vec{r},t) + \beta|\psi(\vec{r},t)|^2\psi(\vec{r},t), \qquad (1)$$

where we consider $\hbar = 1$, $m = 0.5$. The height of the stadium is taken as $h = 1$ and its maximal length is $l = 2$ (see fig. 1). Thus, the average level spacing is $\Delta \approx 4\pi/A \approx 7.04$, where A is the billiard area. At $\beta = 0$ the numerical methods of quantum chaos allow to determine efficiently about a million of eigenenergies of linear modes and related eigenmodes [17]. For comparison, we also consider the case of a rectangular billiard with approximately the same area as the stadium and with the golden mean ratio $l/h = (1+\sqrt{5})/2, h = 1$. We note that

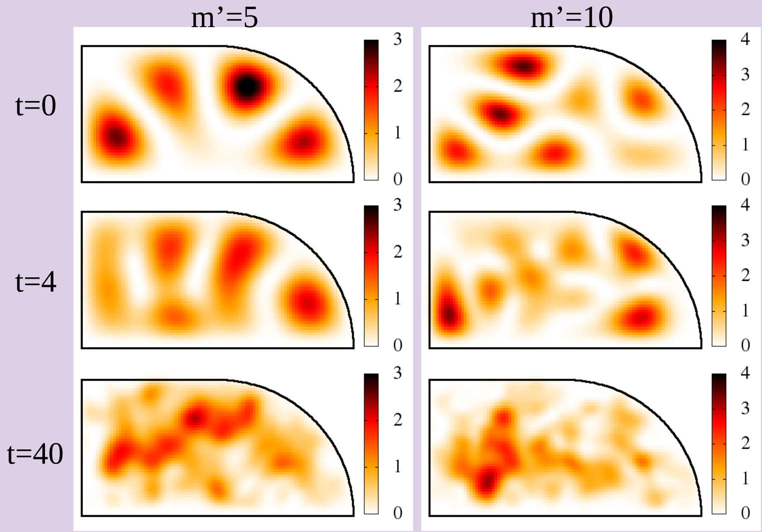
BEC thermodynamics in Bunimovich stadium

In such a case we should have the standard Bose-Einstein distribution ansatz over energy levels E_m [27]:

$$\rho_m = 1/[\exp[(E_m - E_g - \mu)/T] - 1], \tag{2}$$

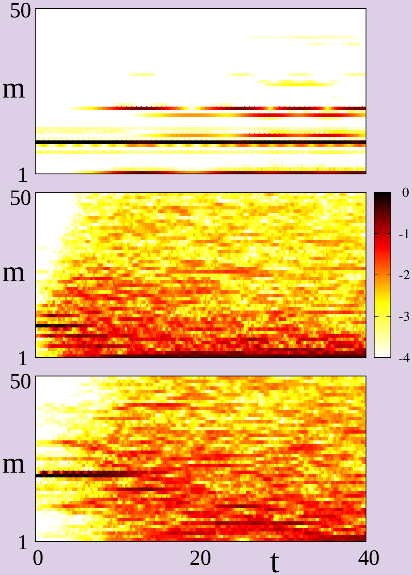
where $E_g = 13.25$ is the energy of the ground state, T is the temperature of the system, $\mu(T)$ is the chemical potential dependent on temperature. The values E_m are the eigenenergies of the stadium at $\beta = 0$. The parameters T and μ are determined by the norm conservation $\sum_{m=1}^{\infty} \rho_m = 1$ (we have only one particle in the system) and the initial energy $\sum_m E_m \rho_m = E$. The entropy S of the system is determined by the usual relation [27] $S = -\sum_m \rho_m \ln \rho_m$. The relation (2) with normalization condition determines the implicit dependences on temperature $E(T)$, $S(T)$, $\mu(T)$.

BEC time evolution



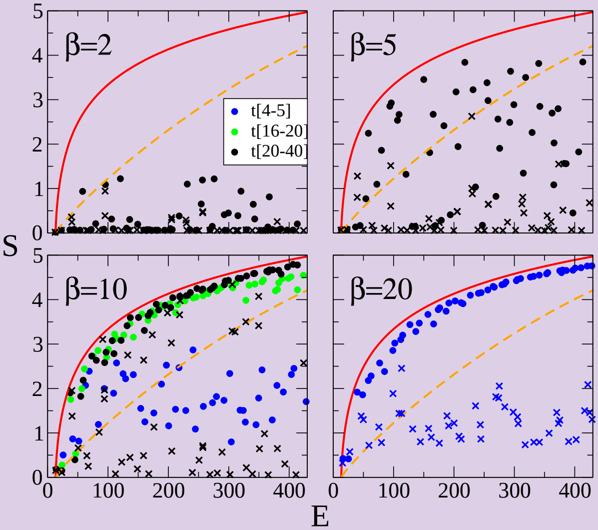
$\beta = 10$

BEC time evolution



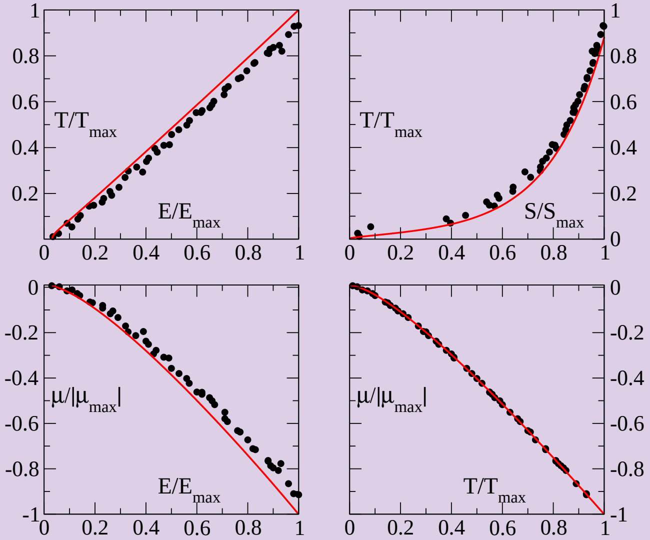
Top $m' = 10, \beta = 2$; middle-bottom $m' = 10, 20$; $\beta = 10$; color for $\log_{10} \rho_m(m')$

Bose-Einstein anzats



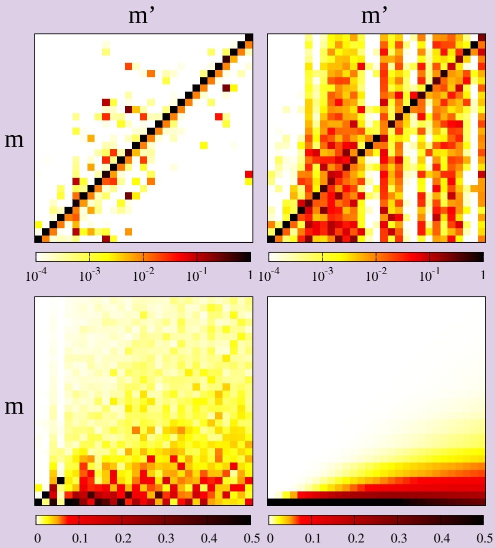
first 50 states; stadium (dots) rectangular (X); time intervals: [4,5] (blue), [16,20] (green), [20,40] (black); Bose-Einstein anzats (red), classical equipartition (orange)

Bose-Einstein anzats



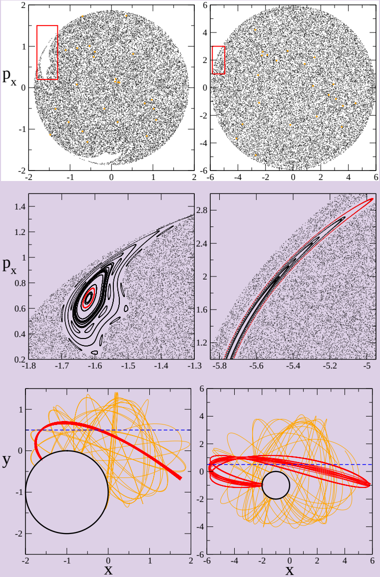
rescaling $E_{max} = 414$, $S_{max} = 4.8$, $T_{max} = 387$, $|\mu_{max}| = 1500$, for $m' = 50$; $\beta = 10$

Distribution of probabilities



$1 \leq m, m' \leq 30$, $\beta = 2, 5, 10$ (top left, top right, bottom left); Bose-Einstein anzats (bottom right)

BEC in Sinai oscillator



$H = (p_x^2 + p_y^2)/2 + x^2/2 + y^2$, disk $r_d = 1$ at $x = y = -1$; $E = 2, 18$ (left, right)

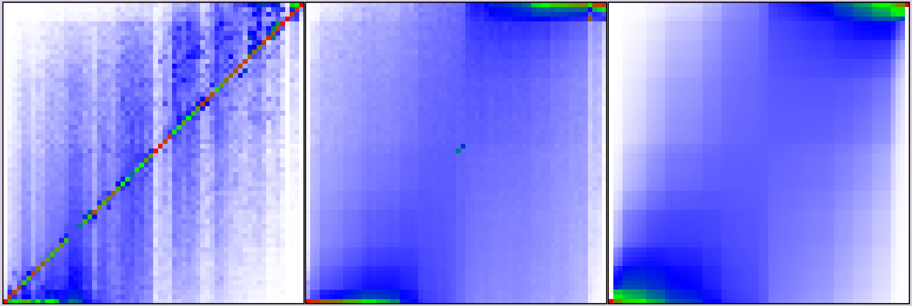
Nonlinear chains with disorder

$$i\frac{\partial \psi_{n_x n_y}}{\partial t} = E_{n_x n_y}\psi_{n_x n_y} + \beta|\psi_{n_x n_y}|^2\psi_{n_x n_y} + (\psi_{n_x+1 n_y} + \psi_{n_x-1 n_y} + \psi_{n_x n_y+1} + \psi_{n_x n_y-1}). \quad (6)$$

Periodic boundary conditions are used for the $N \times N$ square lattice with $-N/2 \leqslant n_x, n_y \leqslant N/2$. However, here we use the extended version of this model assuming that

$$E_{n_x n_y} = \delta E_{n_x n_y} + f(n_x^2 + n_y^2), \quad -W/2 \leqslant \delta E_{n_x n_y} \leqslant W/2\ (M3). \quad (7)$$

This is the $M3$ model with random values of energies $\delta E_{n_x n_y}$ in a given interval.



8×8 sites, $f = 1$, $W = 2$, $\beta = 1, 4$ (left, center); quantum Gibbs anzats (right); $t = 2 \times 10^6$ (Ermann, DS NJP (2013))

DT in Bose-Hubbard systems

Similarly as in Ref. [28], we consider a one-dimensional Bose-Hubbard ring containing L sites. The quantum many-body Hamiltonian of this system reads $\hat{H} = \hat{H}_0 + \hat{U}$ with

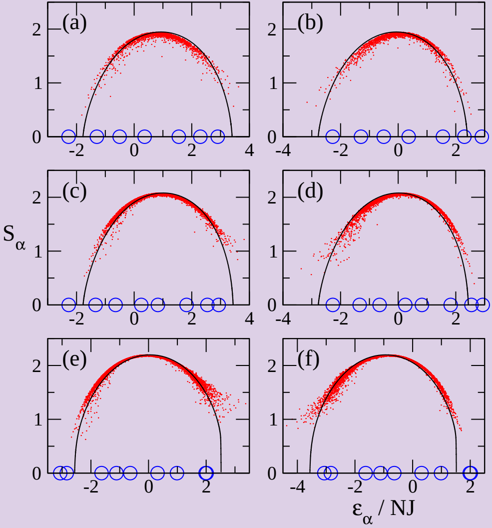
$$\hat{H}_0 = -J \sum_{l=1}^{L} (\hat{a}_l^\dagger \hat{a}_{l-1} + \hat{a}_{l-1}^\dagger \hat{a}_l) + \sum_{l=1}^{L} \epsilon_l \hat{a}_l^\dagger \hat{a}_l \,, \quad (1)$$

$$\hat{U} = \frac{U}{2} \sum_{l=1}^{L} \hat{a}_l^\dagger \hat{a}_l^\dagger \hat{a}_l \hat{a}_l \,, \quad (2)$$

where $\hat{a}_l^\dagger$ and $\hat{a}_l$ respectively denote the creation and annihilation operators associated with site l and where we formally identify $\hat{a}_0 \equiv \hat{a}_L$ and $\hat{a}_0^\dagger \equiv \hat{a}_L^\dagger$. The on-site energies ϵ_l ($l = 1, \ldots, L$) are fixed but randomly selected with uniform probability density from the interval $-W/2 \leq \epsilon_l \leq W/2$. Evidently, the single-particle Hilbert space of this finite system is L dimensional,

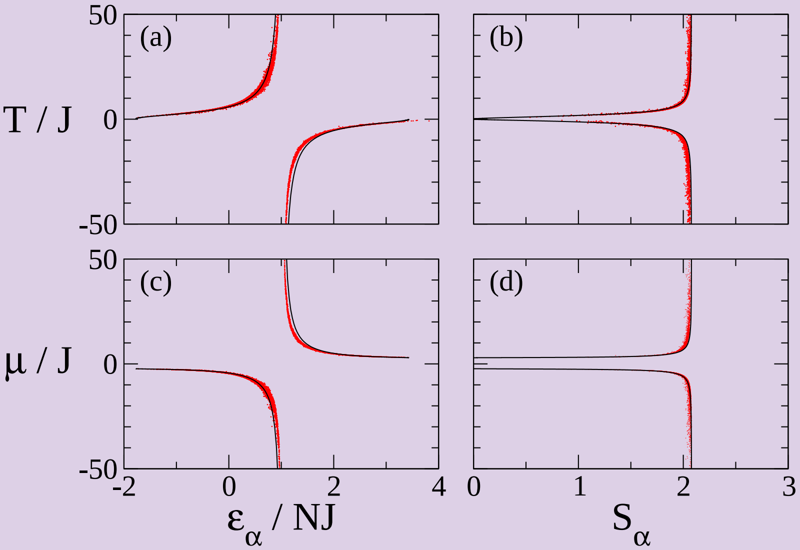
Schlagheck, DS (2015)

DT in Bose-Hubbard systems



$L = N = 7$ (top), 8 middle, 9 bottom; $U = 0.5J, -0.5J$ (left, right); $W = 4J$, red points are many-body eigenstates; black curve is Bose-Einstein anzats

DT in Bose-Hubbard systems



$L = N = 8$, $U = 0.5J$, $W = 4J$

Discussion

Conditions of dynamical thermalization?

not too weak and not too strong interactions

Åberg criterion: two-body matrix element should be larger than the spacing between directly coupled states

S.Åberg PRL **64**, 3119 (1990)
(see also R5)

References + Refs. therein:

R1. M.Mulansky, K.Ahnert, A.Pikovsky and D.L.Shepelyansky, "Dynamical thermalization of disordered nonlinear lattices", Phys. Rev. E v.80, p.056212 (2009)
R2. L.Ermann and D.L.Shepelyansky, "Quantum Gibbs distribution from dynamical thermalization in classical nonlinear lattices", New J. Phys. v.15, p.123004 (2013)
R3. L.Ermann, E.Vergini and D.L.Shepelyansky, "Dynamical thermalization of Bose-Einstein condensate in Bunimovich stadium", Europhys. Lett. v.111, p. 50009 (2015)
R4. P.Schlageck and D.L.Shepelyansky, "Dynamical thermalization in Bose-Hubbard systems", arXiv:1510.01864 (2015)
R5. D.L.Shepelyansky, “Quantum chaos & quantum computers" Physica Scripta v. T90, p.112 (2001)