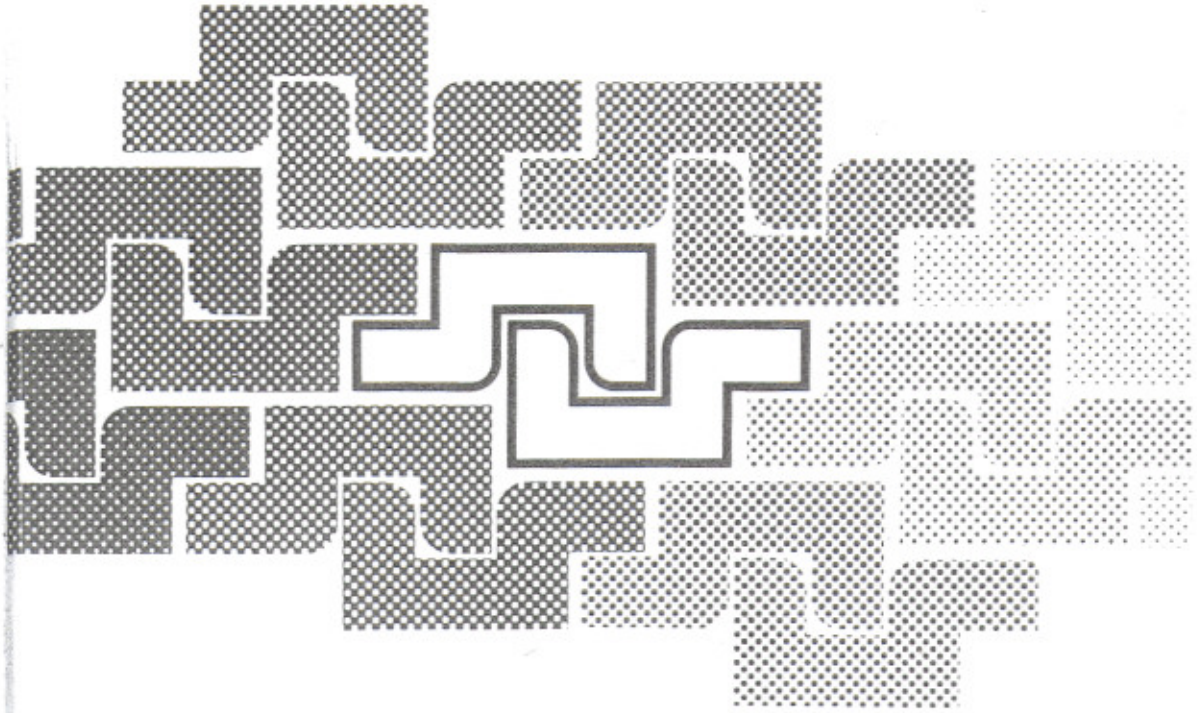

Quantum Chaos – Quantum Measurement

Edited by

P. Cvitanović, I. Percival and A. Wirzba

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QUANTUM MEASUREMENT

LOCALIZATION AND DELOCALIZATION OF QUANTUM CHAOS

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INTRODUCTION

A great success in understanding of the properties of quantum chaos has been achieved by investigation of the dynamics described by maps. A wellknown example is the quantum standard map [1]. In this model quantum effects lead to localization of classical chaos that is a dynamical version of Anderson localization [2]. Intensive investigations of the model allowed to establish the connection between the localization length l and the classical diffusion rate D: $l = D/2\hbar^2$ [3]. In this paper it will be shown that such maps can describe the dynamics of real physical systems. Also the investigation of maps has led to a discovery of systems for which quantization doesn't suppress classical diffusion.

KEPLER MAP

One of the systems where quantum localization of chaos takes place is highly excited Hydrogen atom in a microwave field. The pioneer experiment of Bayfield and Koch [4] had shown that ionization of such atom happened for relatively small field strength. The theoretical investigation of this problem (see [5] and Refs. there in) has established its close connection with the problems of classical and quantum chaos. Moreover, the extensive numerical and analitical investigations showed that in the case of high microwave frequency ($\omega n^3 > 1$, here and below we use atomic units) the dynamics of the system, which originally is ruled by the continuous Hamiltonian equations, can be described by the Kepler map [6]:

$$\bar{N} = N + k\sin\phi, \quad \bar{\phi} = \phi + 2\pi\omega(-2\omega\bar{N})^{-3/2} \tag{1}$$

Here $k = 2.58\epsilon/\omega^{5/3}$, $N = E/\omega$ has the meaning of the number of absorbed or emited photons (E is the energy of the electron), ϵ is the field strength, ϕ is the phase of microwave field at the moment when the electron passes near the nucleus. The bar denotes the new values of the variables after one orbital period.

The physical reason due to which the motion can be quite accurately [6] described by the simple area-preserving map is the following: when the electron is far from the nucleus mircowave field leads only to a small fast oscillations which doesn't modify the avarage energy and the Coulomb trajectory of the electron. The change of energy happens only at perihelion where the Coulomb singularity leads to a sharp increase of the electron velocity. Ionization takes place when the energy of the electron becomes positive after a pass near the nucleus $N > 0$. Then the electron goes to infinity and

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never returns back. Therefore for the map (1) ionization is equivalent to absorbtion of trajectories with $N > 0$.

The Kepler map (1) can be locally reduced to the standard map [7]. For that one needs to linearize the second equation in (1) near the resonant (integer) values of ωn^3 that gives:

$$\bar{N} = N + k \sin \phi, \quad \bar{\phi} = \phi + T\bar{N} \tag{2}$$

with $T = 6\pi\omega^2 n^5$. After quantization the variables (N, ϕ) become operators with commutation rule $[N, \phi] = -i$, the fractional part of N is constant and the system is locally equivalent to the quantum standard map (quantum kicked rotator). On this basis we come to the conclusion that diffusive excitation takes place if $K = kT = 49\epsilon_0{\omega_0}^{1/3} > 1$, where $\epsilon_0 = \epsilon {n_0}^4$, $\omega_0 = \omega {n_0}^3$ and n_0 is the principal quantum number of initially excited level. The diffusion rate is equal to $D = k^2/2$ and according to [3, 6] the localization length for the steady-state distribution, measured in the number of photons, is equal to $l_\phi = D = 3.33\epsilon^2/\omega^{10/3}$. The difference from the localization length for a quasienergy eigenfunction is connected with strong fluctuations at the tail (see [3]). If the localization length is less than the number of photons required for ionization $N_I = 1/2{n_0}^2\omega$ then the ionization rate will be exponentially small: $W \sim \exp(-2N_I/l_\phi)$. In the opposite case $l_\phi = D > N_I$ the delocalization takes place and the process of ionization is close to the classical one. Numerical simulations with the quantum Kepler map [8] reproduce the 10%-threshold for ionization obtained in the laboratory [9].

SUMETSKY-KUCHIEV-SUSHKOV (SKS) MAP

Another physical problem dynamics of which in some approximation can be reduced to a simple map is highly excited hydrogen atom in a homogeneous magnetic field. During last years many interesting results were obtained for this problem [10] - [12]. However, here we will concentrate on the analyse of the properties of extended orbits in this system. The first analitical results for such states were obtained in 1982 by Sumetsky [13] but in the further investigations it somehow happened that this paper has been avoided and unrefered. However, recently after observation of long-living states in the continuum [12] the interest to such extended states has been renewed [14]. The physical reason for that is that ionized trajectories are always extended. Due to that it's possible to expect that the approach of [13] can help to understand the dynamics for positive energies.

For magnetic quantum number $m = 0$ the Hamiltonian of the system has the form [10] - [14]:

$$\mathcal{H} = \frac{{p_z}^2}{2} - \frac{1}{(z^2 + \rho^2)^{1/2}} + \frac{{p_\rho}^2}{2} + \frac{\rho^2\omega^2}{2} \tag{3}$$

Here z is the direction along the magnetic field H, ρ is the perpendicular direction, $\omega = H/2c$. For extended orbits the electron can be far from the nucleus, so that $z >> \rho$. Then the first two terms in (3) give the Coulomb energy ($E_\| \approx -1/2n^2$) and the last two give the energy of Larmor rotation ($E_\perp = \omega n_L$), where n_L is Landau level number and n is the principal quantum number. The sum of these terms is equal to the total energy $E = const$. The classical dynamics depends only from one dimensionless parameter $\kappa = 2E/(H/c)^{2/3}$. The orbit is extended if $z \sim n^2 >> \rho \sim (n_L/\omega)^{1/2}$ that leads to the condition: $n_L << \omega n^4$. The condition of hard chaos has the form $\omega n^3 >> 1$ or $|\kappa| << 1$.

Let's now consider the orbits with so small Larmor radius ρ that for it the Coulomb energy near the nucleus is bigger than the Larmor energy:

$$\frac{1}{\rho} >> \rho^2\omega^2, \quad n_L << \frac{1}{\omega^{1/3}} \tag{4}$$

Then far from the nucleus the motion is described by the approximate Hamiltonian:

$$\mathcal{H} \approx \frac{{p_z}^2}{2} - \frac{1}{z} + \frac{{p_\rho}^2}{2} + \frac{\rho^2}{2}(\omega^2 + \frac{1}{z^3}) \tag{5}$$

and near the nucleus the electron moves over a parabolic trajectory. The general solution of the equations with the Hamiltonian (5) have the form of linear combinations of Bessel functions [13], [14]:

$$\rho = \sqrt{\mid \tau \mid}\,(aJ_{1/6}(\mid \tau \mid) + bJ_{-1/6}(\mid \tau \mid)) \tag{6}$$

where $\tau = \omega t$ and a, b are arbitrary real constans. Far from the nucleus ($\tau >> 1$) this solution describes the usual Larmor oscillations and near the nucleus ($\tau << 1$) it gives the motion over parabola, since there $z = (9/2)^{1/3}t^{2/3}$. Therefore (6) is also functionally correct near the nucleus. The junction with real parabolic solution for small z shows that the parameter b changes sign after the passage near the nucleus ($b \longrightarrow -b$ with $-\tau \longrightarrow \tau$). From this junction and the direct solution (6) it follows that the change of $E_\parallel$ happens in the region near the nucleus. Let's introduce the phase of Larmor motion θ in such a way that at the turning point for $z << 1$ radius $\rho \sim \cos(\theta)$ (at the turning point for $z >> 1$ $\rho \sim \cos(\theta - \omega T/2)$, where T is the Kepler period with given $E_\parallel$) and the conjugated action (Landau quantum number n_L). Then from the junction and the equation (6) we obtain the SKS map for one orbital period of the electron around the nucleus:

$$\bar{n}_L = n_L\,(7 + 4\sqrt{3}\sin(2\theta))$$

$$\bar{\theta} = \operatorname{arccotan}(\operatorname{cotan}(\theta + \frac{\pi}{6}) + 2\sqrt{3}) + \frac{\pi}{6} - 2\pi\omega(-2(E - \omega\bar{n}_L))^{-3/2} \tag{7}$$

The last term in the second equation gives the free change of Larmor phase during the Kepler period $T = 2\pi(-2E_\parallel)^{-3/2}$ and the first one gives its change during the passage near the nucleus. An example of numerical test of applicability of this map is shown on Figs. 1,2 for $\kappa = -0.03$. The full curves are the theoretical lines from (7), the dots are from the numerically obtained phases of Larmor motion at the turning point with $z >> 1$ ($y = \ln(n_L)$, $T(n_L) = 2\pi(-2(E - \omega n_L))^{-3/2}$). The map is area-preserving for any function $T(n_L)$.

There are few interesting features of the derived SKS map. One of them is that the change of n_L after one orbital period is quite big ($1/13.928 < \bar{n_L}/n_L < 13.928$) [14] and that the map doesn't contain small parameters. Another property is that for $E < 0$ and small values of n_L ($\omega n_L << \mid E \mid$) the second equation in (7) doesn't depend on n_L. Then the θ-motion has a simple fixpoint attractor if the equation $\bar{\theta} = \theta$ has a solution. Since, $\partial\bar{\theta}/\partial\theta = n_L/\bar{n}_L$ the value of n_L will grow exponentially in this case. If the solution is absent then the motion is stable and the action n_L remains small. The regions of stability are determined from the conditions: $\bar{\theta} = \theta$ and $\partial\bar{\theta}/\partial\theta = 1$ (touching). This gives, in agreement with [13], that the motion is stable for $q + 1/3 < (-2E)^{-3/2}H/c < q + 2/3$, where $q > 0$ is an integer. However, the direct numerical iteration of the SKS map shows that the measure of the stable regions is quite small.

Since the change of the energy happens during the passage near the nucleus therefore the map describes also the dynamics in the continuum ($E > 0$) (for extended states). No regions of stability were observed in this case and variations of θ looks to be chaotic. For $E > 0$ the value of the maximal ionization rate Γ_{max} can be obtained by the calculation of the relative number of the trajectories lossed after one iteration. For the initial state

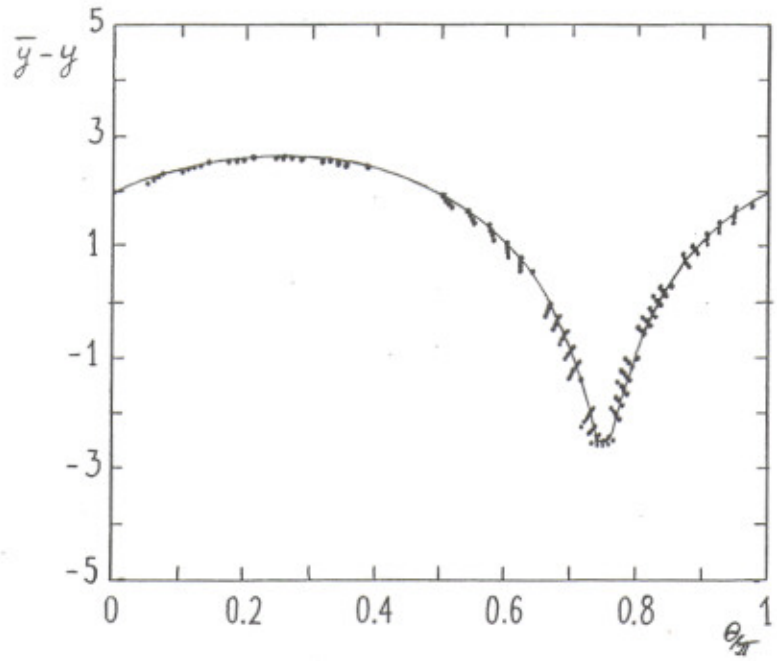
$\bar{y}-y$
5
3
1
-1
-3
-5
0
0.2
0.4
0.6
0.8
1
θ/π

Fig. 1

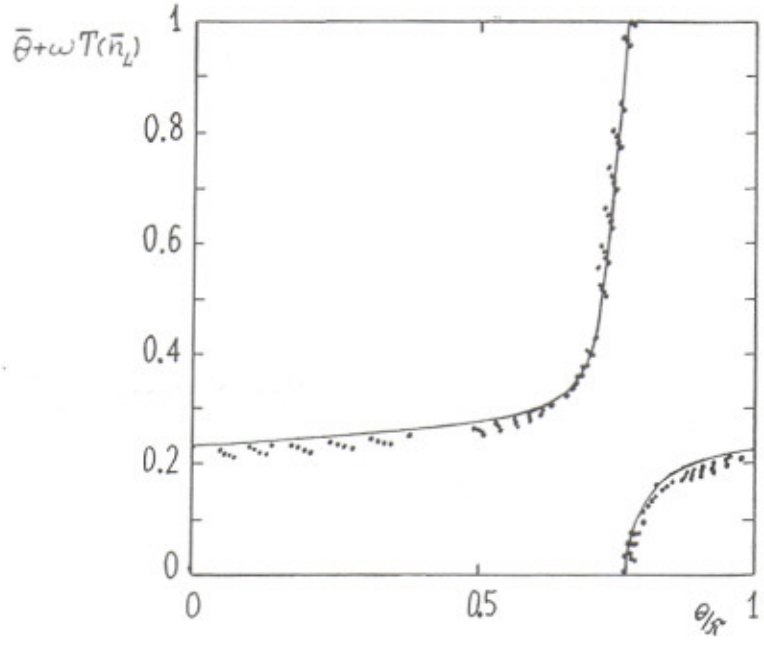
$\bar{\theta}+\omega T(\bar{n}_L)$
1
0.8
0.6
0.4
0.2
0
0
0.5
1
θ/π

Fig. 2

n_L at the ionization border ($E - \omega n_L = E_{||} \approx 0$ and homogeneous destribution in θ) the loss of probability is equal to $\Delta\theta/\pi$, where $\Delta\theta$ is the size of the interval for which $\bar{n}_L/n_L < 1$. From (7) $\Delta\theta/\pi = 1/6$ and therefore $\Gamma_{max}n^3 = \Gamma_{max}/\Delta E = 1/12\pi << 1$. This expression for Γ_{max} is in agreement with the quantum computations of [14]. The small value of Γ_{max} can explain the observation of narrow lines in the continuum [12].

It is interesting to ask how fast will be ionization in the case $\ln\kappa >> 14$. In this case it's possible to assume that the motion is chaotic for n_L in the interval $E/\omega < n_L < \omega^{-1/3}$ where the right inequality corresponds to the condition that the Larmor frequency becomes much less than the Kepler frequency of z-motion. Then the phase θ becomes complitely random and the dynamics in n_L can be described by the Fokker-Plank equation for y ($< \Delta y >= 3.218$, $< (\Delta y)^2 >= 6.893$). Since the total interval of diffusion in y is of the order of $\ln\kappa$ therefore the ionization time (in the number of orbital periods) will be of the order of $\ln^2(\kappa)$. However, in the real physical time the main contribution in life-time comes from the time of the last orbital period. Further investigations are required for a better understanding of ionization process. Due to the exponential growth of n_L quantization will not lead to localization of states which will be ergodic inside the chaotic component.

HARPER MAP

Since 1979 [1] it was considered that quantum effects leads to a localization of chaos. In [15] it was found an example of chassically chaotic system in which the quantization does not suppress the diffusion and can lead even to a much faster excitation. The classical dynamics is given by the Harper map:

$$\bar{p} = p + K\sin x, \quad \bar{x} = x - L\sin\bar{p} \tag{8}$$

where K and L are positive parameters and bars are for the new values of the variables after one iteration. The quantized motion of the kicked Harper model (8) is described by the following Hamiltonian:

$$H = L\cos(\hbar\hat{n}) + K\cos x\,\delta_1(t) \tag{9}$$

where we used units for which $\hbar$ is the dimensionless Plank constant, $\hat{n} = -id/dx$, $p = \hbar n$ and $\delta_1(t)$ is a periodic delta function of period one. If in (9) to put unity instead of $\delta_1(t)$ we will obtain the well known Hamiltonian of the Harper model, which in some approximation describes a motion of an electron in a 2D cristal in the presence of perpendicular magnetic field (see [16] and Refs. there in). The classical motion of the Harper model is integrable that easily explains the transition from localized ($K < L$) to delocalized ($K > L$) states in n for the quantum case (we consider states with fixed quasimomentum in x-direction).

For the kicked Harper model the classical dynamics looks completely different: starting from K, $L \approx 4$ there are no noticeable regions of stability and excitation is characterized by diffusion in p and x directions with different diffusion rates. For $K = L$ dymanics is symmetric. Numerical investigations of the quantum dynamics [15] showed that for $K = L$ the second moment $< (\Delta n)^2 >$ grows diffusively with time (for irrational $\hbar/\pi$ and fixed quasimomentum β in x direction). During some time interval t^* the excitation was the same as in the classical case but the assimptotical diffusion rate was different from the classical value (usually approximately 2 times less). However these results were obtained for not very small values of $\hbar \sim 0.5$ and what will be the ratio between the quantum and classical diffusions for $\hbar \longrightarrow 0$ is an open question. On the same grounds as for the kicked rotator [3] it's possible to expect that t^* scales as $1/\hbar^2$.

For $K > L$ in all tested cases excitation goes in a ballistic way with $< (\Delta n)^2 > /t^2 = \gamma = const$ and for $K < L$ in the most cases there was suppression of excitation [15]. In the corresponing classical cases the phase space is completely chaotic and excitation is diffusive. However, as it was mentioned in [15], there were some cases for which the excitation was ballistic also for $K < L$. Here are two examples:

1) $K = 3.1$, $L = 6$, $\hbar = 2\pi/7.618...$, $\gamma \approx 0.3$ and for exchanged values of K and L $\gamma \approx 0.5$;

2) $K = 7$, $L = 8$, $\hbar = 2\pi/4.618...$, $\gamma \approx 0.35$ and for exchanged values of K and L $\gamma \approx 0.01$.

The values of γ were determined over 5000 kicks for $\beta = 0$ and initially excited level $n = 0$.

For the case $K = L$ the recent results obtained in [17] show that the diffusive excitation is connected with a Cantor spectrum of quasienergies which on small scales has the same type of level clustering as in the Harper model [16]. However, as it is seeing from the above examples the situation for $K \neq L$ is different from the Harper model and its understanding requires further investigations.

It is interesting to mention that in the systems without timereversability a ballistic excitation can take place in the symmetric case. For example, in the system with the Hamiltonian:

$$H = L(\cos(p) - \frac{A}{2}\sin(2p)) + K(\cos(x) - \frac{B}{2}\sin(2x))\delta_1(t) \qquad (10)$$

the ballistic excitation takes place for $K = L = 2$, $A = B = 1.5$, $\hbar = 2\pi/7.618...$ with $\gamma \approx 0.1$ for the same conditions as for the two above examples. It is interesting to note that such possability is not a priori excluded by the general mathematical theorem [18].

I kindly aknowledge O.Sushkov for stimulating discussions of the problem of H-atom in a magnetic field.

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