Nonlinear Dynamics

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Nonlinear Waves 2

Dynamics and Evolution

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Quantum Localization of Dynamic Chaos

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Investigation of dynamical quantum systems which are chaotic in the classical limit, is carried out. It is shown that quantum effects lead to a localization of chaos with the localization length determined by the classical diffusion rate. These effects play a significant role in the ionization of highly excited hydrogen atom by microwave field.

1. INTRODUCTION

The nature and properties of dynamic chaos in classical mechanics is now well understood /1-3/. At the same time, investigation of simple models has shown that the dynamics of classically chaotic quantum systems has a number of peculiarities /2,4,5/. The most interesting one is the quantum diffusion limitation /4-6/. An example of physical system in which these effects play a significant role is a highly excited hydrogen atom (n ~ 100) in a monochromatic field of microwaves /7/. For the field strength exceeding some critical value, the motion of a classical electron becomes chaotic which leads to the diffusive ionization of atom /8-11/. Such a picture of ionization is in agreement with the experiment /7/ and a more detailed check of this situation is also known /12/. However, under some conditions, a limitation of quantum diffusion takes place in this system that leads to a sharp decrease of ionization probability in comparison with the classical case /13-15/. Below it will be shown that quantum localization of chaos also takes place for diffusive molecular excitation considered in /16,17/.

This limitation of diffusion is the result of the localization of quasi-energy eigenfunctions (QEE), which is analogous /18/ to the well-known Anderson localization /19/. In such an approach the serial number of the unperturbed level plays the role of a spatial coordinate. However, in spite of the usefulness of this analogy, we need to stress two important differences between them. The first one is the absence of randomness in the dynamical system, and the second one is the fact that QEE localization occurs in a quite different class of systems than that considered in solid-state problems.

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2. THE QUANTUM ROTATOR MODEL

In order to investigate the motion of quantum systems which are chaotic in the classical limit, we chose the model of a kicked quantum rotator with Hamiltonian

$$H = \frac{\hat{n}^2}{2} + k \cos\theta \, \sigma_T(t) \qquad (1)$$

where $\hat{n} = -i\frac{\partial}{\partial\theta}$, $\sigma_T(t)$ is the periodic delta function with dimensionless period T, θ is the phase variable, $V(\theta) = k\cos\theta$ is the external perturbation and $\hbar = 1$ /4-6,18/. Here $H_0(n) = \frac{n^2}{2}$ determines the energies of unperturbed levels n. The dynamics of the corresponding classical system is determined by the equations of motion with Hamiltonian (1), where n, θ are canonically conjugated action-phase variables. After integration over a period T we obtain the standard map /1,3/:

$$\bar{p} = p + K\sin\theta, \quad \bar{\theta} = \theta + \bar{p} \qquad (2)$$

where $\bar{p}$ and $\bar{\theta}$ are the values of the variables p, θ after a period T, $p = Tn$, $K = kT$. For $K \leq K_{cr} = 0.9716...$ /20/ the change of n is finite $|\Delta n| \leq (\frac{k}{T})^{1/2}$, but for $K > K_{cr}$, $(\Delta n)^2$ grows according to the diffusion law with the rate

$$D = \frac{(\Delta n)^2}{\tau} = \frac{D_0(K)}{T^2},$$

where $D_0(K)$ is the diffusion rate in p in the standard map (2) and τ is the number of periods. The quasi-classical condition has the form $K >> 1$, $T << 1$. K = const. Within the chaotic component, the dependence of the diffusion rate on K may be approximately described by the following expression

$$D_0 \approx \begin{cases} \frac{K^2}{2}\left(1+2J_2(K)+2J_2^{\,2}(K)\right), & K \geq 4.5 \\ 0.30\ (\Delta K)^3, & K < 4.5 \end{cases} \qquad (3)$$

where $J_2(K)$ is the Bessel function and $\Delta K = K - K_{cr}$. For $K \geq 4.5$ the dependence of D_0 on K has the form of oscillations which decay as K grows. The limit value $D_0 = \frac{K^2}{2}$ corresponds to the quasi-linear approximation, when phases $\theta(\tau)$ in (2) are random and independent. For $K \to K_{cr}$ we use in (3) the empirical formula which was obtained from numerical experiments /6/. The value of exponent $\eta \approx 3$ in power law $D_0 \propto (\Delta K)^{\eta}$ is close to that given in /22/.

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Numerical experiments /4-6,18,23,24/ with quantum standard map (1) have shown that in the course of time $\langle n^2 \rangle$ stops growing. This means that the external field effectively excites only a finite number of unperturbed levels ($\Delta n \sim \ell$). It is natural to interpret this effect as the result of the QEE localization which is analogous to the Anderson localization in one-dimensional lattice /18,6,23,14/. For the number of excited levels and the localization length of QEE, the following theoretical estimate has been obtained in /5,6/:

$$\ell = \alpha D \sim \Delta n \sim \tau_D \tag{4}$$

where α is an undetermined numerical constant. The derivation of (4) may be done in the following way. Let one unperturbed state contain ℓ QEE with quasi-energies ν_i. Since all these ν_i are distributed within the interval $[0,2\pi]$, its average spacing is equal to $\Delta\nu \sim \frac{1}{\ell}$ (here we consider the case when the unperturbed levels are uniformly distributed in this interval). If we initially excite one unperturbed state, then the diffusion will continue during finite time τ_D until the discreteness of the QEE spectrum becomes effective. According to the uncertainty relation, we have $\tau_D \sim \frac{1}{\Delta\nu} \sim \ell$. After this time the number of diffusively excited levels will be equal to $\Delta n \sim (D\tau_D)^{1/2} \sim \ell$. From this relation we obtain equation (4). The condition for its applicability is $D \gg 1$.

3. LOCALIZATION OF QEE

Let us consider an equation for the eigenfunction with quasienergy ν /18/:

$$u_n^- = e^{i(\nu - TH_0(N))} u_n^+, \quad u^+(\theta) = e^{-iV(\theta)} u^-(\theta). \tag{5}$$

Here $u^{\pm}(\theta)$ are the values of function u before and after a kick $\sigma(t)$ and $u_n^{\pm}$ are the Fourier coefficients of $u^{\pm}(\theta)$. After simple transformations equation (5) may be rewritten in the form:

$$\hat{H}_{ss} u = \{\cos\frac{\hat{V}}{2} \tan(\frac{\nu}{2} - \frac{T\hat{H}_0}{2}) \cos\frac{\hat{V}}{2} - \frac{1}{2}\sin\hat{V}\}u = 0 \tag{6}$$

where $u = e^{\pm i\frac{V}{2}} u^{\pm}$. After the Fourier transform $e^{-i\frac{V}{2}} = \sum_r W_r^- \cdot e^{i(r\theta+\varphi r)}$ it is easy to see that the Hamiltonian H_{ss} corresponds to an one-dimensional lattice with interacting sites and energy $E = -\sum_r W_r^- W_{-r}^- \sin\varphi_r \cdot \cos\varphi_{-r}$. In such an approach, quasi-energy ν determines the potential of interaction and the eigenvalue of energy E plays the role of a parameter.

The serial number of unperturbed level n corresponds to a discrete spatial coordinate in the lattice. Since all eigenfunctions in one-dimensional random lattice are localized /19/, it is natural to expect an exponential localization of QEE in (1). If $\cos\frac{V}{2} \neq 0$, we may introduce $u = \cos\frac{V}{2}\, u$ and divide equation (6) by $\cos\frac{V}{2}$. After that we reduce the problem to the case with Hamiltonian $\hat{H}_{ss} = \tan(\frac{\nu}{2} - \frac{T}{2}\hat{H}_0) - \tan\frac{\hat{V}}{2}$. This procedure was implicitly used in /18/. However, it is necessary to stress that this approach leads to the appearance of a nonphysical singularity which does not allow for an analysis of the wide class of potentials with $|V(\theta)| \geq \Pi$.

The form of equation (6) is convenient for exploiting the analogy with solid-state problems. However, for numerical experiments it is more convenient to rewrite equations (5), (6) as follows. We introduce

$$\bar{u} = e^{\pm i\frac{V}{2}}\, u^{\pm}/g,$$

where $g(\theta)$ is an arbitrary real function and will consider the case when g and V are even functions of θ. Then we obtain

$$\sum_r \bar{u}_{n+r}\, W_r \sin(\chi_n + \varphi_r) = 0, \qquad (7)$$

where $e^{-i\frac{V}{2}g} = \sum_r W_r e^{i(r\theta+\varphi_r)}$, $\chi_n = (\nu - TH_0(n))/2$. If only W_r with $|r| \leq N$ differ from zero, then a column of 2N known values of $\bar{u}_n$ determines the value of QEE for arbitrary n. The recursive computation of $\bar{u}_n$ from (7) may be considered as the motion of a dynamical system in the discrete time n. The dynamics of the 2N-component vector is determined by a transfer matrix M_n, the expression for which may be easily obtained from equation (7). It may be shown that the dynamics in n is Hamiltonian and there are N positive and N negative Lyapunov exponents (LEX) $\gamma_i^+ = -\gamma_i^- \geq 0$ (see, for example, /3/). Asymptotically, the localization length of QEE is determined by the minimal positive LEX $\gamma_1 = 1/\ell$ /23, 24/. The fact that γ_1 is different from zero, leads to exponential localization of QEE and to a discrete spectrum of quasi-energies.

Taking g = 1, we obtain in (7) $W_r = J_r(k/2)$, $\varphi_r = \frac{\Pi r}{2}$, $\chi_n = (\nu - Tn^2/2)/2$. where $J_r(k/2)$ is the Bessel function. Due to the fast decay of W_r for $|r| > k/2$, it is possible to use a finite number of sites $N \sim k/2$. The results of numerical experiments /23/, obtained by the LEX method (dots in Fig. 1.), show that in the quasi-classical region $T \leq 1$; $5 \leq k \leq 75$; $1.5 \leq K \leq 29$ ($T/4\Pi$ is a typical irrational number), the localization length is satisfactorily described by the formula (straight line):

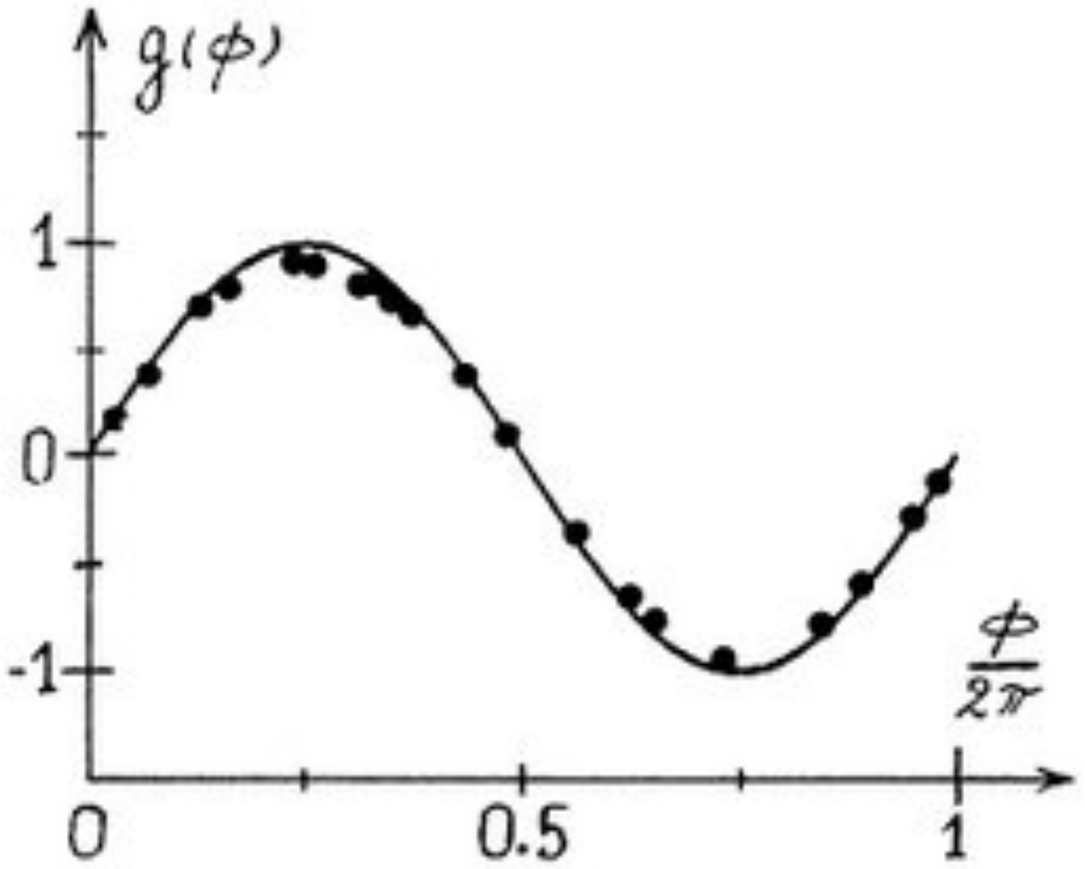

Figure 2.

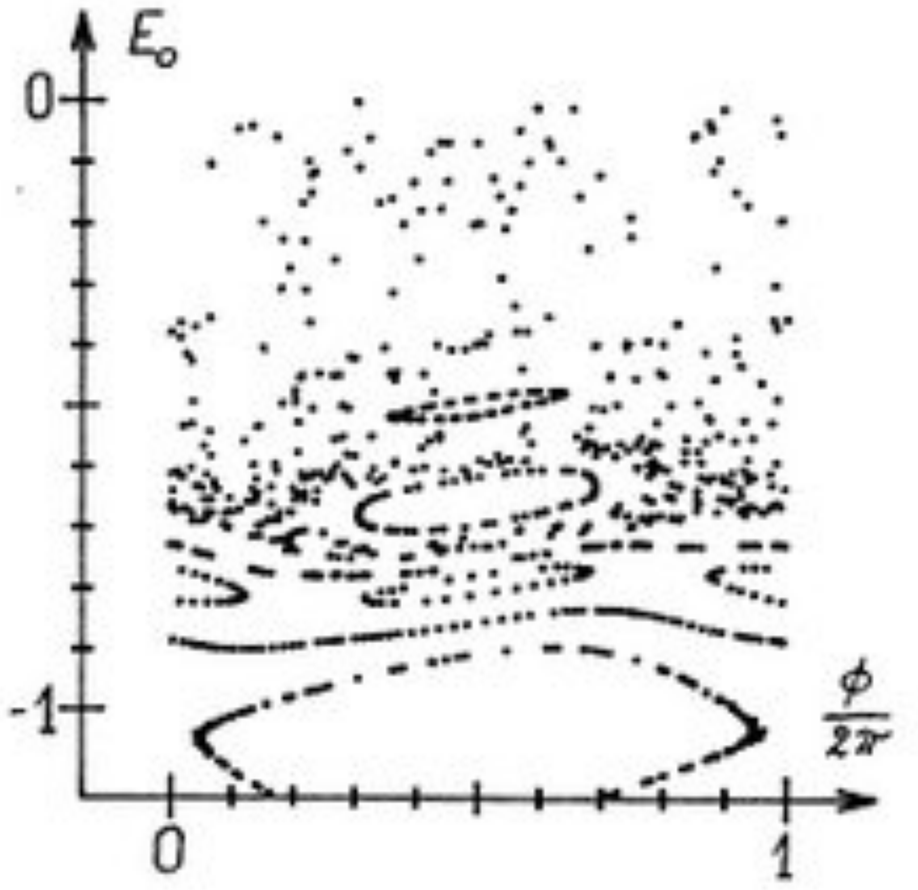

Figure 3.

For the quantization of (10) /26/ we note that according to the usual relation $\hat{E} = -i \frac{\partial}{\partial t}$, operator $\hat{N} = -i \frac{\partial}{\partial \phi}$ $(-\infty<\phi<\infty)$. Since the perturbation is periodic in ϕ, then, besides quasi-energy, a new integral of motion will appear (quasi-momentum). For an unperturbed level n_0 the quasi-momentum is equal to the fractional part of $N_0 = -N_I$. External perturbation leads to the excitation of high harmonics of quasi-momentum. Since $\phi \propto t$, this quasi-momentum corresponds to the quasi-energy in the initial system (1). Thus, the quantized map (2) describes the diffusion over harmonics of quasi-energy in (9). The serial number of a harmonic gives the number of absorbed photons and is equal to $N_\phi = N - N_0$. The quantization gives $N_\phi = -i \frac{\partial}{\partial \phi}$ $(0\leq\phi\leq 2\pi)$. As the result we obtain the quantum map for a wave function

$$\overline{\psi} = \exp(-i\hat{H}_0)\ \hat{P}\ \exp(-ik\cos\phi)\ \psi \qquad (11)$$

where $\hat{H}_0 = 2\pi[-2\omega(N_0+\hat{N}_\phi)]^{-1/2}$ and $\hat{P}$ is the projection operator on bounded states $(N < 0)$. As in model (1), quantum effects lead to the localization of diffusive excitation. Due to homogeneity of diffusion the localization is also homogeneous and exponential with the length $\ell_\phi = D$. The steady-state distribution has the form /6,26/:

$$\overline{f}_{N_\phi} = \frac{1}{2\ell_\phi}\ (1+x)\ e^{-x}, \quad x = \frac{2|N_\phi|}{\ell_\phi}, \quad \ell_\phi = 3.333\ \frac{\varepsilon^2}{\omega^{10/3}}\ . \qquad (12)$$

An example of such a distribution is shown in Fig. 4 for $n_0 = 100$, $\varepsilon_0 = 0.04$, $\omega_0 = 3$. The solid line is the result of numerical simulation of quantum model (9), crosses (+) indicate the probability in interval $[N_\phi - \frac{1}{2}, N_\phi + \frac{1}{2}]$, dots were obtained by iterating the quantum map (1).

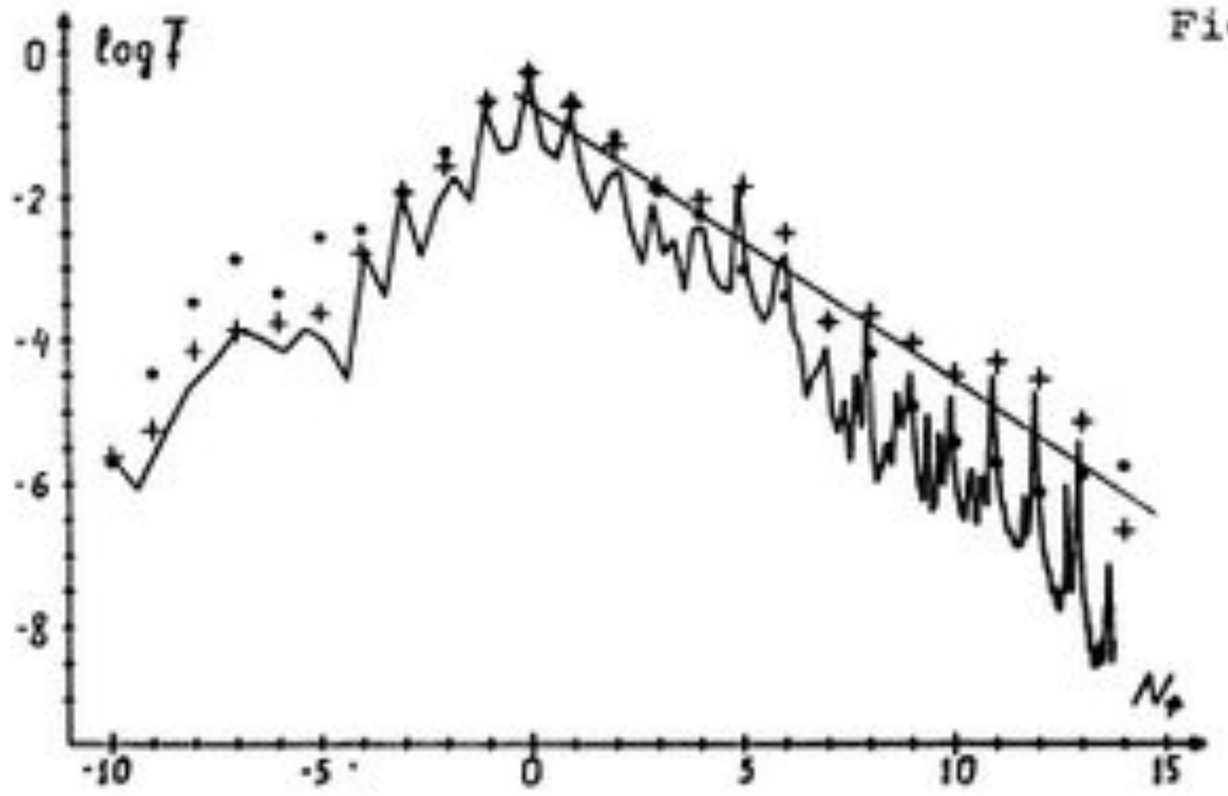

Figure 4.

The straight line is the result of the least-square fit for the maxima of $\overline{f}_{N_\phi}$ with $N_\phi > 0$. For the case in Fig. 4, the experimental value of ℓ_ϕ is 1.6 times larger than the theoretical one (12).

If the localization length is comparable with the number of photons required for ionization $\ell_\phi \simeq N_I = -N_0$, then the delocalization takes place and the process of excitation is close to the classical one. The results obtained are in agreement with /14,15/ and allow to describe the excitation of high levels and to explain the observed /14/ multi-photon peaks in distribution over levels.

According to (11), the one-photon ionization rate (in number of orbital period) is equal to $\gamma_{1\phi} = (\frac{k}{2})^2$ that is in agreement with the standard perturbation theory ($k \ll 1$). For $k \gg 1$ and $N_I \ll k$, the distribution of electrons in the continuum is $\propto J_{N\phi}^2(k)$. In the localized regime $N_I > \ell_\phi > k > 1$ the ionization rate is equal to

$$\gamma_\phi \sim \sum_{N=N_I-k}^{N_I} \overline{f}_N \sim k f_{N_I}$$

(the loss of probability after one kick). In physical time we obtain

$$\Gamma_\phi \sim k f_{N_I} (k\omega)^{3/2} \sim \frac{\omega^{5/4}}{\sqrt{n_0}} (\frac{\varepsilon_q}{\varepsilon_0})^{3/2} \exp(-2(\frac{\varepsilon_q}{\varepsilon_0})^2) \qquad (13)$$

where $\varepsilon_q = \frac{\omega_0^{7/6}}{\sqrt{6.66 n_0}}$ is the boundary value of the delocalization. For $\varepsilon_0 > \varepsilon_q$ the ionization rate is determined by the classical diffusion and for $\omega_0 \sim 1$ it is much larger than the usual one-photon ($\omega_0 = n_0/2$) ionization rate at the same field intensity: $t_I \gamma_{1\phi} \sim (n_0/2)^{-4/3}$ (/15/, Fig. 1).

It is interesting to note that the localization length in the number of photons (12) can be rewritten in the form $\ell_\phi = 2\Pi^2 \mu^2 \varepsilon^2 \rho^2$, where

$\mu = \frac{0.411}{\omega^{5/3} n^3}$ and $\rho = n^3$ are one-photon matrix element and density of states in the hydrogen atom (9) (see details in /25,26/). In such a form this formula can be applied to a more general problem of excitation of a system with a constant density of states in a monochromatic field. Such a system has been considered as a model of collisionless molecular dissociation /17/.

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