3. Blackwell et al., Proc. 9th Int. Conf. on Plasma Physics and Contr. Nucl. Fusion (Baltimore), IAEA, Vienna, 1, 227 (1983).
4. T. Ohkawa, Phys. Lett., 67, 35 (1978).
5. B. B. Kadomtsev and O. P. Pogutse, in: Plasma Physics and Controlled Nuclear Fusion Research, IAEA, Vienna, Vol. 2, 69 (1985).
6. Kailhacker et al., Plasma Phys. Contr. Nucl Fusion, 26, 49 (1984).
7. T-10 Group, Proc. 12th European Conf. Contr. Fusion and Plasma Physics, Budapest, Vol. 1 (1985), p. 38.
8. M. Murakami et al., Plasma Physics and Controlled Fusion, 28, No. 1A, 17 (1986).
9. R. J. Bickerton, Plasma Physics and Controlled Fusion, 28, No. 1A, 55 (1986).
10. B. Coppi, Comments Plasma Phys. Controlled Fusion, 5, 261 (1980).
11. H. F. Furth, Invited paper on 12th European Conf. on Contr. Fusion and Plasma Physics, Budapest (1985).
12. J. B. Taylor, Phys. Rev. Lett., 33, 1139 (1974).
13. D. Biskamp, Preprint IPP 6/258, Max-Planck Institute fur Plasmaphysik (1986).

LOCALIZATION OF DYNAMICAL CHAOS IN QUANTUM SYSTEMS

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Numerical experiments with a simple quantum model show that the localization length of dynamical chaos is determined by the diffusion velocity in the classical limit. We investigate the localization phenomenon near the critical value of the parameter of the model. We determine the conditions for delocalization in the case of nonuniform diffusion.

1. Introduction

The phenomenon of dynamical chaos, i.e., random (unpredictable) motion of a fully deterministic (dynamical) system in classical mechanics, has been studied thoroughly and is, in the main, understood (see, e.g., [1, 2]). Besides an important applied value in the problem of stability of motion, the dynamical chaos is of considerable principal interest in physics since it changes appreciably our traditional concepts about the nature and mechanism of randomness and statistical laws by generalizing them (under definite additional conditions) to extremely simple systems (only two degrees of freedom for a conservative Hamiltonian system, and only one degree of freedom in the case of periodic external perturbation). For the discussion below it is significant that the source of dynamical chaos is, in the final result, the continuity of the phase space in the classical mechanics; the mechanism which is used to transform this continuity into the chaotic nature of motion is associated with a strong (exponential) instability of the trajectories [3]. A characteristic feature of the dynamical chaos is the diffusion in the phase space and, of course, the continuous spectrum of motion.

In view of the above fundamental value of the dynamical chaos, a natural and important question arises: to what extent is this phenomenon conserved in the more exact quantum mechanics? The answer to this problem was given by Krylov [4]: in quantum mechanics, this chaos is impossible by virtue of the discreteness of the spectrum of any system bounded in the phase space, as well as by virtue the discreteness of the phase space itself. We understand here a free evolution of the quantum system (its state vector $\psi(t)$) without intermediate measurements.

It is possible that exactly for this reason, the majority of works on "quantum chaos" whose review is given in [1, 3] studied simply features of the quantum behavior under the conditions of chaos in the classical limit. However, there is a principal question: how can one accommodate the absence of chaos in quantum mechanics with the correspondence principle which requires the transition to classical dynamics, including the chaotic one. This problem was formulated and solved in [3].

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The idea of the solution consists of the introduction of characteristic time scales of quantum evolution at which different properties of classical chaos are manifested. At present, two such scales are known. The first and the shorter (logarithmic) scale is determined by the fast spreading of the wave packet as a result of exponential instability of classical trajectories [3, 5]. Here, the dimensionless time τ is measured in the units of the periods of motion, and $q \propto \hbar^{-}$ is the characteristic quantum parameter of the problem (below we set $\hbar = 1$). According to the Ehrenfest theorem, during this very short time scale the wave packet moves along a classical trajectory and, therefore, the quantum dynamics has all the properties of classical chaos.

The second, considerably longer (powerlike) and therefore more important time scale τ_d is of the order $\ln \tau_d \sim \ln q$, and determines the diffusion and relaxation processes in quantum mechanics. This scale was observed in numerical experiments [6] and explained in [3]. The aim of the present work is to study in more detail the diffusion scale τ_d.

2. The Model

As in the previous works [6-8], we use a model described by the so-called standard mapping. In the classical limit the mapping $(I, \theta) \to (\bar{I}, \bar{\theta})$ has the form (in dimensionless quantities)

$$\bar{I} = I + k \sin\theta, \quad \bar{\theta} = \theta + T\bar{I}, \tag{1}$$

where (I, θ) are the action-angle variables, T is the period of the mapping, and k is the perturbation parameter. In a quantum system, the mapping for the wave function $\psi \to \bar{\psi}$ takes the form [6]

$$\bar{\psi} = \exp\left(i\frac{T}{2}\frac{\partial^2}{\partial\theta^2}\right)\exp(-ik\cos\theta)\,\psi. \tag{2}$$

The action is quantized (I = n is integral), and the transition to the classical limit corresponds to $k \to \infty$, $T \to 0$, $K = kT = \text{const}$. The last parameter determines fully the dynamics of the classical model (1). Depending on the value of the parameter K, there exist two qualitatively different regimes: the finite ($|\Delta I| \lesssim \sqrt{k/T}$) and infinite motion. The critical value of K which divides both regimes is equal to $K_{cr} = 0.9716...$ [9]. It should be noted that in both regimes there exist both regular and chaotic components of motion, and the measure of the first tends to zero with increasing K [2].

For $K > K_{cr}$, the motion with a single chaotic component can be described as a diffusion with respect to I with velocity (see [10] and Sec. 5 below)

$$D_{cl} \equiv \frac{\langle(\Delta I)^2\rangle}{\tau} = \frac{D_0(K)}{T^2} \approx \frac{1}{T^2}\begin{cases}(K^2/2)\,(1+2J_2(K)+2J_2^2(K)) & (K \geqslant 4{,}5)\\ 0{,}30\,(K-K_{cr})^3 & (K \leqslant 4{,}5)\end{cases}, \tag{3}$$

where τ is the number of iterations of the mapping, and $J_2(K)$ is the Bessel function.

In the quasiclassical region the diffusion law (3) is conserved within the diffusion time scale which for the model under consideration is of the order of $\tau_d \sim D_{cl} \sim k^2$ [3]. For $\tau \ll \tau_d$, the quantum dependence of D_q, in accordance with the correspondence principle, repeats well all the details of the classical diffusion, including those which are not described by the simple relation (3). This is illustrated in Fig. 1 where the circles show numerical data of the classical model (1) [11]. The full line corresponds to the theory (3) [10], and the crosses are the numerical results for the quantum model (2) [12] with $k \approx 40$.

For $\tau \gg \tau_d$, the quantum diffusion stops and is transformed into steady-state oscillations [13] which have been investigated up to $\tau = 5 \cdot 10^4$ [14]. This indicates the discreteness of the quasienergy spectrum of the model (2) [3]. If only for this reason, the "quantum chaos" in the entire quasiclassical region is not a true one as in the classical limit. We shall call it a pseudochaos. We note that the deviation from the true chaos begins already during a much shorter time scale τ_s which manifests itself, in particular, in the absence of local instability of the quantum motion for $\tau > \tau_s$ [15], and in the absence of residual correlations [15, 16] (see also [1]).

For $T \gtrsim 1$ the motion is essentially quantum [1]. Nevertheless, pseudochaos is conserved also in this region under the additional conditions $k \gtrsim 1$. However, its statistical properties, in particular, the velocity of initial diffusion, differ, generally speaking, from the region $T \lesssim 1$ even for $k \gg 1$. In the opposite case $k \ll 1$, the ordinary perturbation theory

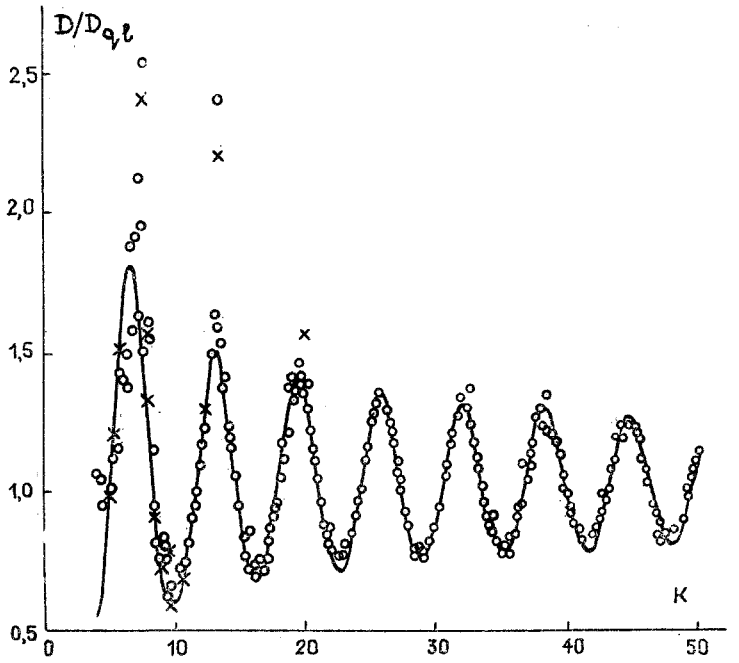

Fig. 1

holds (in the absence of resonance) which indicates that the corrections to the unperturbed states are small, i.e., the quantum dynamics is regular independently of the character of motion in the classical limit. This mechanism of quantum stabilization of the classical chaos, associated with the discreteness of the quantum spectrum, was considered by Shuryak [17].

In the case of a classical nonlinear resonance, the condition of quantum stabilization takes the form $k/T \lesssim 1$ which has the following simple physical meaning: the number of levels captured in the resonance is $\sqrt{k/T} \lesssim 1$ [17]. If both parameters are small ($k \lesssim 1$, $k/T \lesssim 1$), all unperturbed states are conserved. This is the region of global applicability of the perturbation theory. For $k \lesssim 1$ but $k/T \gtrsim 1$, the unperturbed states are conserved only between resonances. Inside the resonances, the unperturbed states are mixed. However, the motion remains regular and one can go over to new, resonant, quantum numbers and their corresponding quasienergy eigenfunctions. Finally, in the quasiclassical region for $k \gtrsim 1$ and $k/T \gtrsim 1$ (but $kT \lesssim 1$), the mixing takes place for all unperturbed states but the region of mixing does not exceed the width of an individual resonance.

The above described quantum dynamics of the model takes place only for irrational values of the parameter $T/4\pi$. In the case of rational $T/4\pi$, a phenomenon of quantum resonance takes place which is specific for the present model [18]. The quasienergy spectrum becomes continuous, and the action $|n|$ increases in proportion to τ ($\tau \to \infty$) for any k. This feature of the model is associated with the character of the unperturbed spectrum $E_n = n^2/2$ and is not typical for the quantum dynamics. For this reason, the dynamics of the model (2) is determined by the quantity T taken modulo 4π.

3. The Principle of Quantum Localization of Chaos

Following [3] we consider, first qualitatively, the mechanism of diffusion restriction in the model (2). It is based on the following simple physical arguments. According to the correspondence principle, the quantum motion in the quasiclassical region should be, in some sense, close to classical, at least over some characteristic time scale. This is clear for the shortest time scale τ_s, i.e., until the spreading of the wave packets which move along classical trajectories. For $\tau \gg \tau_s$ the quantum dynamics, of course, does not coincide fully with the classical one but, as numerical modelling shows [3, 6, 15], the classical diffusion in the system continues. If the diffusion were unrestricted as in the classical limit, the spectrum of quasienergies would be continuous. It is important, however, that in the case of purely discrete spectrum with average level density ρ_0, the classical diffusion can continue during the time

$$\tau \sim \tau_d \sim \rho_0. \qquad (4)$$

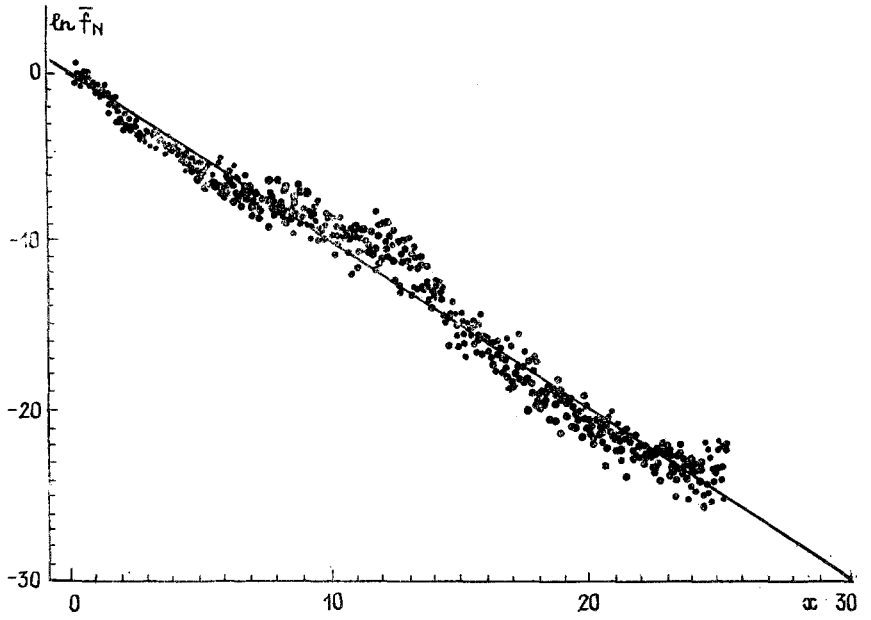

Fig. 2

This follows immediately from the uncertainty principle since for $\tau \ll \rho_0$ the system does not resolve ("does not feel") the discrete character of the spectrum. The estimate (4) also determines the diffusion time scale for the model (2). We should note that the density ρ_0 in (4) is determined only by the eigenfunctions represented in the given state of the system, whose number is always finite.

To estimate the quantity ρ_0 we consider initially the evolution of an arbitrary unperturbed state. As a result of classical diffusion during the time τ_d, some number $\Delta n \sim (D_c l \tau_d)^{1/2}$ of the neighboring unperturbed states will be effectively excited. This means that the exact eigenfunctions of the system are superpositions of several (of the order of Δn) unperturbed states. Vice versa, any unperturbed state is represented by this number of eigenfunctions. Hence $\rho_0 \sim \Delta n \sim \tau_d$ according to (4) and [3]

$$\tau_d \sim D_{cl}, \quad l \sim \Delta n \sim D_{cl}, \tag{5}$$

where l is the effective number of unperturbed states which are involved in the steady-state oscillations after the diffusion terminates. In other words one can say that the quantity l determines the size of the localization region of the eigenfunctions in the unperturbed basis (with respect to n). The quantity l will be called the localization length. A notable feature of relations (5) is that they establish a relationship between the quantum characteristics of motion (diffusion scale τ_d and localization length l) with the diffusion velocity D in the classical limit.

The estimate (5) for τ_d is clearly independent of the initial state of the system, in the same way as the localization length of the eigenfunctions. As far as the localization length l of the steady-state oscillations is concerned, the estimate (5) is valid only in the case when the size of the initial state $l_0 \ll l$. If $l_0 \gtrsim l$, the size of the state remains, in order of magnitude, unchanged.

Rechester et al. [10] noted an analogy between the above localization in the momentum space (with respect to n) and the well-known Anderson localization in a random potential (see, e.g., [20]). The most important difference between the two phenomena is due to the fact that the model (2) which we consider here does not contain any random parameters. By continuing this analogy one can note that the quantum resonance corresponds to the delocalized Bloch states in a period potential [18, 19]. We also note that the localization mechanism in both systems is, generally speaking, quite different depending on the parameters of motion. For example, for the model (2) for $K \gtrsim 1$ and $k > 1$, the localization of the spectrum is associated with the termination of classical diffusion as a result of interference effects. If $K \lesssim 1$ ($k > 1$), the localization is determined by quantum tunnelling to the classically inaccessible region.

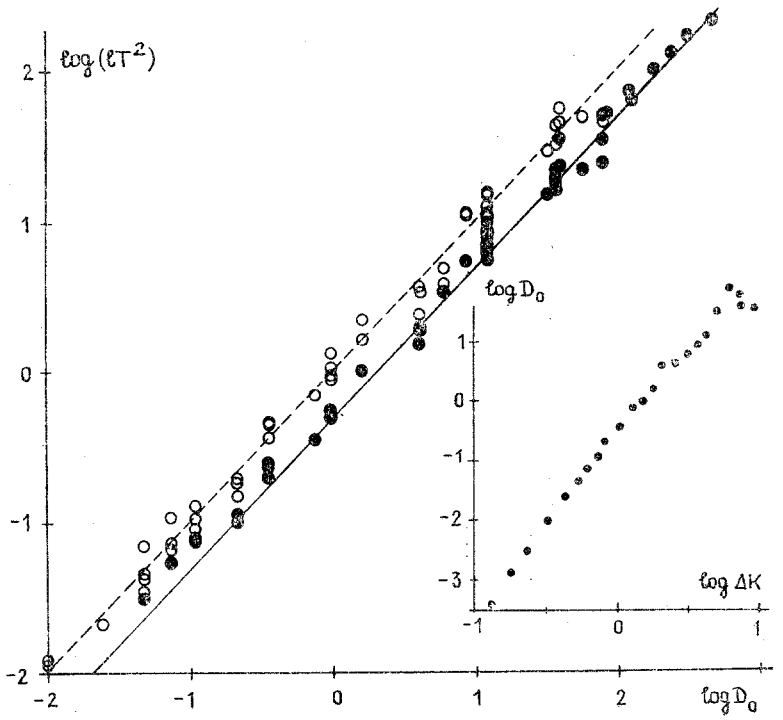

Fig. 3

4. Quantum Localization Under Uniform Diffusion

Below we shall consider the quantum localization associated with the termination of quantum diffusion, i.e., for the model (2) we shall assume that $K > K_{cr}$ and $k \gg 1$. We consider first the case of uniform (with respect to n) diffusion for $K \gg 1$. In this case one can also practically neglect the stable component of the motion [21]. To improve the estimate (5) it is necessary, first of all, to determine more accurately the diffusion scale τ_d and (or) the localization length l. As we have explained, this is done most conveniently just for the quantity l since the localization law of the eigenfunctions predicted in [22] by analogy with the Anderson localization can be approximately represented in the form of the following expression:

$$|\varphi_m(n)| \propto e^{-|n-m|/l+\xi_{nm}}, \tag{6}$$

where the random quantity ξ_{nm} characterizes fluctuations around the mean exponential localization and $\langle \xi_{nm} \rangle = 0$.

Let us suppose that at the initial moment of time, the system is in the state with n = 0. The steady-state oscillations which are established for $\tau \gg \tau_d$ are characterized by a time-averaged distribution

$$\overline{f}(n) = \overline{|\psi(n,\tau)|^2} = \sum_m |\varphi_m(0)\,\varphi_m(n)|^2. \tag{7}$$

We note that $\overline{f}(n)$ is an analog of the density–density correlator in a solid [20]. Noting that, on average,

$$\langle |\varphi_m(n)|^2 \rangle \approx \frac{1}{l_s} e^{-2|n-m|/l_s}, \tag{8}$$

we obtain from (7)

$$f(n) \approx \frac{1}{2l_s} \exp\left(-\frac{2|n|}{l_s}\right)\left(1+\frac{2|n|}{l_s}\right), \quad \overline{n^2} = l_s^2. \tag{9}$$

The quantity l_s can differ from the localization length of the eigenfunctions l. The reason is due to the fluctuations ξ_{nm}. For example, in the case of Gaussian fluctuations with $\langle (\Delta\xi_{nm})^2 \rangle = D_\xi |\Delta n|$ we have (m = 0)

$$|\varphi_0(n)| \sim \int_{n/l}^{\infty} \exp\left(-\frac{n}{l} + \xi - \frac{\xi^2}{2D_\xi n}\right) d\xi. \tag{10}$$

Evaluating the integral we obtain

$$\frac{1}{l_s} = \frac{1}{l} - \frac{D_\xi}{2}, \quad D_\xi l \ll 1, \quad \frac{1}{l_s} = \frac{1}{2l^2 D_\xi}, \quad D_\xi l \gg 1. \tag{11}$$

An analogous phenomenon is well known in solids [20]. An example of the steady-state distribution is shown in Fig. 2 for k = 10, T = 0.5, K = 5, $x = 2n/l_s$, $\bar{f}_N = \bar{f}(n)2l_s/(1 + x)$; the straight line is $\bar{f}_N = e^{-x}$. Within the statistical fluctuations, the law (9) holds in a vast range which includes about 10 orders of magnitude of the variation of $\bar{f}_N$. A typical large-scale structure is associated, clearly, with strong fluctuations of the quantity ξ_{nm} (see (10)).

In numerical experiments, the localization length l_s was determined from a steady-state distribution using relation (9). According to (5), the relation

$$\alpha = l_s/D_{cl} = l_s T^2/D_0 \tag{12}$$

should not depend on the parameters of the model. Figure 3 shows our numerical data for the dependence $l_s T^2$ on D_0 (circles) on a log–log plot. It is seen that in a range of almost four orders of magnitude, the expected dependence (12) indeed holds, and the average value $\langle\alpha\rangle = 1.04 \pm 0.03$ (the error is statistical) is very close to unity. The quantum perturbation parameter k changes in the interval 5-120, $T \leq 1$ and the localization length l_s = 9-180. Figure 3 also shows the data obtained in [23] for the localization length of the eigenfunction: $l^{-1} = \langle \ln|\varphi_m(n)|/n\rangle$ (points). The parameters vary in the intervals: k = 5-75, $T \leqslant 1$, l = 8-1200. In the calculation of l, Shepelyanskii [23] used a special method which makes it possible to obviate the determination of full eigenfunctions. Despite the scatter of points, the obtained data clearly show that $l_s \approx 2l$. From this relation we determine the diffusion velocity ξ_{nm} and the velocity of the wavefunction $D_\xi = 1/l = 2/D_c l$. A direct calculation of D_ξ by the method of [23] confirms this relation.

In fact, it is seen from Fig. 3 that the localization law (12) remains in force also in the region $K \sim 1$ where the classical diffusion becomes considerably nonuniform and large regions of stable motion are formed. We note that in the range of values of K in the insert of Fig. 3, the dependence of D_0 on $\Delta K = K - K_{cr}$ is very complicated (see (3)).

5. Quantum Localization in the Critical Region

For $K \to K_{cr}$, the velocity of classical diffusion in the model (1) sharply falls as a result of formation of a specific critical structure [9, 24]. According to [24], $D_0 \propto (\Delta K)^3$ ($\Delta K = K - K_{cr}$). This law agrees with the old measurements of D_0 [21] and was verified with more accuracy by us in the interval K = 1.1-2.8 where the diffusion velocity can be described by the empirical formula (3) (see insert in Fig. 3).

Expression (3) gives the average diffusion velocity during a time much shorter than the time of the transition between neighboring integral resonances (approximately equal to $130(\Delta K)^{-3}$). The local diffusion velocity in this region becomes sharply nonuniform, again because of the formation of a critical structure. Nevertheless, the law of quantum localization (12) holds satisfactorily also in this region under the additional condition that the quantum parameter p is sufficiently large. In the opposite case, the character of the steady-state distribution sharply changes (Fig. 4, k = 10, K = 1.5; the period of the resonant structure $2\pi/T \approx 42$). Here, the nonuniformity of diffusion leads to a nonmonotonic variation of $\bar{f}(n)$. Clearly pronounced are four resonances which correspond to "plateaux" of $\bar{f}(n)$. The lowest plateau is determined by the rounding errors in the calculation. The plateaux inside the resonances are formed as a result of rapid oscillations with respect to n. Between resonances, in the regions of critical structure, the diffusion velocity and, alongside, the localization length decrease sharply and this leads to a fast fall of $\bar{f}(n)$. This case will be called nonuniform localization.

An estimate for k_{cr} can be obtained as follows. Using a method described in Sec. 3, we represent the localization length in the form

$$l^2 \sim k^2 + (D_0/T^2)\,\tau_d \sim \tau_d^2. \tag{13}$$

Here, the term k^2 characterizes the contribution of the first period of perturbation (for the initial state n = 0) which is important for $D_0 \to 0$ ($K \to K_{cr}$). It is seen from (13) that two regimes of quantum localization are possible, depending on the ratio of both terms. If $k^2 \ll D_0\tau_d/T^2$, i.e., the contribution of the first period of the perturbation can be neglected,

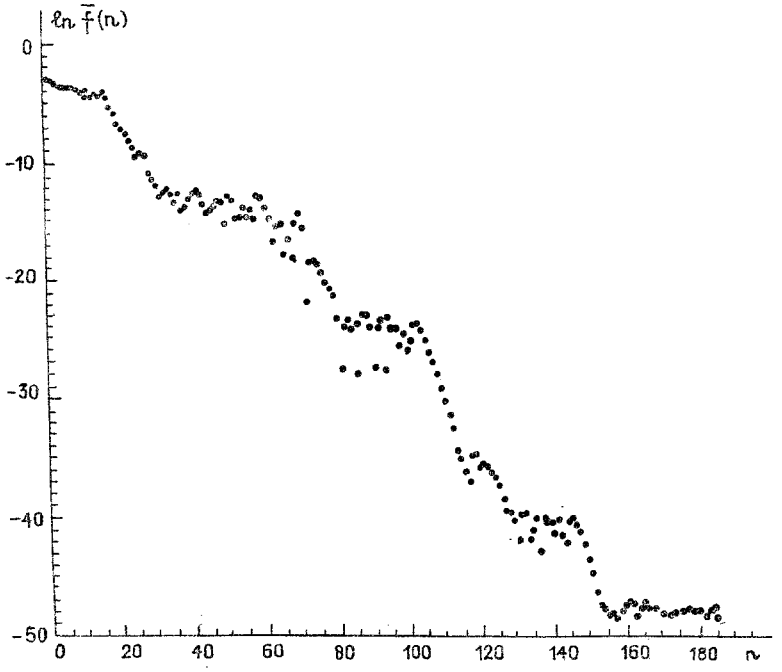

Fig. 4.

then $l \sim D_0/T^2$, and we return to previous expression (12). This is the regime of fast diffusion which leads to a smooth steady-state distribution (Fig. 2). This regime is conserved even for $K \to K_{cr}$ if the quantum parameter k is sufficiently large (13). In the opposite limit of slow diffusion $D_0\tau_d/T^2 \ll k^2$, the localization becomes nonuniform (Fig. 4). The boundary between the two regimes is given by the estimate $k^2 \sim D_0\tau_d/T^2$ or

$$D_0 k_{cr}/K^2 \approx 1, \tag{14}$$

where the empirical value of the nonuniformity parameter is taken from the numerical data. The boundary (14) can be associated with the characteristics of the critical structure for $K \to K_{cr}$. A simple resonant theory of such structure is described in [25]. According to this theory, the above-critical perturbation $\varepsilon = \Delta K > 0$ disrupts all small-scale features of the critical structure up to the features with characteristic time of motion $\tau_\varepsilon \sim \varepsilon^{-1}$. We then find from (14) $\tau_\varepsilon \sim k_{cr}^{1/3}$, where we used the estimate $D_0 \sim \varepsilon^3$ (3) and we put $K \approx 1$.

If, instead of the quantum parameter k we use $\hbar$, and understand the transition to the classical limit as $\hbar \to 0$, the last estimate takes the form $\tau_\varepsilon \sim \hbar^{-1/3}$ and coincides formally with the result of [26] obtained by another method. However, the interpretation of this result is here completely different. First, in [26], the time $\hbar^{-1/3}$ is in no way associated with the magnitude of the perturbation ε. Second, Fishman et al. [26] take it as the diffusion scale τ_d. This quantity, however, is much larger (see (13)):

$$\tau_d \sim k_{cr} \sim \hbar^{-1} \gg \tau_\varepsilon \sim k_{cr}^{1/3} \quad (k_{cr} \gg 1).$$

6. Delocalization

If the velocity changes during the diffusion process, the character of quantum localization can be affected considerably. In particular, delocalization is possible when the quantum diffusion continues without limit. This phenomenon was studied in [3] using the model (2) in which the parameter $k(\tau)$ depends explicitly on time. In the case of the power-law dependence $k = k_1\tau^\mu$ ($k_1 > 1$), the delocalization sets in for $\mu \geqslant 1/2$.

Below we consider the example of nonuniform diffusion with respect to n by taking $D_c l(n) = D_1 n^\beta$. If β is sufficiently large, i.e., the diffusion velocity increases faster than n, the number of unperturbed states (Δn) associated with diffusion before some time τ will increase faster than τ. Since for number of quasienergy eigenstates and their density $\rho_0 \gtrsim \Delta n \sim \tau$, the equality (4) is not reached and localization does not take place. This also determines the conditions of delocalization.

Let us suppose that initially $n = n_0 \sim 1$. We then obtain from the diffusion equation $D_1\tau \sim n^{2-\beta} < D_1 n$. This inequality also gives the delocalization boundary: $\beta \geqslant 1$, $D_1 \gtrsim 1$, and

the condition $D_1 \gtrsim 1$ is equivalent to the requirement that $k \sim \sqrt{D_1} \gtrsim 1$ for any τ (the Shuryak boundary, see Sec. 2). We note that the restriction $\beta \geqslant 1$ coincides precisely with the above condition $\mu \geqslant 1/2$ for an explicit time dependence of diffusion velocity. If the initial $n_0 \gg 1$, the localization is possible also with $\beta > 1$ for the length of uniform localization (5) with $D_{c\ell} = D_{c\ell}(n_0) < n_0$. Hence, the delocalization condition [27] $D_1 \gtrsim n_0^{1-\beta} \ll 1$. In the opposite case, the system "does not feel" the increase of $D_{c\ell}$ with n. For $n_0 \sim 1$, this condition reduces to $D_1 \gtrsim 1$. The described mechanism of delocalization is important in the excitation of the hydrogen atom by a monochromatic field [27].

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LITERATURE CITED

1. G. M. Zaslavskii, Stochasticity of Dynamical Systems [in Russian], Nauka, Moscow (1984).
2. A. Lichtenberg and M. Lieberman, Regular and Stochastic Dynamics [Russian translation], Mir, Moscow (1984).
3. B. V. Chirikov, F. M. Izrailev and D. L. Shepelyansky, Sov. Sci. Rev., 2C, 209 (1981).
4. N. S. Krylov, Works on the Foundation of Statistical Physics [in Russian], Izd. Akad. Nauk SSSR, Moscow (1950).
5. G. P. Berman and G. M. Zaslavsky, Physica, 91A, 450 (1978).
6. G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, Lect. Notes Phys., 93, 334 (1979).
7. T. Hogg and B. A. Huberman, Phys. Rev. Lett., 48, 711 (1982).
8. J. D. Hanson, E. Ott, and T. M. Antonsen, Phys. Rev., 29A, 819 (1984); Phys. Rev. Lett., 53, 2187 (1984).
9. J. M. Greene, J. Math. Phys. 20, 1183 (1979).
10. A. B. Rechester, M. N. Rosenbluth, and R. B. White, Phys. Rev., 23A, 2664 (1981).
11. A. B. Rechester and R. B. White, Phys. Rev. Lett., 44, 1586 (1980).
12. D. L. Shepelyanskii, Candidate's Dissertation, Novosibirsk (1981).
13. B. V. Chirikov, Usp. Fiz. Nauk, 139, 360 (1983).
14. G. Casati, Private Commun. (1985).
15. D. L. Shepelyanskii, Physica, 8D, 208 (1983).
16. D. L. Shepelyanskii, Teor. Mat. Fiz., 49, 117 (1981).
17. É. V. Shuryak, Zh. Eksp. Teor. Fiz., 71, 2039 (1976).
18. F. M. Izrailev and D. L. Shepelyanskii, Teor. Mat. Fiz., 43, 417 (1980).
19. S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev., 29A, 1639 (1984).
20. I. M. Lifshits, S. A. Gredeskul, and L. A. Pastur, Introduction to the Theory of Disordered Systems [in Russian], Nauka, Moscow (1982).
21. B. V. Chirikov, Phys. Rep., 52, 263 (1979).
22. S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. Lett., 49, 509 (1982).
23. D. L. Shepelyanskii, Phys. Rev. Lett., 56, 677 (1986).
24. R. S. MacKay, J. D. Meiss, and I. C. Percival, Physica, 13D, 55 (1984).
25. B. V. Chirikov and D. L. Shepelyansky, Physica, 13D, No. 3, 395 (1984); B. V. Chirikov, Proc. Int. Conf. Plasma Physics, Lausanne, Vol. 2 (1984), p. 761.
26. S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. Lett., 53, 1212 (1984).
27. G. Casati, B. V. Chirikov, and D. L. Shepelyanskii, Phys. Rev. Lett, 53, 2525 (1984).