results are presented in the figure. One can see how the harmonics $nk_0 + mk_1$ with all possible n and m arise so that the spectrum becomes dense and practically may be considered continuous. However, in the strict sense of the term there are only discrete components in the spectrum and the field remains quasiperiodic in space. This paradox can be resolved if we consider a quasiperiodic field as an object (a two-dimensional torus) in a phase space. It is convenient to consider a cross-section of the torus. For eq.(1) this means that we take the points $u_\ell = |a(\ell L, t)|$, $L = 2\pi/k_0$, $\ell = 0, 1, 2, \ldots$ and mark them on the plane $(u_\ell, u_{\ell+1})$. These points form a closed-line cross-section of the torus. One can see this line fold, wind and stretch as the time grows. For large t one cannot distinguish this line from a strange attractor, as well as the spectrum cannot be distinguished from the continuous one.

Using space-time analogy, we may reformulate the problem described above as a spatial development of temporal chaos. For quasimonochromatic waves in the medium with convective instability the 'dual' form of the Ginzburg-Landau equation is used

$$\frac{\partial a}{\partial x} = a + (1 + i\bar{C}_1)\frac{\partial^2 a}{\partial t^2} + (-1 + i\bar{C}_2)|a|^2 a$$

Thus, all the above results can be directly applied to this case.

Nonlinear and Turbulent Processes in Physics (Kiev, Naukova Dumka v. 1, p. 227 (1988))

CHAOS AND IONIZATION OF THE HYDROGEN ATOM IN A MONOCHROMATIC FIELD

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We consider ionization of highly excited Hydrogen atom by a linear polarized monochromatic electric field. For extended states with magnetic quantum number $m = 0$ and parabolic quantum numbers $n_1 \gg n_2$ it is possible to describe the classical process of excitation by the one-dimensional Hamiltonian[1,2]

$$H = -\frac{1}{2n^2} + \varepsilon n^2 \cos\omega t \left[\frac{3}{2} - 2\sum_{s=1}^{\infty} \frac{J_s'(s)}{s} \cos(s\lambda)\right] \tag{1}$$

where n, λ are action and conjugate phase and atomic units are used. Instead of continuous equations it is more convenient to describe the motion by a map. To this end, we use the unperturbed expression for coordinate Z and integrate the Hamiltonian equations over an orbital period of the electron. After that we find the generating function of the map:

$$G(\bar{N}, \phi) = \bar{N}\phi + 2\pi(-2\omega\bar{N})^{-1/2} + k\,A(\bar{\varkappa})\cos\phi$$

Here $N = E/\omega = -1/(2\omega n^2)$, ϕ is the phase conjugated to N and equal to the value of ωt at the moment of passage through the perihelion, $k = 0.822\pi\,\varepsilon/\omega^{5/3}$ and $A(\varkappa) = \frac{\varkappa^{2/3}}{0.411} \cdot J_\varkappa'(\varkappa)$ where $J_\varkappa(\varkappa)$ is Anger function (for integer $\varkappa$ it coincides with Bessel function and for $\varkappa \to 0: J_\varkappa'(\varkappa) = \varkappa/2$, $\bar{\varkappa} = \omega(-2\omega\bar{N})^{-3/2}$).

For $\varkappa \gg 1$ the function $A \to 1$ and the map has very simple form:

$$\bar{N} = N + k\sin\phi\,, \quad \bar{\phi} = \phi + 2\pi\omega(-2\omega\bar{N})^{-3/2} \tag{2}$$

where the bar denotes the values of variables after an orbital period. A check of this map was made in the following way. We numerically solve the continuous equations (1) and fix the values of ωt in the perihelion. The function $g(\phi) = (\bar{N} - N)/k$ was determined from the three successive values of ϕ. The comparison of numerical results (points) with the theoretical dependence $g(\phi) = \sin\phi$ (line) is shown in Fig. 1 for initial conditions with $\varepsilon_0 = \varepsilon n_0^4 = 0.04$, $\omega_0 = \omega n_0^3 = 1.5$.

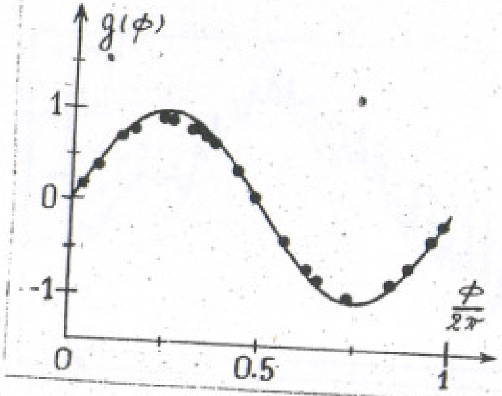

Fig. 1.

Therefore, the map (2) give satisfactory description of motion up to $\omega_0 \approx 1$.

The map (2) can be locally approximated by the standard map [3,4] with parameters k, $T = 6\pi\omega^2 n_0^5$ and $K = kT = \varepsilon_0/\varepsilon_c$, where $\varepsilon_c \approx 1/(49\,\omega_0^{1/3})$. The global diffusion takes place for $K > 1$ [3] or $\varepsilon_0 > \varepsilon_c$ that is in agreement with[1]. During the diffusion process the value of K increases, the phases ϕ become random and independent and the diffusion rate on $\mathcal{N}$ is equal to $D = k^2/2$. An example of phase-plane for $E_0 = \mathcal{N}\omega n_0^2$ and ϕ at $\varepsilon_0 = 0.04$, $\omega_0 = 3$ is shown in Fig. 2 (6 stable and 1 unstable trajectories). It is important to note that ionization appears as the result of the last kick which carries a trajectory from $\mathcal{N} < 0$ to $\mathcal{N} > 0$ after that it extends to infinity.

For quantization of (2) we note that according to the usual relation $\hat{E} = -i\frac{\partial}{\partial t}$ the operator $\hat{\mathcal{N}} = -i\frac{\partial}{\partial \phi}$ $(-\infty < \phi < \infty)$. Since the perturbation is periodic in ϕ then a new integral of motion will appear (quasi-momentum), besides quasi-energy. For an unperturbed level n_0 the quasi-momentum is equal to the fractional part of $\mathcal{N}_0 = -1/(2\omega n_0^2)$. External perturbation leads to excitation of high harmonics of quasi-momentum. Since $\phi \propto t$, this quasi-momentum corresponds to the quasi-energy in the initial system (1). Thus, the quantized map (2) describes the diffusion over harmonics of quasi-energy in (1). A serial number of harmonic gives a number of absorbed photons and is equal to $\mathcal{N}_\phi =$

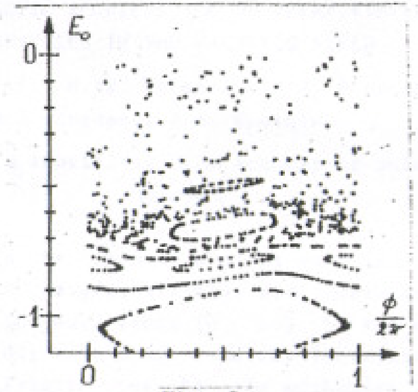

Fig. 2.

$= \mathcal{N} - \mathcal{N}_0$. The quantization gives $\hat{\mathcal{N}}_\phi = -i\frac{\partial}{\partial\phi}$ $(0 \le \phi \le 2\pi)$. As the result we obtain the quantum map for a wave function

$$\overline{\Psi} = \exp(-i\hat{H}_0)\,\hat{P}\,\exp(-ik\cos\phi)\,\Psi \qquad (3)$$

where $\hat{H}_0 = 2\pi[-2\omega(\mathcal{N}_0 + \hat{\mathcal{N}}_\phi)]^{-1/2}$ and $\hat{P}$ is the projection operator on bounded states ($\mathcal{N} < 0$). Quantum effects lead to localization of diffusive excitation. Due to homogeneity of diffusion the localization is also homogeneous and exponential with the length $\ell_\phi = D$ [4]. The steady-state distribution has the form

$$\overline{f}_{\mathcal{N}_\phi} = \frac{1}{2\ell_\phi}(1+x)e^{-x}, \quad x = \frac{2|\mathcal{N}_\phi|}{\ell_\phi}, \quad \ell_\phi = 3.33\,\frac{\varepsilon^2}{\omega^{10/3}} \qquad (4)$$

An example of such distribution is shown in Fig. 3 for $n_0 = 100$, $\varepsilon_0 = 0.04$, $\omega_0 = 3$. The solid line is the result[2] of numerical simulation of quantum model (1), crosses (+) indicate the probability in the interval $[\mathcal{N}_\phi - \frac{1}{2}, \mathcal{N}_\phi + \frac{1}{2}]$, points were obtained by iterating the quantum map (3). The straight line is the result of least-square fit for the maxima of $\overline{f}_{\mathcal{N}_\phi}$ with $\mathcal{N}_\phi > 0$. For the case of Fig. 3 the experimental value of ℓ_ϕ is in 1.6 times larger then the theoretical one (4).

If the localization length is comparable with the number of photons required for ionization $\ell_\phi \approx \mathcal{N}_I \approx -\mathcal{N}_0$ then the delocalization[2] takes place and the process of excitation is close to the classical one. The results obtained are in agreement with[2] and allow to describe the excitation on high levels and to explain the observed[2] multiphoton peaks in distribution over levels.

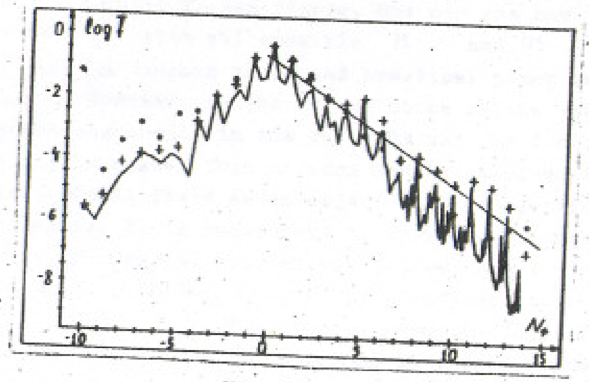

Fig. 3.

According to (3) the one-photon ionization rate (in number of iterations) is equal to $\gamma_\varphi = (k/2)^2$ that is in agreement with the standard perturbation theory ($k \ll 1$). For $k \gg 1$ and $N_I \ll k$ the distribution of electrons in the continuum $\sim J^2_{N_\varphi}(k)$ that agrees with[5]. The equation close to (3) has been obtained in [5]. However, in[5] the artificial condition $\bar{\Psi} = \psi$ was used and consideration was made only for one iteration.

In the localized regime $N_I > \ell_\varphi > k > 1$ the ionization rate is equal to $\gamma_\varphi \sim \sum_{N=N_I-k}^{N_I} \bar{f}_N \sim k \bar{f}_{N_I}$ (the loss of probability after one kick). In physical time we obtain

$$\Gamma_\varphi \sim k \bar{f}_{N_I} (k\omega)^{3/2} \sim \frac{\omega^{5/4}}{\sqrt{n_0}} \left(\frac{\mathcal{E}_q}{\mathcal{E}}\right)^{3/2} \exp\left(-2\left(\frac{\mathcal{E}_q}{\mathcal{E}}\right)^2\right) \qquad (5)$$

where $\mathcal{E}_q = \dfrac{\omega^{7/6}}{\sqrt{6.66}\, n_0}$ is the delocalization border.

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STOCHASTIC ACCELERATION OF RELATIVISTIC PARTICLES IN THE MAGNETIC FIELD

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While moving in the field of a plane wave and in the transverse magnetic field charged particles are accelerated. Particles moving before the slow wave front ($\omega < kc$) in the magnetic field may collide repeatedly with a wave thereby increasing their energy. In the nonrelativistic case the same mechanism leads to the particle escape from a potential well [1] and to the interaction loosening. In so doing, the particle acceleration is always stochastic. In the relativistic case for a group of particles with the initial velocity close to the phase velocity of a wave the confinement is possible in the potential well. This confinement is accompanied by nonlimited regular acceleration of particles along the wave front [2-4]. The particles with another initial conditions can be accelerated only stochastically [5].

This paper studies dynamics of charged particles in the field of a fast electromagnetic wave ($\omega > kc$) and in the transverse magnetic field. In the fields of such a configuration there is no observed the capture and regular acceleration of particles. The Hamiltonian H which describes the charged particle interaction with a linearly polarized wave with a vector potential $\vec{A} = \vec{e}_y A \sin(kx - \omega t)$ in the transverse magnetic field $\vec{B} = \vec{e}_z B_0$, can be written as

$$\mathcal{H} = \left\{m^2c^4 + c^2p^2 + e^2\left[B_0x + A\sin(kx - \omega t)\right]^2\right\}^{1/2} \qquad (1)$$

The equations of particle motion in the given field are

$$\dot{x} = c^2p/\mathcal{H} \quad ; \quad \dot{p} = -e^2\left[B_0 + kA\cos(kx-\omega t)\right]\left[B_0x + A\sin(kx-\omega t)\right]/\mathcal{H} \qquad (2)$$

The Hamiltonian (1) is a function of time

$$\dot{\mathcal{H}} = -e^2A\omega\cos(kx-\omega t)\left[B_0x + A\sin(kx-\omega t)\right]/\mathcal{H} \qquad (3)$$

If $A \neq 0$ a particle moving along a Larmor circumference begins interacting with a wave; for ultrarelativistic particles with the