Renormalization Chaos and Motion Statistical Properties

In a recent Letter[1] Geisel, Zacherl, and Radons presented numerical data on the spectral density $S(\omega)$ of chaotic motion in a simple conservative model with two freedoms. They interpreted the spectrum as a power law with the exponent α in the range 0.7-1.1. Since this model is reduced to a two-dimensional canonical map, it is instructive to compare the results of Ref. 1 with those of Refs. 2-5 (see also Chirikov[6]). In these papers, for three different maps, the *average* exponent varies in the range $0.4 \leq \alpha \leq 1$ depending on the map's parameters. An example is given in Fig. 1.[2] Here, the statistics of Poincaré recurrences is shown, which is at *average* also a power law whose exponent p is related to the spectrum exponent α by $\alpha = 2 - p$.[4,5] Averaging over different values of the parameter λ (see Fig. 1) gives $\langle \alpha \rangle \approx 0.5$ (Ref. 2) which is compatible with the results of Ref. 1 as well as with the theoretical prediction, $\alpha = 0.5-0.7$.[4,5] Notice that the characteristic motion time scale $\tau \sim \omega^{-1} \sim 10^3$ in Ref. 1 is much shorter than that in Refs. 2, 4, and 5 ($\tau \sim 10^5-10^7$) and especially in Ref. 3 ($\tau \sim 10^8$).

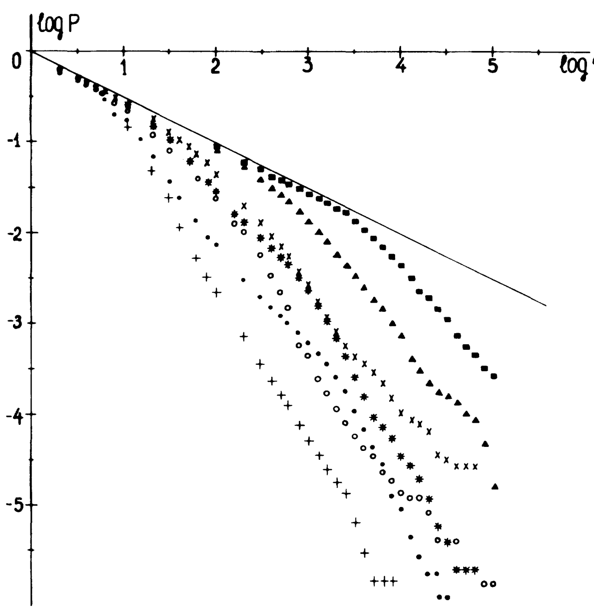

FIG. 1. Statistics of Poincaré recurrences in time larger than τ to line $y=0$ in "wisker" map $\bar{y} = y + \sin x$, $\bar{x} = x - \lambda \times \ln|\bar{y}|$: 10^7 iterations for each $\lambda = 1$ (plusses), 3 (filled circles), 5 (open circles), 7 (asterisks), 10 (crosses), 30 (filled triangles), 100 (filled squares). The straight line $P(\tau) = 1/\sqrt{\tau}$ corresponds to free diffusion in the layer $|y| \lesssim \lambda$.

The most interesting peculiarity of the earlier results is in their evidence for irregular oscillations of the local exponent $\alpha(\omega)$ as ω varies, these oscillations depending on the map's parameters. In other words, the spectrum turned out to be no simple power law at all. In Refs. 4 and 5 this effect was related to the rotation number at the chaos border and was interpreted as an indication for the so-called renormalization chaos.

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