

Synchronization in Rings of Saturn



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12 Feb 2009, BINP: Theory seminar in memory of
Boris Chirikov (06/06/1928 - 12/02/2008)



DS. A.S.Pikovsky *et al.* arxiv:0812.4372 - 23 Dec 2008

- Synchronization
- Experiments: Zero-resistance states
- Numerical simulations, Kapral collision model
- Coulomb interactions, rotating Wigner crystal
- Rings of Saturn

2D Electron transport in a microwave field

- **EPJB 60, 225 (2007)**

Synchronization, zero-resistance states and rotating Wigner crystal

A.D.Chepelianskii (ENS, rue d'Ulm, Paris)

A.S.Pikovsky (Univ. of Potsdam)

and DS

- **Discussions:**

A.Bykov (Inst. of Physics of Semiconductors, Novosibirsk)

F. von Oppen (Freie Univ. Berlin)

J.Schmidt and F.Spahn (Univ. of Potsdam)

- **Support:**

ANR PNANO projects MICONANO, NANOTERRA of French government

Hero of the talk

- Christian Huygens
(1629 - 1695)



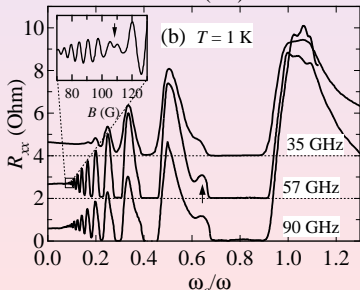
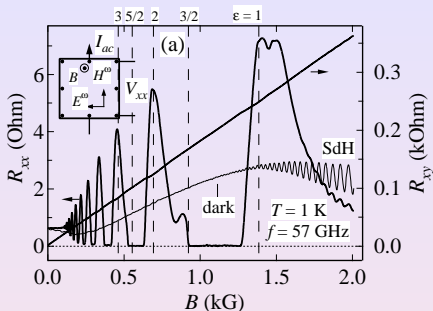
Experiments on Zero-Resistance States (ZRS)

- High mobility 2DEG in a microwave field

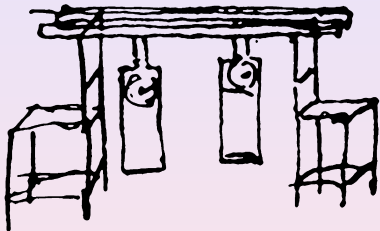
*M.A.Zudov, R.R.Du, J.A.Simmons, J.I.Reno
PRB **64**, 201311 (2001)

*R.G.Mani, J.H.Smet, K. von Klitzing, V.Narayanamurti, W.B.Johnson, V.Umansky
Nature **420**, 646 (2002)

*M.A.Zudov, R.R.Du, I.N.Pfeiffer, K.W.West
PRL **90**, 046807 (2003)
(Fig. image)

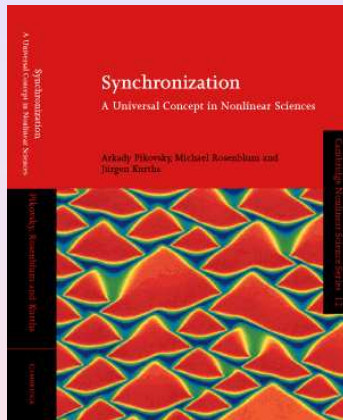


Synchronization



Ch. Huygens (1665) “sympathy of two clocks”

A. Pikovsky, M. Rosenblum, and J. Kurths, Cambridge University Press (2001)



Fireflies in Siam



Numerical studies

Classical 2D electron dynamics with short range and Coulomb interactions.

Short range interactions: the **Nosè-Hoover thermostat (Hoover (1999))** combined with interactions treated in the frame of the mesoscopic multi-particle collision model **(Kapral (2004))**:

$$\partial \mathbf{q}_i / \partial t = \mathbf{p}_i / m, \quad \partial \mathbf{p}_i / \partial t = \mathbf{F}_i + \mathbf{f}_{Li} + \mathbf{f}_{ac} - \gamma \mathbf{p},$$

$$\partial \gamma / \partial t = [\langle \mathbf{p}^2 \rangle / (2mT) - 1] / \tau^2$$

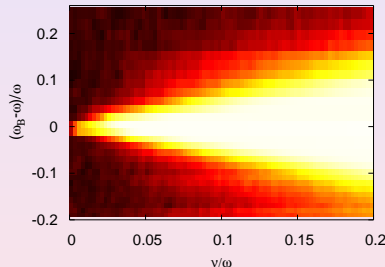
where $\mathbf{q}_i, \mathbf{p}_i$ are the coordinate and the momentum of particle i , $\mathbf{f}_{Li} = e[\mathbf{p}_i \times \mathbf{B}] / mc$ is the Lorentz force, \mathbf{F}_i is an effective force produced by particles collisions, τ is the relaxation time in the thermostat and $\langle \mathbf{p}^2 \rangle$ means average over all N particles.

Another choice, used for short range and Coulomb interactions, is to equilibrate the heating induced by the microwave field we introduce in Eq. an energy-dependent dissipation with $\gamma = \gamma_0(E - E_F) / E_F$ for $E = p^2 / 2m > E_F$ and $\gamma = 0$ for $E < E_F$. In such a way the dynamics remains Hamiltonian for $E < E_F$ while above E_F the dissipative processes are switched on as it is usually the case for 2DEG; thus E_F plays a role of Fermi energy.

In experiment electrons are at a Landau level $n_L \approx 100$.

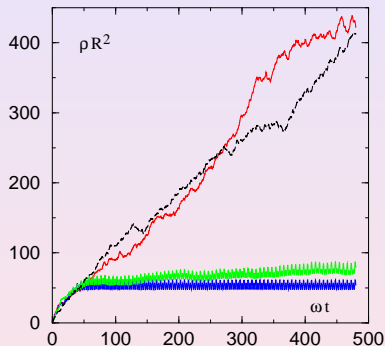
Kapral collision model

Diffusion rate density D/D_0 as a function of frequency detuning $(\omega_B - \omega)/\omega$ and rescaled microwave field strength ν/ω with $\nu = f/(mv_T)$ where $v_T = \sqrt{2T/m}$ is the thermal velocity and D_0 is the diffusion rate in absence of microwave at $\omega_B = \omega$. The particle dynamics is described by the NH thermostat at temperature T with short range interactions treated in the MMPCM formalism (see Eqs. above). The system parameters described in the text are: $N = 1000$, $N_c = 4 \times 10^4$, $\omega\Delta t = 0.2$, $\omega\tau = 10$, $\omega t = 500$, $L/r_B = 10$, $D_0/D_c = 0.12$ (with $D_c = v_T^2/\omega$, $\rho = N/L^2$ and r_B taken at $\omega_B = \omega$, thus a number of particles inside a Larmor circle is $N_B = \pi r_B^2 \rho = \pi \rho v_T^2/\omega^2 = 10\pi$, ω is kept constant). Color is proportional to D/D_0 (black maximum $D/D_0 \approx 1.2$; white minimum $D/D_0 = 0$).



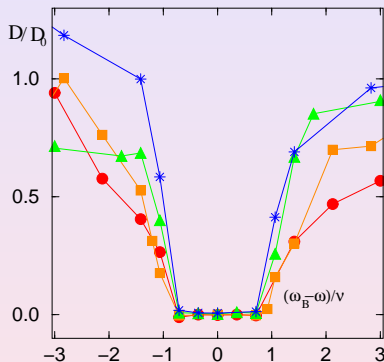
Coulomb interactions

Dependence of electron square displacement R^2 , rescaled by electron density ρ , on the rescaled time ωt . Here the Larmor frequency is $\omega_B = \omega$ at microwave field strength $f = 0$ (red top curve); $f/(mv_F\omega) = 0.059$ ($fa/E_F = 0.02$) for $\omega_B = \omega$ (blue bottom curve), $\omega_B = 0.875\omega$ (second from top black dashed curve), and $\omega_B = \omega$ with impurity scattering mean free path $l_i = 96r_B$ (second from bottom green curve). Total number of electrons is $N = 100$ and $N_B = \pi\rho v_F^2/\omega^2 = 34.7$. The linear fit gives the diffusion rates $D/D_c = 0.089, 0.068, 0.0040, 9 \times 10^{-6}$ with $D_c = v_F^2/\omega$ (respectively for curves from top to bottom ordered at $\omega t = 400$).



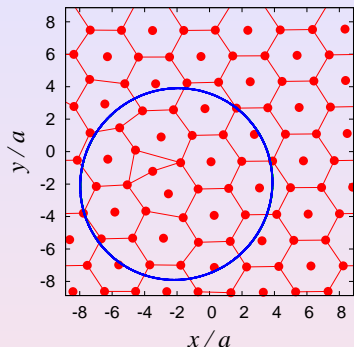
Coulomb interactions: diffusion rate

Dependence of rescaled diffusion rate D/D_0 on the rescaled frequency difference $(\omega_B - \omega)/\nu$. Here $\nu = f/mv_F$, D_0 is diffusion rate in absence of microwave at $\omega_B = \omega$, $fa/E_F = 0.02$ and number of electrons in a Larmor circle is $N_B = 2$ (stars), 8 (triangles), 34.7 (squares), 138.8 (points) with $D_0/D_c = 0.054, 0.089, 0.12, 0.14$ and $D_0/v_F a = 0.20, 0.35, 0.53, 0.64$ respectively. Total number of electrons is $N = 100$, $L = \sqrt{N/\rho} \approx 17.72a$.



Rotating Wigner crystal

Instant image of the rotating Wigner crystal formed by $N = 100$ electrons (points) in a periodic cell with $L = \sqrt{N/\rho} \approx 17.72a$, $\omega t = 480$, $\omega_B = \omega$, $fa/E_F = 0.02$ and $N_B = 34.7$ (bottom curve in one of previous Fig.); the circle shows an orbit of one electron for $240 \leq \omega t \leq 480$; lines are drawn to adapt an eye showing a hexagonal crystal with a defect.



Synchronization domain of ZRS phase:

$$|\omega_B - \omega| \leq 0.8f/mv_F, \quad (1)$$

For experiment conditions the relative size of ZRS plateau is

$\Delta\omega/\omega \approx 2\nu/\omega \approx fv_F/\omega E_F$ and with $E_F \sim 100K^0$, $v_F \sim 3 \times 10^7 \text{ cm/s}$ and

$\omega/2\pi = 35\text{GHz}$ this gives $\Delta\omega/\omega \approx 0.1$ if the field strength acting on an electron is

$f/e \approx 5\text{V/cm}$. The coherent rotation of electrons in the crystal creates a rotating current in 2D plane which in its turn generates a magnetic field $B_W \sim \mu_0 e v_F \rho \sim 1\text{G}$ parallel to 2DEG and rotating in the plane with a frequency close to ω .

Coulomb crystals in rf-traps

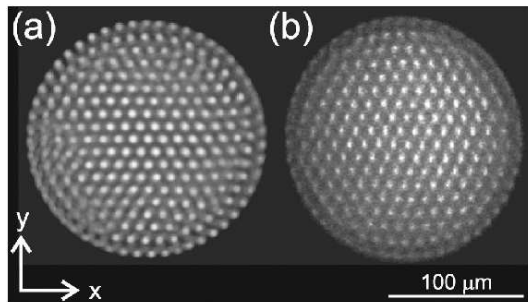
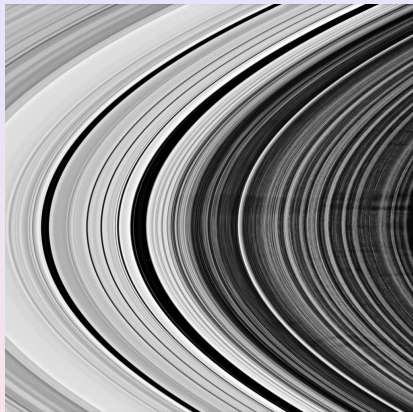
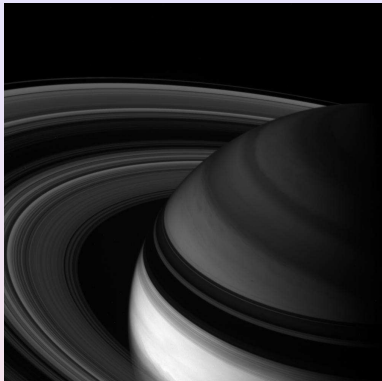


FIG. 3. Images of Coulomb crystals. (a) Time averaged image based on data from MD simulations of Coulomb clusters with 2685 ions at $\Gamma \sim 400$ (temperature: ~ 5 mK). The averaging time is 10 ms. (b) Image from experiments with clusters containing ~ 2700 ions.

Mainz experiments: G.Werth et al. Phys. Rev. A. **56**, 4023 (1997); Eur. Phys. J. D **18**, 295 (2002)

Aarhus experiments: A.Mortensen, E.Nielsen, T.Matthey, and M.Drewsen, Phys. Rev. Lett. **96**, 103001 (2006) [Fig. image]

Synchrony Conjecture for Planetary Rings



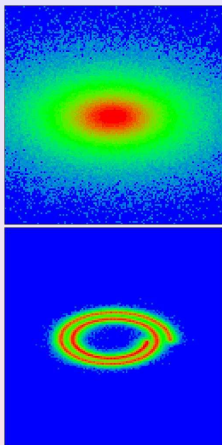
NASA Cassini images of Saturn Ring (2007)

Features to explain: enormously long life time (10^{12} rotations)
and very sharp edges (10m)

Ch.Huygens coded in 1655:

“It (Saturn) is girdled by a thin flat ring, nowhere touching, inclined to the ecliptic”

Synchronization in rings of Saturn



Density distribution of particles in the ring in the plane of local epicycle velocities ($-3 < v_x/v_{ep} < 3$; $-3 < v_y/v_{ep} < 3$) obtained by the numerical simulations with $N = 1000$ particles inside the spacial square box $S = 5r_s \times 5r_s$ where r_s is the epicycle radius; the particle density is $\rho = N_p/S = 40/r_s^2$. The rotation frequency ratio is $\Omega/\Omega_s = \Omega/2\omega = 1.15$ (top panel) and 1 (bottom panel, synchronized regime); the dimensionless force amplitude of Mimas is $\epsilon = 0.64$. The number of Kapral cells is $N_{cel} = 100 \times 100 = 10^4$, the Kapral collisions are done after time $\tau_K = 0.5/\Omega_s$; the relaxation time of Nosè-Hoover dynamics is $\tau_H = 16/\Omega_s$. The data are averaged over time interval $0 \leq t \leq 10^4/\Omega_s$. Density is proportional to color (red/gray for maximum, blue/black for minimum).

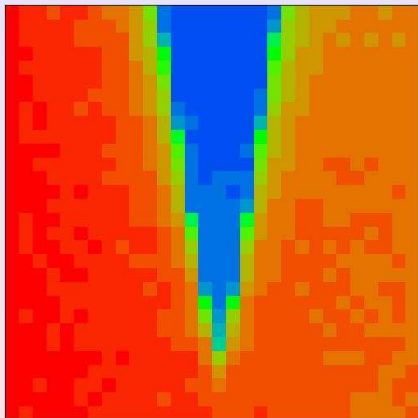
Hill equations in local box:

$$\partial x/\partial t = v_x; \quad \partial y/\partial t = v_y + V_s; \quad \partial v_x/\partial t = 2\Omega v_y + F_x(t)/m_p;$$

$$\partial v_y/\partial t = -\Omega v_x/2; \quad V_s = -3\Omega x/2; \quad F_x(t)/m_p = \epsilon v_{ep} \sum_m \delta(t - mT_{mimas});$$

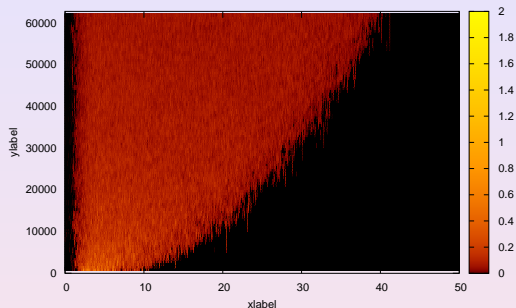
$$\epsilon \approx 0.6, \quad v_{ep} \sim 0.5 \text{ cm/s}, \quad \Omega_s = 1.52 \times 10^{-4} \text{ s}^{-1}, \quad \Omega_s/\omega = 2, \quad R_B = 117580 \pm 10 \text{ km}$$

Synchronization in rings of Saturn



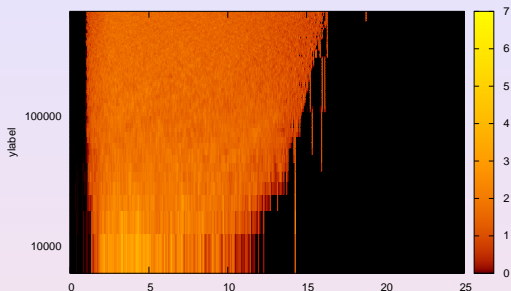
Dependence of the rescaled diffusion rate D/\tilde{D}_0 shown by color on the rescaled frequency $\Omega/2\omega$ (horizontal axis) and driving force strength ϵ (vertical axis) for the range $0.85 \leq \Omega/2\omega \leq 1.15$ and $0 \leq \epsilon \leq 0.7$. The color is proportional to D/\tilde{D}_0 with red/gray for maximum value ($D/\tilde{D}_0 = 1.26$) and blue/black for minimum ($D/\tilde{D}_0 = 4 \times 10^{-4}$), here \tilde{D}_0 is the diffusion rate at $\epsilon = 0, \Omega = \Omega_s$ (also $D/D_0 = 1.7 \times 10^{-5}$ with $D_0 = r_s^3 \Omega_s$). Data are obtained for time $t \leq 10^4/\Omega_s$, $N = 1000$, $N_{cel} = 200 \times 200$, other parameters are as in previous Fig.

Synchronization in rings of Saturn



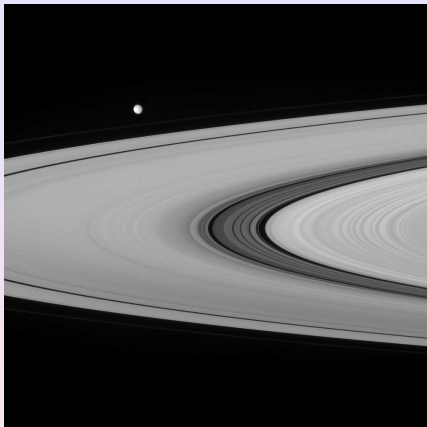
Dependence of density of particles (in arbitrary units) on time $\Omega_S t$ (vertical axis) and position in the ring x/r_S (horizontal axis) for $\epsilon = 0.6$, $\Omega/2\omega = 1.1$ and zero frequency gradient $g = 0$. There are $N_{cel} = 1200 \times 120$ in the whole space box $S = 50r_S \times 5r_S$; $\tau_K = 0.5/\Omega_S$, $\tau_H = 4.5/\Omega_S$. Initially there are $N = 60$ particles in the left box $S = 5r_S \times 5r_S$ and this number is kept constant during the computations till the finite moment of time $\Omega_S t = 6.28 \times 10^4$ when there are 305 particles in total.

Synchronization in rings of Saturn



Dependence of density of particles (in arbitrary units) on time $\Omega_S t$ (vertical axis in logarithmic scale) and position in the ring x/r_S (horizontal axis) for $\epsilon = 0.6$, the frequency gradient in space is $g = 0.002$ with $\Omega/2\omega = 1.05$ at $x = 0$ and $\Omega/2\omega = 1$ at $x/r_S = 25$ corresponding to the outer B ring edge. There are $N_{cel} = 1200 \times 120$ in the whole space box $S = 50r_S \times 5r_S$ (only half is shown); $\tau_K = 0.5/\Omega_S$, $\tau_H = 4.5/\Omega_S$. Initially there are $N = 60$ particles in the left box $S = 5r_S \times 5r_S$ and this number is kept constant during the computations till the finite moment of time $\Omega_S t = 6.28 \times 10^5$ when there are 300 particles in total.

Summary



Mimas and the Great Division
(NASA Cassini Mission
image of Sept 7, 2007;
[http://photojournal.jpl.nasa.gov/
catalog/PIA09750](http://photojournal.jpl.nasa.gov/catalog/PIA09750))

A generic mechanism of
SYNCHRONIZATION INDUCED SELF-ASSEMBLY is proposed



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СПЕЦИАЛЬНЫЕ ПРИЛОЖЕНИЯ: Личный вклад ТЕХЗОНА ЖИЛПЛОЩАДЬ ОТДЫХ ДЕНЬГИ ОБРАЗОВАНИЕ СТИЛЬ

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Наука

9 фев

ПЕРВАЯ НОВОСТИ ПОЛИТИКА БИЗНЕС ФИНАНСЫ ОБЩЕСТВО КОММЕНТАРИИ КУЛЬТУРА АФИША НАУКА АВТОМОБИЛИ СПОРТ

ДЕЗЕРТИР ГЛУХОВ ФИНАНСОВЫЙ КРИЗИС УБИЙСТВО МАРКЕПОВА КУБОК УЕФА ЛИГА ЧЕМПИОНОВ ФОНД ПОМОЩИ

КОСМИЧЕСКИЙ ОБЗОР

ПДД в кольце Сатурна

— 25.12.08 10:31 —

ТЕКСТ: АРТЕМ ТУНЦОВ
ФОТО: NASA/JPL/SPACE SCIENCE INSTITUTE



У многих колец Сатурна невероятно четкие края – при ширине «трека» в десятки тысяч км «обочина» простирается лишь на 10 м. Объяснение довольно просто: частицы, движущиеся по кольцу, четко соблюдают порядок перестроения из ряда в ряд и практически не сталкиваются, несмотря на очень плотный «трафик».

ПОИСК

ПАРТНЕРЫ

Коммерсант
Пули из прошлого
В Москве застрелен один из руководителей антитеррористического центра Чечни и экс-вице-м...

ДЕНЬГИ
Грабительский потребности
Взять деньги у банка в долг дорого, и улучшения ситуации ждут.

СПЕЦИАЛЬНЫЕ ПРИЛОЖЕНИЯ
В Паттайе все ок, но больше не поедем

Я врач и знаю, как сейчас экономит на боль...

Меня уже достали эти «мудеса» с «Аськой»