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QUANTUM LOCALIZATION OF DYNAMICAL CHAOS

DIMITRI L. SHEPELYANSKY

Institute of Nuclear Physics
630090 Novosibirsk, USSR

Abstract

The discovery of the quantum limitation of diffusive excitation in the model of the kicked rotator [1] had shown that quantization can completely change the properties of classically chaotic motion. First understanding of this unusual phenomenon was obtained on the grounds of a simple estimate based on the uncertainty relation [2,3]. As the result, it was shown that the time of diffusive excitation and the number of excited unperturbed levels is proportional to the classical diffusion rate. Another approach to the problem was developed in [4], where the interesting analogy between the kicked rotator model and Anderson localization in a one-dimensional random potential was established. In such considerations the quantum number of the unperturbed level plays the role of a spatial coordinate.

M.-J. Giannoni, A. Voros and J. Zinn-Justin, eds.
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According to this analogy each quasi-energy eigenfunction is exponentially localized near some unperturbed level n_0: $\psi_n \sim \exp(-|n-n_0|/l)$.

The use of the transfer matrix technique [5,6] allowed to determine numerically the value of the localization length l and to check the theoretical prediction [2,3,5,7]: $l \sim D$. This numerical method doesn't require a computation of exact eigenfunctions and allows to have more than 10^6 unperturbed levels. According to the results obtained in [5] the localization length for quasi-energy eigenfunctions is given by the expression: $l = D/2$ (here the diffusion rate D gives the square of the number of excited unperturbed levels after one period of perturbation). The numerical coefficient $\frac{1}{2}$ is in agreement with the exactly solvable Lloyd model. However, for the localization length l_s in the steady-state distribution, numerical data give the larger value $l_s = D$ that is connected with strong fluctuations in eigenfunctions [5,7].

The destruction of this localization can be achieved by introducing, instead of the perturbation periodic in time, a quasi-periodic perturbation with two or three frequencies. This makes the system analogous to 2- or 3-dimensional solid state, and in the semiclassical limit leads to an exponentially large localization length or unlimited diffusive excitation, correspondingly [8,9].

The investigation of the kicked rotator model has allowed to understand the process of microwave ionization of highly excited hydrogen atoms and to determine under what conditions this process can be described by classical mechanics [10].

A detailed description of the questions considered above can be found in [11].

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