PHYSICAL REVIEW A VOLUME 47, NUMBER 2 FEBRUARY 1993

Classical stabilization of the hydrogen atom in a monochromatic field

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(Received 22 May 1992)

We report the results of analytical and numerical investigations on the ionization of a classical atom in a strong, linearly polarized, monochromatic field. We show that the ionization probability decreases with increasing field intensity at field amplitudes much larger than the classical chaos border. This effect should be observable in real laboratory experiments.

PACS number(s): 32.80.Rm, 31.50.+w, 42.50.Hz

The behavior of a hydrogen atom under the interaction of a monochromatic, linearly polarized, electric field constitutes one of the basic problems in both classical and quantum physics. Even though, at least at first glance, the problem looks very simple, the appearance of classical chaotic motion introduces a rich variety of different regimes, and so far we do not have a satisfactory understanding, not even for the classical problem.

The one-dimensional (1D) model in which the electron moves along a straight line in the direction of the field has been studied in great detail in the past ten years. This model correctly describes the excitation of real atoms initially prepared in very extended states along the field direction. For such a model it has been found [1] that the classical ionization mechanism, at least for $\omega_0 \gtrsim 1$ is connected to the appearance of chaotic motion for $\epsilon_0 > \epsilon_{0c} \approx \frac{1}{50}\omega_0^{-1/3}$, which in turn leads to unlimited diffusion and ionization. Here $\epsilon_0 = \epsilon n_0^4$ and $\omega_0 = \omega n_0^3$ are the rescaled intensity and frequency and n_0 is the initial value of the action variable (in the following we will use atomic units).

In the quantum case the picture is substantially modified by the so-called quantum localization phenomenon, which leads to a suppression of classically chaotic diffusion due to quantum interference effects. According to localization theory [2,3], the field threshold for ionization is higher than the classical one and it increases with ω_0. This picture is valid if $\omega_0 \gtrsim 1$ and provided that several photons are necessary to reach continuum.

It has also been shown [3] that, as far as the excitation in energy is concerned, the 1D model describes the main essential features of the two-dimensional case and even of the real 3D atom, due to Coulomb degeneracy, which leads to a slow change of orbital momentum and conjugated phases, making the system effectively described by the one-dimensional Kepler map [3]. The above predictions have been confirmed by laboratory experiments [4].

Recent investigations [5–8] have shown that for high frequencies (i.e., photon energy larger than the unperturbed binding energy) and high intensities the photoionization rate decreases with increasing field intensity. Several mechanisms have been put forward to explain this unexpected behavior; the basic idea is that, by increasing the laser field intensity, the amplitude of the electron wave function is reduced near the nucleus, which in turn leads to a reduction of the ionization rate. It has also been shown [6,7] that for high magnetic quantum number m stabilization should be observable. Experimental observations of the suppression of ionization in strong laser fields have been reported recently [8]. However, up to now, the stabilization of the hydrogen atom was considered as a purely quantum effect only.

The purpose of the present Rapid Communication is to show, via numerical computations, that stabilization is a feature of the classical motion, to support this conclusion with analytical arguments and to discuss the dependence of the stabilization field threshold on the parameters of the problem. This remarkably implies that stabilization may occur even at small frequencies, much less than the frequency for one-photon ionization. We surmise that the classical stabilization phenomenon observed here is at the root of the corresponding quantum behavior.

In a previous paper [9] we have studied the classical ionization mechanism in a linearly polarized field for states with zero value of the projection of the orbital angular momentum along the field direction ($m=0$). Due to the symmetry of the Hamiltonian, m is an integral of the motion and the orbits lie in a plane. We have shown that if the orbital momentum l is large enough, the atoms remain stable until the size of the electron oscillations in the free field is comparable with the distance from the nucleus. No stabilization is present here; however, the field threshold value for ionization can be very high (for example, for $n_0=24$, $\omega_0=50$, $l/n_0=0.75$, we have $\epsilon_I=\epsilon_0/n_0^4 \approx 6\times 10^6$ V/cm).

The situation is qualitatively different for $m \neq 0$. Indeed, for $m=0$ the oscillations are always in the plane of the unperturbed orbit, and, as they become sufficiently large, collisions may take place with the nucleus and ionization will follow. Instead, if $m \neq 0$, collisions with the nucleus can be avoided and the stabilization phenomenon is not *a priori* excluded.

The Hamiltonian of our problem is more conveniently written by means of cylindrical coordinates in a reference frame oscillating with frequency ω;

$$H=\frac{p_z^2}{2}+\frac{p_\rho^2}{2}+\frac{m^2}{2\rho^2}-\frac{1}{\left[\rho^2+\left[z-\frac{\epsilon}{\omega^2}\sin(\omega t)\right]^2\right]^{1/2}}, \quad (1)$$

where ϵ and ω are the field strength and frequency. The properties of the motion can be understood on the following grounds. For large frequencies $\omega_0=\omega n_0^3 \gg 1$ and for large oscillation amplitude of the nucleus $\epsilon/\omega^2 \gg n_0^2$ one can consider the oscillating nucleus as a charged thread with a linear charge density σ slowly dependent on z: $\sigma(z)=[\omega^2/(\pi\epsilon)]/[1-(z\omega^2/\epsilon)^2]^{1/2}$. Therefore, for z and ρ smaller than ϵ/ω^2 one can approximately describe the averaged motion by means of the Hamiltonian

$$H_{\rm av}=\frac{p_z^2}{2}+\frac{p_\rho^2}{2}+\frac{m^2}{2\rho^2}+2\sigma(z)\ln\left[\frac{\rho\omega^2}{\epsilon}\right]. \quad (2)$$

The expression for the potential in (2) is valid as long as $\rho \lesssim \epsilon/\omega^2$, while for larger ρ values the potential is approximately zero. The conditions under which expression (2) gives a good description of the motion (1) will be discussed below.

From Eq. (2) it is easily found that the minimum of the potential (for $z \ll \epsilon/\omega^2$) is at $\bar{\rho}=\sqrt{\pi\epsilon/2}(m/\omega)$. The frequency of the oscillations in ρ near $\bar{\rho}$ is $\Omega \sim \omega^2/(\epsilon m)$ and the energy required for ionization (the depth of the potential) is approximately

$$I \approx \frac{2\omega^2}{\pi\epsilon}L, \quad L=\ln\left[\left[\frac{2\epsilon}{e\pi}\right]^{1/2}\frac{1}{(\omega m)}\right]. \quad (3)$$

The minimum distance $\rho_{\rm min}$ between the nucleus and the electron can be found from the condition $m^2/(2\rho_{\rm min}^2=I$ and gives

$$\rho_{\rm min}=\frac{m}{2\omega}\left[\frac{\pi\epsilon}{L}\right]^{1/2}, \quad (4)$$

which grows with the field strength ϵ. The physical reason for this growth of $\rho_{\rm min}$ with ϵ is quite clear: the large amplitude of the field oscillations leads to a decrease of the attractive Coulomb force, and the repulsive centrifugal potential leads to an increase of the minimal distance between the nucleus and the electron.

The averaged Hamiltonian (2) gives a good description of the electorn motion if the frequency of the oscillations in ρ is much smaller than the frequency of nuclear oscillations ($\Omega \ll \omega$); this condition is fulfilled for a sufficiently strong field, namely,

$$\epsilon > \epsilon_{\rm stab}=\beta\frac{\omega}{m}, \quad (5)$$

where β is some numerical constant. The same estimate for the stabilization border $\epsilon_{\rm stab}$ can be obtained from the condition that the change in energy ΔE, due to one passage of the electron at the distance $\rho_{\rm min}$ from the nucleus, must be smaller than I. In fact, the change of the momentum is $\Delta p \approx \Delta t/\rho_{\rm min}^2 \approx \omega/(\epsilon\rho_{\rm min})$, which gives $\Delta E \approx \omega^2/(\epsilon^2\rho_{\rm min}^2)$, that is, less than I if Eq. (5) is satisfied. Therefore, for $\epsilon > \epsilon_{\rm stab}$ the frequency of nucleus oscillations is larger than the other frequencies of the motion (2) and $H_{\rm av}$ is a constant of the motion with adiabatic accuracy [$\approx \exp(-{\rm const}\times\omega/\Omega)$].

In order to check the above estimates we investigated the process of ionization of the classical atom by numerical solution of the exact Hamiltonian system (1). The initial distribution of classical trajectories was chosen to model a quantum state with fixed values of principal quantum number n_0 (action), orbital momentum l, and its projection on the field direction m (magnetic quantum number). Therefore the classical trajectories had the same initial value of n,l,m and the phases conjugated to n and l were homogeneously distributed in the interval $[0,2\pi]$. The field was smoothly switched on and off during a number of field periods $T_{\rm sw}=\omega_0$, and the total interaction time (number of field periods) was chosen as $T_{\rm int}=500\omega_0$ (so that the physical interaction time was always fixed and equal to 500 unperturbed periods of the electron). Different values of ω_0 were considered, from $\omega_0=1$ up to $\omega_0=30$. We numerically investigated the dependence of the ionization probability $W_{\rm ion}$ (or stabilization probability $W_{\rm stab}=1-W_{\rm ion}$) on the field strength ϵ. The ionization probability $W_{\rm ion}$ was determined as the relative number of trajectories with positive energy after the field pulse (the total number of trajectories for each run was taken equal to 100). The numerical results for $W_{\rm stab}$ are presented in Fig. 1. The most remarkable fact is the appearance of a large fraction of nonionized trajectories with increasing field intensity. It has been found that the increase of $T_{\rm sw}$ in a few times leads to practically complete ionization, even for fields where stabilization takes place in Fig. 1. Indeed, if $T_{\rm sw}$ is larger than ω_0, then during the switching the electron will be able to come close to the nucleus and then will be easily ionized. We remind the reader that for relatively small fields the change of the electron's energy after one passage near the nucleus is $2.6(\epsilon/\omega^{2/3})(1-m^2/l^2)^{1/2}$ [3], and this ex-

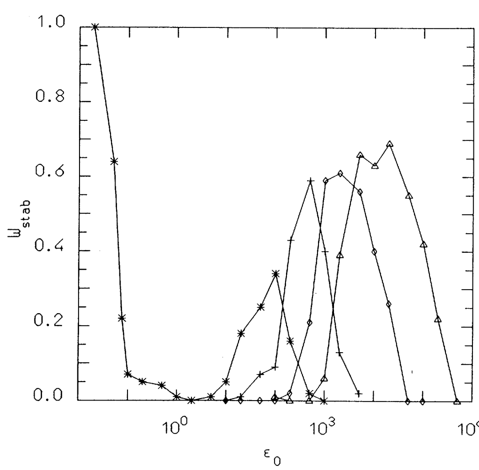

FIG. 1. Dependence of stabilization probability $W_{\rm stab}$ ($W_{\rm stab}=1-W_{\rm ion}$) on field strength ϵ_0 for $\omega_0=1$ (*), 3 (+), 10 (◇), and 100 (△) with initial values $m/n=0.25$, $l/n=0.3$. The interaction time is equal to $500\omega_0$ field periods. (Sets of numerical points are jointed by segments to guide the eye.) For the case $\omega_0=1$ we have computed $W_{\rm stab}$ also for small ϵ_0, down to the chaos border.

plains why there is practically complete ionization for $\epsilon_0 \approx 1-10$ [10]. The dependence of the stabilization border $\epsilon_{\rm stab}$ defined as the value of ϵ for which $W_{\rm stab}=0.2$ is presented in Fig. 2. It is seen that the dependence of the stabilization border on ω is approximately linear, in agreement with Eq. (5) with the numerical constant $\beta \approx 12$. Another feature of Fig. 1 is the disappearance of stabilization for very big field values. This effect can be understood on the grounds of the expression (4) for $\rho_{\rm min}$. Indeed, since in the initial distribution $\rho < 2n_0^2$, the orbits cannot be captured by the potential in (2) if $\rho_{\rm min} > 2n_0^2$. This leads to the estimate for the destabilization border $\epsilon_{\rm destab} \approx 16L\omega_0^2/(\pi m^2 n_0^2)$, which is in agreement with the data of Fig. 1. However, let us mention that for $\epsilon > \epsilon_{\rm destab}$ stability is possible for orbits with bigger initial distance from the nucleus.

The dependence of the 20% ionization border $\epsilon_{\rm stab}$ on m is presented in Fig. 3. It is seen that for $m > 0.03$ the numerical data are in approximate agreement with Eq. (5), with $\beta = 12$. However, for smaller m values, $\epsilon_{\rm stab}$ becomes practically independent of m. This can be understood by taking into account that for small m the value of $\epsilon_{\rm stab}$ becomes so large that the period of oscillations in ρ $[T_\rho \approx (\epsilon/\omega^2)/v = (\epsilon/\omega^2)/(\omega/\sqrt{\epsilon})]$ is comparable with the time of interaction with the field. Direct observation of the orbits nonionized after the field pulse shows indeed that they suffer only few collisions with the line $\rho=0$.

We have also analyzed the dependence of $W_{\rm stab}$ on the relative value of the parameters n_0, l, m. For example, for $\epsilon_0=5000$, $\omega_0=10$, and a fixed value of the ratio $l/n_0=0.3$, we have found that the probability $W_{\rm stab}$ decreases with m; this is in agreement with the results of [9], in which it is shown that there is no stabilization for $m=0$. Moreover, for fixed m/n_0 the probability $W_{\rm stab}$ decreases with increasing l. However, for a more accurate understanding of the properties of the motion over the whole range of parameter values n_0, l, m, more detailed analytical and numerical investigations are needed. In Fig. 4 we present two typical orbits in the (ρ,z) plane. It is seen that the minimum distance between the electron

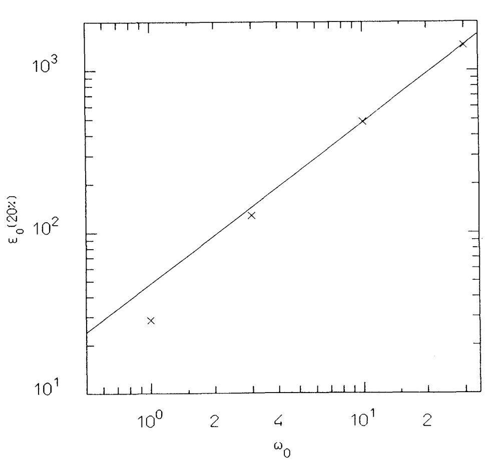

FIG. 2. Dependence of the 20% stabilization border on ω_0 for $m/n_0=0.25$, $l/n_0=0.3$. The straight line corresponds to the analytical estimate (5) with $\beta=12$ (least-squares fit).

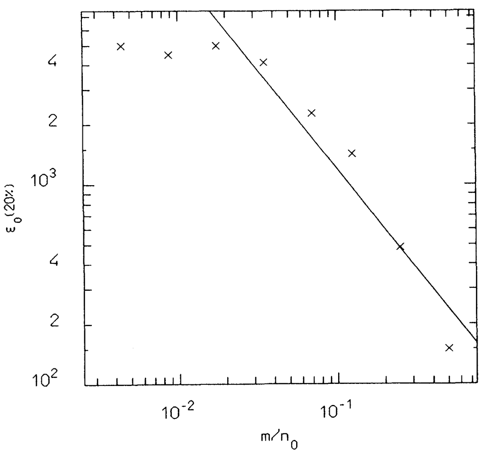

FIG. 3. Dependence of the 20% stabilization border on m/n_0 for $\omega_0=10$ and $m/l \simeq 5/6$. The straight line corresponds to the analytical estimate (5) with $\beta=12$.

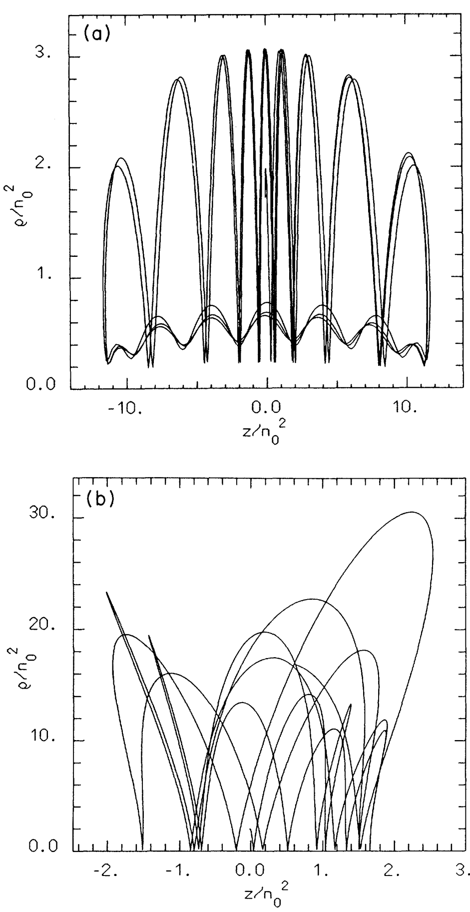

FIG. 4. Examples of long-living trajectories for $\epsilon_0=10^4$, $\omega_0=30$. (a) $m/n_0=0.125$, $l/n_0=0.15$; numerically $\rho_{\rm min}/n_0^2 \approx 0.15$ and the ionization time is larger than 15 000 microwave periods; (b) $m/n_0=0.035$, $l/n_0=0.04$, $\rho_{\rm min}/n_0^2 \approx 0.04$, and the trajectory was ionized approximately after 50 000 microwave periods.

and the nucleus is in agreement with the estimate (4). However, the numerical analysis reveals other interesting features. Indeed, for the case of Fig. 4(a), $\rho_{\min}$ is relatively large, and "collisions" with the line $\rho=0$ do not change much the energy E_ρ of the motion in the ρ direction, so that this change happens only near $|z|=\epsilon/\omega^2$ where the approximation (2) is no longer valid. Probably, the motion is such cases can be integrable. For small m ($\rho_{\min}$) the collisions lead to the change of E_ρ and after many of them the orbit is ionized [Fig. 4(b)]. Such a type of motion can correspond to chaotic ionization. Unfortunately it is difficult to construct an analytical theory of the motion due to the absence of simple analytical expressions for the averaged potential.

Returning back to the stabilization border (5) we need to mention that it was derived under the assumption that $\rho_{\min}<\epsilon/\omega^2$, which gives $\epsilon>m^2\omega^2$. Another condition for the described picture is the nonapplicability of the Kepler-map picture, which is $\epsilon>\omega^{4/3}$ [3]. Since we are interested in fields of the order of $\epsilon\approx\beta\omega/m$, all this implies $m\omega^{1/3}\ll 1$. The case of opposite inequality, which is possible for very high ω, seems to be less interesting. Indeed, for $\epsilon<m^2\omega^2/2$ the atom will remain stable due to the large distance between the electron and the nucleus (as it was in [9]). For $m^2\omega^2/2<\epsilon<\epsilon_{\rm destab}$ a significant fraction of the orbits will remain stable, due to condition (5), and for $\epsilon>\epsilon_{\rm destab}$ fast ionization will take place. As a consequence, for $m\omega^{1/3}\gg 1$ stabilization does not take place, but atoms remian stable up to very strong fields. Further numerical investigations are required for this regime.

Finally we would like to mention when the above classical picture can be applied to real atoms. The sufficient condition is that the depth of the potential I must be larger than the energy of the photon. This gives $\epsilon>\omega\ll 1$ and together with (5) gives the stabilization condition $m\gg\beta$. This obviously does not rule out the existence of stabilization, of pure quantum origin, for $m<\beta$ (or $\epsilon>\omega$). We think that the classical stabilization discussed above can be observed in real experiments with Rydberg atoms. For example, for experiments [4] with $\omega/(2\pi)\approx 20$ GHz, $n_0\approx 70$, $m\approx 35$, the stabilization border (5) corresponds to $\epsilon\approx 2\times 10^3$ V/cm, which requires the use of powerful sources of microwave radiation.

Note added. After submission of this paper results [11] that numerically confirm the absence of stabilization in the regime $m\omega^{1/3}\gg 1$ and the existence of stabilization, even for $\alpha=\epsilon/\omega^2<n_0^2$, were obtained. In addition, arguments based on the analogy between stabilization and channeling of relativistic particles in a crystal support the conclusion that stabilization takes place also in the quantum case when $\omega>I$.

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