Quantum chaos: diffusion photoeffect in hydrogen

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D. L. Shepelyanskiĭ. *Quantum chaos: diffusion photoeffect in hydrogen.* The process of ionization of a highly excited hydrogen atom in a monochromatic electromagnetic field is an example of an unusual photoeffect in which ionization at a frequency much lower than the ionization energy ($\omega \ll I$) proceeds much more rapidly than one-photon ionization ($\omega > I$).[1] Such rapid ionization is linked with the appearance of dynamic chaos[4] in the classical system at a field intensity higher than the critical value: $\varepsilon > \varepsilon_c = \omega_0^{-1/3} n^{-4}$ (atomic units are employed, $\omega_0 = \omega n^3 \gtrsim 1$), when nonlinear resonances overlap.[5-7] Here there arises the question of the influence of quantum effects on the chaotic motion (quantum chaos), which is also of general physical interest in itself.

In the region of chaos the excitation of the electron is described by the diffusion equation with the diffusion velocity[6,7] $D = \langle (\Delta n)^2 \rangle / \Delta\tau = 2\varepsilon^2 n^3 \omega^{-7/3}$ (for definiteness we confine our attention to the case of a linearly polarized field and initial states with parabolic and magnetic quantum numbers $n_1 \gg n_2 \sim 1$ and $m = 0$, in which the dynamics can be described by the one-dimensional Schrödinger equation[8]; $\tau = \omega t / 2\pi$). Because of the rapid growth of D with the level number n the diffusion ionization proceeds over a characteristic time $\tau_I \sim n^2/D$, while its rate $\Gamma_D \sim \omega/\tau_I$ for $\omega \sim n^{-3}$ is many times higher than the rate of one-photon ionization Γ_P for $\omega = I = (2n^2)^{-1}$: $\Gamma_D / \Gamma_\Phi \sim n^{4/3}/8$.[1]

Figure 1 shows the dependence of the ionization probability (total probability on levels with $n \geqslant 100$ and in the continuum in %) at the time $t = 80\ \pi n_0^3 \approx 2 \cdot 10^{-9}$ s on the frequency of the field obtained by numerical simulation of the classical (1) and quantum (2) equations of motion.[1] The initial level number $n_0 = 66$, the intensity of the field is fixed $\varepsilon = 0.05 n_0^{-4} \approx 14$ V/cm. For $\omega > \omega_0 \approx I$ the numerical data are in excellent agreement with the theoretical rate of one photon ionization (curve). In the interval $\omega_c \lesssim \omega_0 \lesssim \omega_l$ the quantum probability agrees satisfactorily with the classical probability, indicating that diffusion excitation also occurs in the quantum system, the lower limit of the diffusion photoeffect $\omega_c \sim 1$ is determined by the fact that at frequencies less than the Kepler frequency there are no primary resonances in the system, and therefore the motion of the electron is stable.[6]

The upper limit $\omega_l \approx (6\varepsilon^2 n_0^9)^{3/7}$ arises because of the fact that quantum effects localize the chaos,[2,8] which was first observed for a simple model of a quantum rotator.[9,10] As a result of this a stationary distribution over levels of the type $f_n \propto \exp(-2|n - n_0|/l)$ with a localization length $l \approx D$ is established in the system.[2,10,11] An example of localization in hydrogen is presented in Fig. 2 for the parameters $n_0 = 66$, $\omega_0 = 2.5$, $\varepsilon n_0^4 = 0.04$, the distribution f_n was averaged in the interval $80 < \tau \leqslant 120$. One can see from it that the quantum distribution (solid line) is localized, while the classical distribution (the fine dashed curve shows the numerical data) is described satisfactorily by the analytical solution of the diffusion equation (dashed curve).[3]

We note that at high levels peaks corresponding to a

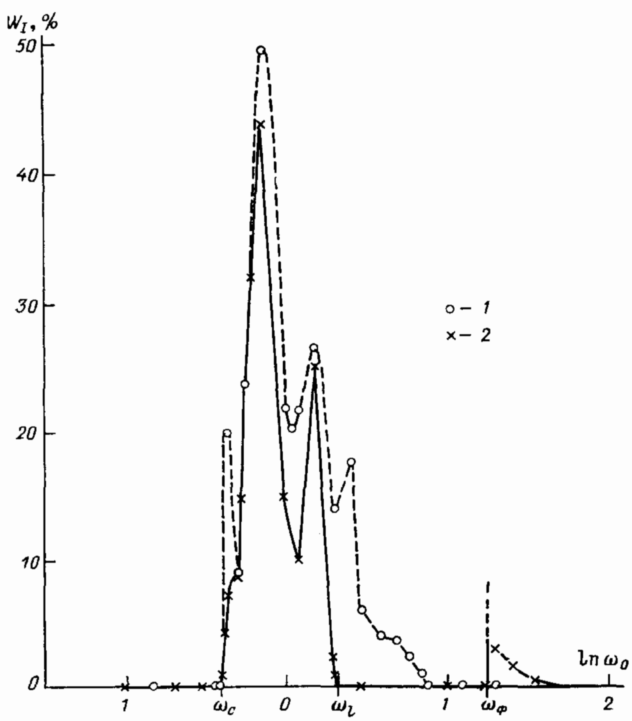

FIG. 1.

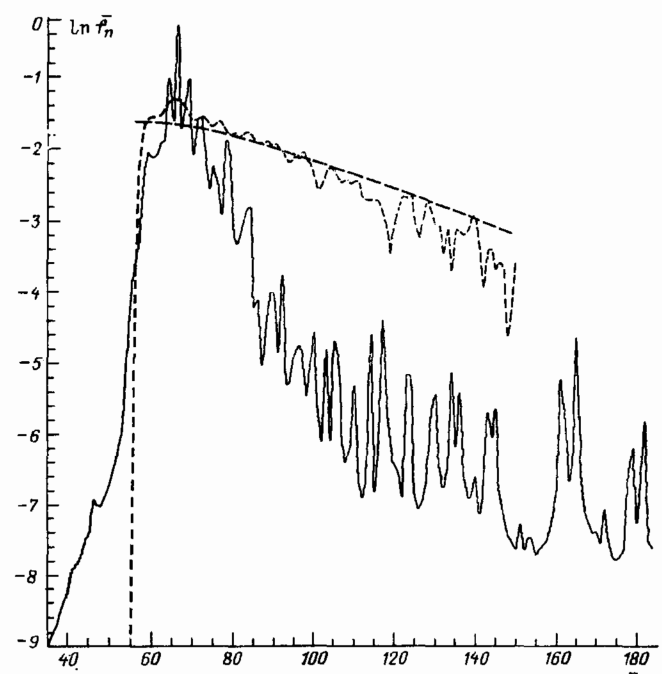

FIG. 2.

chain of one-photon transitions appear in the distribution.[2] If the matrix element of such a transition[6] $\varepsilon z_{nn'} \approx 0.4\,\varepsilon\omega^{-5/3} (nn')^{-3/2} \approx 0.4\ \ \varepsilon\omega^{-5/3} n^{-3} c 1/2n^2 - 1/2n'^2 = \omega \gtrsim n^{-3}$ is much less than the distance between neighboring levels n^{-3}, then its probability is low $\sim (\varepsilon\omega^{-5/3})^2$ and is independent of n. In this case the chain of one-photon peaks is localized, while the rate of ionization equals $\Gamma_I \sim (\varepsilon\omega^{-5/3})^{2k}\,\Gamma_\Phi$, where $k = n_0/2\omega_0$ is the number of peaks while $\Gamma_\Phi \approx (\varepsilon\omega^{-5/3})^2 \omega^{3/2}$ is the transition rate from the last peak into the continuum. For $\omega^{5/3} \ll \varepsilon < \varepsilon_c$ there is no ionization in the classical system, while in the quantum system ionization proceeds owing to tunneling through the invariant curves: $\Gamma_I \sim (\omega/\tau_I)\exp[-Cn_0(\varepsilon_c - \varepsilon_0)/\varepsilon_0]$, where C is a numerical constant.

In the region of chaos with $l \sim n_0$ delocalization occurs in the system because of the growth of D as n increases. In this regime the quantum process of excitation agrees satisfactorily with the classical process.[1-3] Local instability nevertheless does not occur in the quantum system even in the region of delocalization. This leads to the fact that in the numerical simulation when at the time $\tau = \tau_g$ time is reversed ($\psi \to \psi^*$) the total probability (even from the continuum) returns to the initial level n_0 at the moment $\tau = 2\tau_g$ (with an accuracy of 10^{-16}).[3] In the classical system such reversibility does not occur because of the exponential instability of the trajectories.

At the present time only two laboratory experiments have actually been performed,[12,13] in which the electron was located in the region of classical chaos ($n_0 \sim 60$, $\omega_0 \sim 0.5$, $\varepsilon n_0^4 \sim 0.06$). The results of these experiments agree satisfactorily with the data from numerical simulation of the classical dynamics[12,14] and in this case confirm the classical picture of diffusion ionization. The reason for this lies in the fact that the experiments were performed in the region of delocalization $\omega_0 < \omega_l$. To observe quantum localization of chaos the experiments must be performed at a higher frequency: $\omega_0 > \omega_l$. In this region quantum effects play a significant role[1-3] and the classical picture of the ionization process[7,13,14] is not valid.

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