Kramers-map approach for stabilization of a hydrogen atom in a monochromatic field

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The phenomenon of stabilization of highly excited states of a hydrogen atom in a strong monochromatic field is discussed. An approximate description of the dynamics from the introduction of the Kramers map allows one to understand the main properties of this phenomenon through analogy with the Kepler map. The analogy between the stabilization and the channneling of particles in a crystal is also discussed.

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I. INTRODUCTION

During the last few years the phenomenon of stabilization of an atom in a strong laser field has attracted a great deal of attention [1]. While the stabilization of an atom has been clearly demonstrated in the numerical experiments, a clear analytical criterion of stabilization is still lacking. Usually, it is assumed that the stabilization condition is satisfied if the energy of the laser photon is larger than the electron coupling energy and the amplitude of electron oscillations in the field is large with comparison to the Bohr radius [2]. However, recent investigation of the corresponding classical problem demonstrated that stabilization remains also in the classical atom [3,4], where the above conditions are violated. The physical explanation of this phenomenon and the condition of stabilization were given in [3,4], but the detailed explanation of the effect still remains an open problem. For a better understanding of this stabilization, I introduce here a one-dimensional atom model which I will call the Kramers model (keeping in mind that it arose from the Kramers-Henneberger transformation). Numerical analysis of this model has allowed us to construct an approximate Kramers map, which describes the process of energy excitation and gives conditions of classical ionization. In some sense, the obtained Kramers map is quite close to the Kepler map [5], which describes the motion in the limit of relatively small fields. Indeed, even in strong files the change the electron energy occurs only when the electron passes near the nucleus, while far from it the electron follows the Kepler orbit. As will be shown, the amplitude of this energy change above the stabilization border decreases with increasing field strength. In this sense, stabilization means that for strong fields the ionization time, measured in units of electron orbital periods, increases with the field. For sufficiently strong fields the classical atom can become absolutely stable (never ionized), as it was for the case of weak fields when the Kolmogorov-Arnold-Moser (KAM) theory can be applied. Let us mention that our definition of stabilization as the growth of ionization time measured in the number of electron orbital periods is quite different from, and more rigorous than, the usual used definition of the lifetime increase with the field. Indeed, for strong fields the orbital period of the electron can be an increasing function of the field. This can lead to the situation where the electron will be ionized after one orbital period, but due to the increase of the period with the field, this will give a formal increase of lifetime. From my viewpoint such an effect cannot be called "stabilization," since after one turn around the center the electron is still ionized. In contrast, by stabilization we will mean a situation in which the electron being rapidly ionized in a relatively weak field remains unionized in stronger fields for many orbital periods.

The paper is constructed as follows. In Sec. II, a brief description of the Kepler map is given, since the analogy with this map can be useful in the stabilization regime. In Sec. III, a qualitative explanation of stabilization is presented. The numerical analysis of the introduced one-dimensional Kramers model and the derivation of the Kramers map are carried out in Sec. IV. In Sec. V, I discuss the analogy between the stabilization and the channeling of electrons in the crystal. In the conclusion, the possibilities of experimental observation of stabilization of Rydberg atoms are discussed.

II. KEPLER MAP

After the pioneering experiments of Bayfield and Koch in 1974 [6], the problem of microwave ionization of highly excited states of the hydrogen atom has been investigated by many groups (see [7] and references therein). The fast ionization observed in the experiments was really surprising since about 100 photons were required to ionize the atom. The typical experimental conditions were $n_0 \approx 70$, $\epsilon_0 = \epsilon n_0^4 \approx 0.05$, $\omega_0 = \omega n_0^3 \approx 1$, where n_0 is the principal quantum number of an initially excited state and ϵ and ω are the strength and the frequency of microwave fields (here and below, we use atomic units). The classical dynamics depends only on the rescaled values ϵ_0 and ω_0.

For the understanding of the process of ionization in

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linearly polarized fields it is convenient to use the one-dimensional atom model [5,7–10]. The investigations of the one-dimensional model showed that for high microwave frequency ($\omega n^3 > 1$) the dynamics of the system, which originally is ruled by the continuous Hamiltonian equations, can be described by the Kepler map [5],

$$\bar{N} = N + k \sin\phi \ , \quad \bar{\phi} = \phi + 2\pi\omega(-2\omega\bar{N})^{-3/2} \ . \tag{1}$$

Here $k = 2.58\epsilon/\omega^{5/3}$, $N = E/\omega$ has the meaning of the number of absorbed or emitted photons (E is the energy of the electron), and ϕ is the phase of microwave fields at the moment when the electron passes near the nucleus. The bar denotes the new values of the variables after one orbital period.

The physical reasons, due to which the motion can be quite accurately [5] described by the simple area-preserving map, is the following: when the electron is far from the nucleus the microwave field leads only to a small fast oscillation, which do not modify the average energy and the Coulomb trajectory of the electron. The change of energy happens only at perihelion, where the Coulomb singularity leads to a sharp increase of the electron velocity. Ionization takes place when the energy of the electron becomes positive, $N > 0$, after a pass near the nucleus. After that, the electron goes away from the center following an unperturbed hyperbolic orbit. Therefore, it never returns to the nucleus and the next kick never happens. Because of this, there is no back transition from positive energies to negative. In this sense, ionization is equivalent to absorption of trajectories for which N becomes positive.

To find the chaos border in the Kepler map we can linearize the second equation in (1) near the resonant (integer) values of ωn^3, obtaining the Chirikov standard map [11],

$$\bar{N} = N + k \sin\phi \ , \quad \bar{\phi} = \phi + T\bar{N} \ , \tag{2}$$

with $T = 6\pi\omega^2 n^5$. Global chaos appears for $K = kT > 1$, which determines the critical-field strength above which the classical atom is ionized. In this regime excitation is diffusive, with the diffusion rate $D = (\Delta N)^2/\Delta\tau = k^2/2$, where τ measures the number of orbital periods of the electron.

The first numerical and analytical investigations of the quantum one-dimensional atom model [10] showed that quantum effects lead to the suppression of classical diffusion. Indeed, in the quantum case the variables (N,ϕ) become operators with the commutation rule $[N,\phi] = -i$, and the system is locally described by the quantum kicked rotator [12]. The photon number is analogous to the level number in the kicked rotator and the excitation probability decreases exponentially with the number of absorbed photons, so that the ionization rate is proportional to $W_I \sim \exp(-2N_I/l_\phi)$. Here, $N_I = n_0/2\omega_0$ is the number of photons required for ionization and $l_\phi = D = 3.33\epsilon^2/\omega^{10/3}$ is the localization length. For $l_\phi \ll N_I$, quantum ionization is exponentially small in comparison with the classical value. However, for $l_\phi > N_I$ the diffusion is delocalized and the process of ionization is close to the classical one. Here I give the description of only the main features of quantum localization; more details an be found in [5,13].

In the three-dimensional atom, the Coulomb degeneracy leads to a slow motion along the energy surface that also allows one to describe the excitation in energy by the Kepler map with a small change of constant k. The motion along the energy surface has some additional integral of motion that explains the existence of location in the three-dimensional atom [5,14]. Recently, the existence of localization in the three-dimensional (3D) case was reconfirmed in [15].

Quantum localization of classical chaotic ionization has been observed in the microwave experiments with hydrogen [16,17] and rubidium [18] atoms. Numerical simulations with the quantum Kepler map [19] reproduce the 10% ionization threshold obtained in the laboratory [16]. The theoretical prediction for the quantum delocalization border was also observed in skillful numerical simulations [15].

While very successful in the description of energy excitation, the Kepler map cannot, however, be applied for the case of very strong fields. Indeed, in its derivation it was assumed that the change of energy after one kick $k\omega$ is much larger than the energy of free oscillations $\epsilon^2/2\omega^2$. This gives the condition of applicability of the Kepler map picture in the one-dimensional case [5],

$$\epsilon \ll \epsilon_{\rm ATI} \approx 5\omega^{4/3} \ . \tag{3}$$

Let us note that this condition is independent of the initial state, since n_0 does not enter directly in the expression for $\epsilon_{\rm ATI}$. Therefore, the Kepler map gives the correct description of motion even when the field is strong enough, and already one kick ionizes the electron [5]. In the case of strong field $\epsilon > \epsilon_{\rm ATI}$ [above threshold ionization (ATI) in terms of [5]], the process of ionization can be understood in terms of another simple model [5].

In the one-dimensional case for $\epsilon \gg \epsilon_{\rm ATI}$, a collision with the nucleus, being unavoidable, goes in a fast way, similar to an elastic wall leading to a prompt ionization [5]. In the two-dimensional case for zero magnetic quantum number m, such a collision also always takes place if the amplitude of free oscillations $2\epsilon/\omega^2$ is larger than the unperturbed distance between the electron and the nucleus in the perihelion $l^2/2$ (l is the orbital momentum). This gives the condition of prompt ionization for $l > (3/\omega)^{1/3}$ [20]:

$$\epsilon > \omega^2 l^2/4 \ , \tag{4}$$

where the approximate expression $l^2/2$ for the prihileion distance $n^2[1-(1-l^2/n^2)^{1/2}]$ has been used. For $l < (3/\omega)^{1/3}$, ionization is ruled by the Kepler map and for $\epsilon_0 > \omega_0^{2/3}/2.6$, prompt ionization takes place after one orbital period [see (1)]. Therefore, there is no stabilization of a classical atom in the strong field for $m = 0$. For high orbital momentum, however, the atom remains stable up to very high-field values. The physical reason for this is quite simple. Indeed, for $l > (3/\omega)^{1/3}$, the electron passes sufficiently far from the nucleus so that the

Coulomb singularity is smoothed, giving an exponentially small change of energy after one orbital period [5,9].

III. STABILIZATION BORDER

While for the magnetic number $m=0$ ionization always takes place in a sufficiently strong field, the case of nonzero m is much more interesting. Indeed, for the linear polarization of the field, the projection m is an exact integral of motion and the centrifugal repulsive potential created by it provides the possibility of avoiding a collision with the nucleus. To analyze the motion in the strong field, it is convenient to use the oscillating Kramers-Henneberger frame [1] and cylindrical coordinates in which the Hamiltonian has the form

$$H=\frac{p_z^2}{2}+\frac{p_\rho^2}{2}+\frac{m^2}{2\rho^2}-\frac{1}{\left[\rho^2+\left[z-\frac{\epsilon}{\omega^2}\sin(\omega t)\right]^2\right]^{1/2}} . \tag{5}$$

If the frequency of the nuclear oscillations is large enough (the condition will be given later), then to a first approximation the nucleus can be considered as a charged thread with a linear charge density σ slowly dependent on z: $\sigma(z)=\omega^2/\{\pi\epsilon[1-(z\omega^2/\epsilon)^2]^{1/2}\}$. Then, for a small z and ρ, the Hamiltonian of averaged motion takes the form [3,4]

$$H_{\rm av}=\frac{p_z^2}{2}+\frac{p_\rho^2}{2}+\frac{m^2}{2\rho^2}+2\sigma(z)\ln\left[\frac{\rho\omega^2}{\epsilon}\right] . \tag{6}$$

The constant under the logarithm takes into account that for $\rho\gg\epsilon/\omega^2$ the coupling energy becomes much less than ω^2/ϵ. From (6) one easily finds the position of the potential minimum $\bar{p}=m\sqrt{\pi\epsilon/2}/\omega$ and the frequency of small oscillations $\Omega=2\sqrt{2}\omega^2/(\pi\epsilon m)$ (for $z\ll\epsilon/\omega^2$). The depth of the potential or the energy required for ionization of the atom is approximately $I\approx 2\omega^2 L/\pi\epsilon$, with $L=\ln[(2\epsilon/(e\pi\omega^2 m^2)]/2$. The minimal distance between the nucleus and electron is determined by the condition $I=m^2/2(\rho_{\rm min})^2$, giving

$$\rho_{\rm min}=\frac{m}{2\omega}\left[\frac{\pi\epsilon}{L}\right]^{1/2} . \tag{7}$$

Since oscillations of the electron take place in the z direction, Eq. (7) gives the minimal distance not only for averaged motion but also for motion with oscillations. The physical reason for the growth of the minimal distance with the field strength is the following: with the increase of the field the amplitude of the field oscillations grows, leading to the decrease of attractive Coulomb force, while the centrifugal potential remains the same.

The average description of the motion (6) is correct if the frequency of field oscillations ω is much larger than the frequency Ω of oscillations in ρ. In that case, the averaged Hamiltonian (6) is a constant of the motion with adiabatic accuracy, which is usually (see, for example, [11]) of the order of $\exp(-\text{const}\times\omega/\Omega)$, and ionization of the atom does not take place. This gives the stabilization border [3,4]

$$\epsilon>\epsilon_{\rm stab}=\beta\frac{\omega}{m} , \tag{8}$$

where β is some numerical constant. The same estimate can be obtained from the condition that the change of energy ΔE during the collision between the electron and the nucleus is smaller than I. Indeed, the time of collision is approximately $\Delta t\approx\rho_{\rm min}\omega/\epsilon$, the change of the momentum is $\Delta p\approx\Delta t/\rho_{\rm min}^2\approx\omega/(\epsilon\rho_{\rm min})$, and the change of the energy $\Delta E\approx\omega^2/(\epsilon^2\rho_{\rm min}^2)$ is less than I if (8) is satisfied. It is interesting to note that the stabilization border (8) can be written as $v_n=\epsilon/\omega>v_{\rm max}$, where v_n is the typical velocity of the nucleus (in the Kramers-Henneberger frame) and $v_{\rm max}=2/m$ is the maximal velocity of the electron in the atom without the external field. The important feature of the stabilization border (8) is its independence of the value of the initially excited state n_0. This means that stabilization can take place for $\alpha=\epsilon/\omega^2\ll n_0^2$ as well as for $\alpha=\epsilon/\omega^2\gg n_0^2$.

Another condition intrinsically used in the derivation of (6) and (8) is $\rho_{\rm min}<\epsilon/\omega^2$, which gives $\epsilon>m^2\omega^2$. In addition, there are two qualitatively different situations depending on the ratio between m and $(3/\omega)^{1/3}$. For the cases $m\ll(3/\omega)^{1/3}$ (stabilized atom regime) we have $m^2\omega^2\ll 5\omega^{4/3}\ll\beta\omega/m=\epsilon_{\rm stab}$. For small field amplitude (3) the excitation is described by the Kepler map and a complete ionization after one orbital period of the electron takes place for $\epsilon_0>\omega_0^{2/3}/2.6$ [5]. Between this border and the above chaos border $\epsilon_{c0}=1/49\omega_0^{1/3}$, ionization goes diffusively, which is also relatively fast. However, for the stronger field (8), when the Kepler map picture is not valid [see (3)], the atom becomes stable. The case of the opposite inequality is not so interesting. Indeed, for $m\gg(3/\omega)^{1/3}$ (stable atom regime) we have $\beta\omega/m\ll 5\omega^{4/3}\ll m^2\omega^2$ and the atom remains stable (nonionized) up to $\epsilon\sim m^2\omega^2$ as it was in (4) ($l\sim m$). Above this value a significant portion (of order half) of atoms will remain stable since condition (8) is satisfied. Finally, ionization takes place only when the value of $\rho_{\rm min}$ (7) becomes larger than the size of the atom $2n_0^2$ and the electron cannot be captured in the stable region during the switching of the field. This gives the destabilization border

$$\epsilon_{\rm destab}\approx\frac{16L\omega^2 n_0^4}{\pi m^2} . \tag{9}$$

This border is also valid for the case $m\ll(3/\omega)^{1/3}$. Of course, in this case the stabilization can be observed only if the time of field switching $T_{\rm sw}$ is less than or order of one orbital period of the electron. Otherwise, a collision with the nucleus will take place at field strength $\epsilon<\epsilon_{\rm stab}$ and the atom will be ionized.

The results of numerical simulation of the ionization process of system (5) are presented in Fig. 1. The stabilization probability $W_{\rm stab}=1-W_{\rm ion}$ is given for different field strengths ϵ_0 and frequencies ω_0. The ionization probability $W_{\rm ion}$ was defined as the relative part of the trajectories with positive energies after the field pulse. The initial distribution of 100 trajectories (25 for

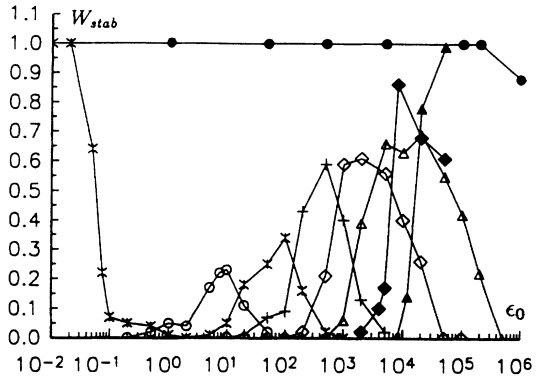

FIG. 1. Stabilization probability $W_{\rm stab}=1-W_{\rm ion}$ is given for different fields strengths ϵ_0 and frequencies [$\omega_0=0.3$ (○), 1 (*), 3 (+), 10 (◇), 30 (△), 100 (◆), 300 (▲), 1000 (●)].

$\omega_0=1000$) corresponded to a quantum state with fixed spherical quantum numbers (fixed actions and equipartition in conjugated phases). The initial value of orbital momentum was equal to $l/n_0=0.3$ and its projection was equal to $m/n_0=0.25$ (at fixed m the change of l in 2 to 3 times did not lead to significant change of the ionization probability). The time of field switching (on-off) measured in the number of field periods was chosen to be equal to $T_{\rm sw}=\omega_0$ (one unperturbed orbital period of the electron). The pulse duration of the field was $T_{\rm int}=500\omega_0$ (500 orbital periods). Here I would like to note that the form of the switching must be chosen in such a way that after the switching the field will not transfer large momentum to the electron (otherwise the electron will not be captured in the potential minimum and will leave the system). For $T_{\rm sw}=\omega_0\gg 1$, the time of switching contains may field periods and this condition is easily satisfied. Another way to satisfy this condition is to make switching in the Kramers-Henneberger frame. In this case, the field in the laboratory frame is given by the second derivative of the field in the Kramers-Henneberger frame and no momentum is transferred to the electron. The data clearly demonstrate the stabilization of the atom for the field strength larger than some critical value. It is convenient to define the stabilization border as the field strength $\epsilon_{\rm stab0}(20\%)$ for which $W_{\rm stab}=0.2$. The dependence of $\epsilon_{\rm stab0}(20\%)$ on ω_0, extracted from the data of Fig. 1, can be well fitted by the theoretical expression (8) with $\beta=12$ in the wide frequency range (see Fig. 2 of [3]). This dependence continues up to $\omega_0=1000$, where we enter into the stable atom regime with $m>(3/\omega)^{1/3}$ and where stabilization disappears in agreement with the above theoretical arguments (see Fig. 1). However, the stability of an atom in this case is of another nature than it was in [20], since the condition (4) is strongly violated. So for such strong fields the stability of an atom is based on the same physical grounds (8) as in the stabilized atom regime for $m\ll(3/\omega)^{1/3}$. The numerical check of the dependence of the stabilization border $\epsilon_{\rm stab0}(20\%)$ on m as well as the destabilization border (9) on ω_0 demonstrates good agreement with the theory [(8) and (9), [3]].

IV. KRAMERS MAP

It is important to stress that according to (8) stabilization can take place even when the size of the electron oscillations $\alpha=\epsilon/\omega^2$ is much less than the unperturbed size of the atom n_0^2. An example of the motion in this case is presented in Fig. 2. In such a case, the electron follows the usual Kepler elliptic orbit and its energy (the size of the orbit) can be changed only during its fast passage near the nucleus. In this sense, we can expect that the motion can be effectively described by some map analogous to the Kepler map.

To construct such a map, let us introduce a simplified one-dimensional Kramers model given by the Hamiltonian,

$$H=\frac{p_\rho^2}{2}+\frac{m^2}{2\rho^2}\frac{1}{\left[\rho^2+\frac{\epsilon^2}{\omega^4}[\sin\gamma+\sin(\omega t)]^2\right]^{1/2}}. \tag{10}$$

This model is obtained from the Hamiltonian (5) by neglecting the changes of z and considering $z=-\epsilon/\omega^2\sin\gamma$ as a constant. In other words, the electron always collides with the line $\rho=0$ at the same z value. The physical reasons for this is the following. The collision of the electron with the nucleus is analogous to a collision of a fast heavy particle with the light electron. In such a collision the change of energy (velocity) takes place mainly in the perpendicular ρ direction, while the velocity in the z direction remains practically the same. According to this physical picture the model (10) mainly presents the changes in the ρ direction. In this sense, it is quite different from the well-known one-dimensional atom model of Eberly [1], which implicitly takes into account the change of energy (velocity) only in the z direction. Also, in [21], the authors considered the velocity change only in z that has led them to a higher stabilization border than (8), while the estimate for $\rho_{\rm min}$ has been found correctly [see (7)].

According to the analogy with the Kepler map, we can expect that the change of the electron energy in the model (10) will take place only when the electron passes near the nucleus and that it will depend only on the phase of the field $\phi=\omega t$ at that moment. Also, if the size of the

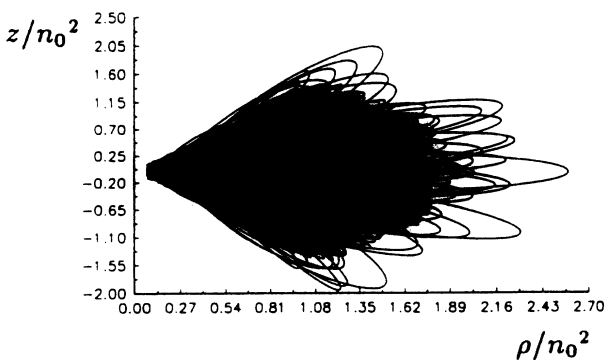

FIG. 2. Example of trajectory for $\omega_0=300$, $\epsilon_0=20000$ with initial $l/n_0=0.3$, and $m/n_0=0.25$; 10^5 field periods are shown.

orbit is much larger than the size of the nucleus oscillations ($\alpha=\epsilon/\omega^2 \ll n_0^2$), then the change of the phase is given by the Kepler law and is the same as in (1). Based on these arguments, we can assume that the dynamics of energy excitation is governed by the Kramers map of the following form:

$$\bar{E}=E+Jh(\phi)\ , \quad \bar{\phi}=\phi+2\pi\omega(-2\bar{E})^{-3/2}\ , \tag{11}$$

with $E=\omega N$, where as in (1) N is the photon number, the maximum change of the energy is given by a constant J, and the unknown function of the kick $h(\phi)$ varies in the interval $[-1,1]$.

To check the validity of this map, I integrated the continuous equations of motion of the model (10) and plotted the change of energy as a function of the field phase at the moment when the value of ρ took one of its minimal values ($p_\rho=0$). Such an approach allows one to find the kick function $h(\phi)$, examples of which are presented in Figs. 3 and 4. The numerical results clearly demonstrate that the function h exists. However, it has a quite unusual property. Indeed, some values of ϕ never appear (even if the number of periods was increased by 20 times). These values of ϕ are approximately equal to $\pi+\gamma, 2\pi-\gamma$ and correspond to those values of the field at which the nucleus passes via the point of collision $z=-\epsilon/\omega^2\sin\gamma$. A closer consideration of motion near these special ϕ values shows that the electron remains during some small time interval (within corresponding phase interval $\Delta\phi$) near the nucleus making one (Fig. 3) or two (Fig. 4) oscillations in ρ of very small amplitude, so that the value of ρ remains practically (but not exactly) the same. This gives correspondingly two (or three) values of the phase ϕ with the same change of ΔE since the value of E was determined in the aphelion. This, of course, puts the question about the derivation of the Kramers map in some other synonymous form. The main properties of the motion, however, can be derived already from the approximate representation (11), where we will define the function h in the empty intervals by connecting the last points at the ends of the interval by a straight line.

Defined in such a way, the Kramers map has properties quite similar to the Kepler map. Indeed, the function h is close to $\cos\phi$, and the approximate chaos border in (11) can be defined by the linearization of the second equation, giving

$$K=6\pi\omega J n^5>1\ , \tag{12}$$

where we used substitution $E=-1/2n^2$. According to this criterion and in agreement with the numerical data the motion is chaotic for the cases of Figs. 3 and 4. If we introduce $k=J/\omega$, which will provide the number of absorbed photons after an orbital period, we will get the same formulas for the diffusion rate $D=k^2/2$, the localization length ($l=D$), and the ionization time $\tau_{\rm ion}=N_I^2/D$, as in the Kepler map. In this sense the most important problem is the definition of the dependence of J on the parameters of the system.

According to the results of the previous section, the amplitude of the kick J must decrease exponentially with the increase of the stabilization (adiabatic) parameter $S=\epsilon m/\omega\sim\omega/\Omega$ [see (7),(8)]. This expectation is in agreement with the results presented in Fig. 5. Indeed, the exponential decrease of J with the field strength, and therefore stabilization, are evident. Let us first discuss the properties of J for nonzero values of γ. Even though the value of energy for the cases of Fig. 5 was quite small,

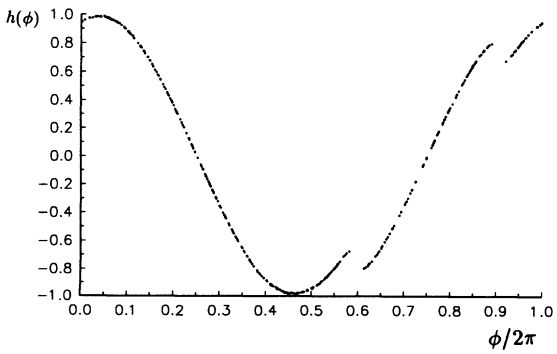

FIG. 3. Example of numerically obtained kick function $h(\phi)$ in Kramers map (11) for $\epsilon=3\times10^4$, $\omega=125$, $m=0.2$, $\gamma=0.6$, $E=-0.125$ (so that effective $n_0=2$), $J=1.1\times10^{-4}$. Nearly 200 orbital periods (points) are shown.

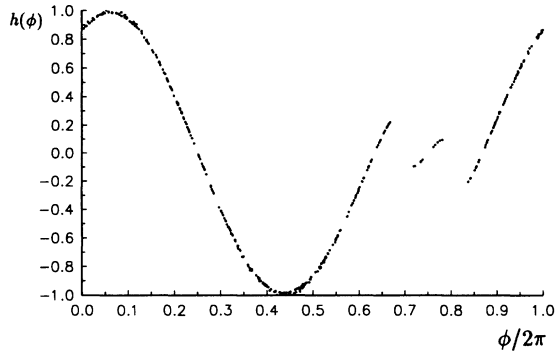

FIG. 4. The same as Fig. 3 with $\epsilon=4\times10^4$, $\gamma=1.2$, and $J=5.8\times10^{-4}$.

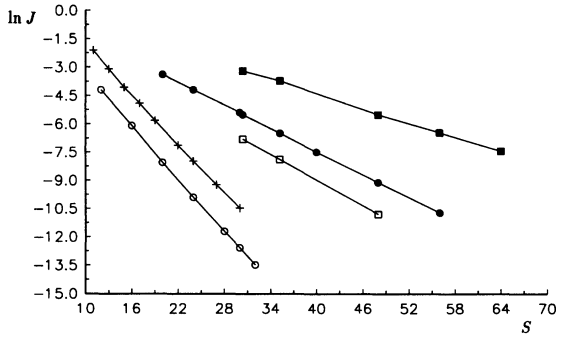

FIG. 5. Dependence of the kick amplitude J in (11) on stabilization parameter $S=\epsilon m/\omega$ for $\omega=125$, $m=0.2$, $\gamma=0$ (○); $\omega=1000$, $m=0.1$, $\gamma=0$ (+); $\omega=125$, $m=0.2$, $\gamma=0.3$ (open squares); $\omega=125$, $m=0.2$, $\gamma=0.6$ (points); $\omega=125$, $m=0.2$, $\gamma=1.2$ (full squares). For all cases, $E=-0.125$. Lines are drawn to guide the eye.

nevertheless there is some dependence of J on energy. An example of such dependence is presented in Fig. 6. We see that $\ln J$ depends on energy E approximately in a linear way and goes to a constant value for $E=0$. This means that in the limit $n_0^2 \gg \epsilon/\omega^2$ the value of J is independent of n_0. This result is consistent with the above arguments that the change of the energy takes place only in the close vicinity of the nucleus. However, in contrast to the Kepler map, it is necessary to have a quite strong inequality $-\alpha E \ll 1$ to neglect the dependence of J on E. We will try to explain this fact later. In the regime of small energies the main change of the phase of the field between collisions [the second equation in (11)] is obviously given by the Kepler law.

To determine the dependence of $J(E=0)$ on the parameters, it is convenient to fix the stability parameter S that allows one to eliminate the strong exponential dependence and to find the factor before the exponent. The numerical results are presented in Fig. 7. The values of $J(E=0)$ were obtained form nonzero energies by linear extrapolation to $E=0$ (see Fig. 6). The numerical data clearly show that for fixed S the value of $J(E=0)$ is independent of the frequency and is inversely proportional to m^2. In principle, the factor $1/m^2$ gives simply the correct dimensionality; however, the lack of dependence on the other dimensionless parameter $\nu=m\omega^{1/3}$ is not so obvious.

Combining all the obtained numerical results, we can present the dependence of the kick amplitude J on the parameters for $|E|\epsilon/\omega^2 \ll 1$ in the following form:

$$J=\frac{g_1 \sin\gamma}{m^2}\exp[-(g_2-g_3\epsilon E/\omega^2)\epsilon m/\omega] , \tag{13}$$

where $g_{1,2,3}$ are some functions weakly dependent on γ. For $\gamma=0.6$, we have it from Figs. 5–7 that $g_1 \approx 0.13$, $g_2 \approx 0.19$, $g_3 \approx 0.08$. The numerical data for other values of γ show that the fitting parameters vary not more than two times for practically the whole interval of γ. For example, $g_1=0.1$ and 0.2, $g_2=0.13$ and 0.21, $g_3=0.045$ and 0.1, for $\gamma=1.2$ and 0.3, respectively.

To understand the numerically obtained formula (13) for J, it is possible to make the following estimate. Taking the partial time derivative from the Hamiltonian (10) we obtain the expression for the change of energy after

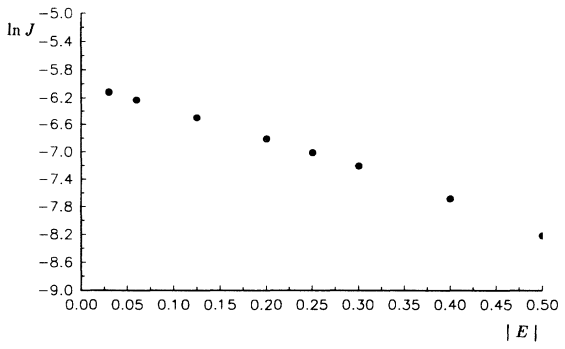

FIG. 6. Example of dependence of J on energy $|E|$ for $\epsilon=2.2\ 10^4$, $\omega=125$, $m=0.2$, $\gamma=0.6$ (points).

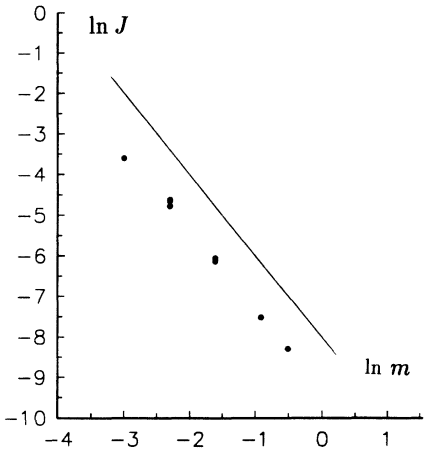

FIG. 7. Dependence of J on m for fixed stabilization parameter $S=35.2$ and $E=0$; nine cases are shown for ω in the interval [10,1000] and m in the integral [0.05,0.6]. The straight line shows the dependence $J \sim 1/m^2$.

one orbital period,

$$\Delta E=\frac{\epsilon^2}{\omega^4}\int \frac{\cos(\eta+\phi)[\sin\gamma+\sin(\eta+\phi)]d\eta}{\rho^{3/2}(\eta)} , \tag{14}$$

where $\eta=\omega t$, and in the denominator we neglected the term with ϵ^2/ω^4 in comparison with ρ^2. We can assume that near the nucleus the time dependence of ρ is the same as for a free electron with momentum m that gives $\rho^2(t)=\rho_0^2+v^2t^2$, where ρ_0 is the minimal distance from the center and v is the velocity of the electron far from the center. For this free motion, with the fixed momentum m, we have the relation $\rho_0^2=m^2/v^2$. For the velocity, it is possible to use the following expression: $v^2=\omega^2/C\epsilon+2E$, where the first term takes into account the fact that the energy must be measured in respect to the minimum of the effective potential [see (6)] and C is some unknown constant. It is easy to see that C determines the minimal distance $\rho_0^2=C\epsilon m^2/\omega^2$ for $E=0$. In principle, the value of C depends on γ.

After the substitution of all these expressions in (14) we obtain the following estimate:

$$J \sim \frac{S^2}{m^2(m^{3/2}\omega^{1/2})}\sin\gamma \exp[-CS/(1+2C\epsilon E/\omega^2)] ,$$
$$h(\phi)=\cos(\phi) , \quad S=\epsilon m/\omega . \tag{15}$$

Of course, the presented derivation is not exact. However, it reproduces quite well the exponential dependence (13) (while the factor before the exponent is not in agreement with the dependence obtained from the numerical simulation). Indeed, numerically, $h(\phi)$ has maxima near 0 and π. Comparison of (15) with (13) gives $g_2=C$ and $g_3=2C^2$. The value of C can be defined directly from the numerical simulation of the one-dimensional Kramers model for different γ. The comparison of the g_2 with C is presented in Fig. 8, showing good agreement with the prediction. The ratios of the numerical value of g_3 (see above) to the theoretical value $2g_2^2$ are equal to 1.13, 1.1, 1.33, respectively, for $\gamma=0.3, 0.6, 1.2$, and are also in

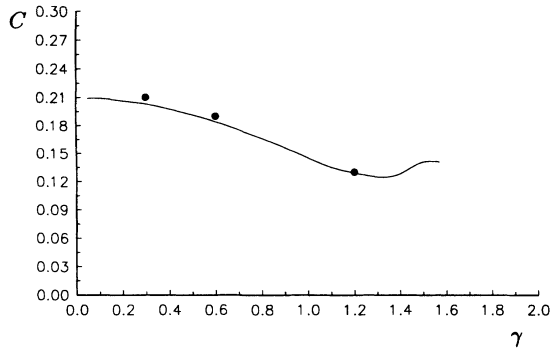

FIG. 8. Dependence of $C=\rho_0\epsilon/S^2$ on γ (full line). Points give values of g_2 to demonstrate theoretical relation $C=g_2$.

good agreement with the theoretical estimate. In future estimates, we will use the expression (13) with the theoretical substitution for g_2 and g_3. Let us also mention that for $\gamma=0$ we find from (14) that $g_2=2C$ (from Fig. 5, the ratio to the theoretical value is approximately 1.1) and $h(\phi)=\sin(2\phi)$, which is quite close to the numerical data. Further theoretical analysis is required to obtain the factor before the exponent in (13).

Let us now once more discuss the conditions under which the Kramers map has been derived. First, in the real two-dimensional (2D) model (5), we fixed the point of collision of the electron with the line $\rho\approx 0$ at $z=-\epsilon/\omega^2\sin\gamma$ that provided the one-dimensional (1D) model (10). Such an approximation allowed us to understand the dynamics of the system and to obtain the simple Kramers map description. Of course, in the 2D model (5), the z point of collision changes from one collision to another. This gives the change amplitude of the kick J, which depends on γ through $C(\gamma)$. However, as follows from Fig. 8, the function $C(\gamma)$ varies no more than twice for the whole interval of variation of γ. Therefore, the Kramers map still gives a correct qualitative description of the dynamics in ρ even in the 2D case (5). To have a more precise description, one needs to take into account the fact that γ changes from one orbital period to another, giving some variation of the kick amplitude J. Indeed, in a first approximation, we can assume that γ changes with some fixed frequency $\gamma=\omega_\gamma t$. In such a case, the dynamics given by the Kramers map is still integrable and the maximal value of ρ is finite (no ionization) for sufficiently large values of the stabilization parameter S, since the kick amplitude J decreases with S exponentially fast. For the dynamics of the model (5) it will mean that the dynamics given by the Hamiltonian (5) averaged over fast field oscillations is integrable and variations of the averaged coupling energy is small (order of J). There is another possibility that the motion described by the averaged Hamiltonian is chaotic. This will give some random variation of γ from collision to collision, which will produce a diffusive growth in energy and finally ionization. Nevertheless, the diffusion rate in energy is of the order of J^2 and is exponentially small for large values of the stabilization parameter S. Therefore, the ionization time will be exponentially large and can be estimated as $t_{\rm ion}\sim n_0^3 E_{\rm coup}^2/J^2$ with $E_{\rm coup}\approx 1/2n_0^2$ for $n_0\gg\epsilon/\omega^2$. This time sharply grows with field strength and stabilization is still present. The understanding of the properties of the Kramers map allows us, as we have seen, to understand the process of excitation in the original models (5) and (10). Further investigations are required to determine the type of motion for the averaged Hamiltonian and the dependence of γ on time.

Second, in (11) we assumed that J is independent of energy. To take into account this dependence, we need to put in the first equation $J=J(\bar{E})$, and in the second equation we need to add the phase shift $\Delta\phi=dJ/d\bar{E}\,f(\phi)$, with $h(\phi)=-df(\phi)/d\phi$. In this way the map will remain canonical. As follows from (15), the map picture cannot be applied to the case with $\epsilon/\omega^2\gg n_0^2$. Indeed, in this case the electron moves near the bottom of the averaged potential and it is difficult to separate the free motion and the kick. However, the stabilization condition in this case is still given by (8). The reason for this is the independence of the stabilization parameter S on n_0. This is also in agreement with numerical data of Fig. 1 (see also [3,4]), where stabilization under the condition (8) takes place both for $\epsilon/\omega^2\gg n_0^2$ and $\epsilon/\omega^2<n_0^2$. However, in the regime $n_0^2\gg\epsilon/\omega^2$, the dependence of J on energy is weak and the Kramers map description works well. The typical structure of the orbits in this case is of the same type as presented in Fig. 2.

While there are still some unclear questions regarding the construction of the Kramers map, the approximate consideration made above and the analogy with the Kepler map allows us to understand the main properties of motion. If the number of photons required for the ionization is large, then, as it was for the quantum Kepler map, it is possible to have diffusive excitation and quantum localization of chaos (let us clarify here that we speak about the localization in the number of photons). This leads to suppression of ionization and, due to this fact, the electron stays in the discrete part of the spectrum, not going very far from the nucleus. In such a case, the localization length in the number of photons is $l_\phi\approx(J/\omega)^2/2$. However, due to high values of the frequency, it is also quite easy to have a situation where one photon can already lead to ionization. In this case for $k=J/\omega<1$, the one-photon ionization rate (per unit of time) is given by perturbation theory and as for the Kepler map (see [5]) it is equal to

$$\Gamma\approx\frac{J^2}{8\pi n^3\omega^2}\ . \tag{16}$$

For $J>\omega$ with probability of approximately one half, the atom is ionized after one orbital period [in (11), as in the Kepler map, the orbit is ionized if, after a kick, $E>0$]. From (16) it is clear that we may have long living states if the field is sufficiently strong. From the quantum viewpoint one of the most interesting cases is the case of small m. In this case, we need to make the substitution $m\to m+1$ since, as is well known, the correct quasiclassical quantization leads to the appearance of the effective centrifugal potential even for zero orbital momentum. This gives the stabilization border $\epsilon>10\omega$ for $m=0$.

Due to the similarity between the Kepler and Kramers

maps, it would be interesting to have a general formula which gives an expression for the kick amplitude for all values of the field strength. However, it is not clear if it is possible to do this. Indeed, the Kepler map can be applied only for fields that are not very strong [see (3)], while the Kramers map works in the other limiting case $\epsilon \gg 10\omega/m \gg 5\omega^{4/3}$ [since we are considering the case $m \ll (3/m)^{1/3}$].

V. CHANNELING ANALOGY

Here I would like to discuss the analogy between the phenomenon of stabilization of an atom in strong fields and the channeling of particles in a crystal (see, for example, [22] and references therein). Let us consider the electron moving in a crystal with the velocity $v < c = 137$ (we will consider the nonrelativistic case). Then, in the frame of the moving electron, its interaction with the protons in the crystal lattice will have approximately the form (5) if only to take into account the interaction with a nearest proton. On the grounds of that analogy, we find that the effective distance between atoms in the crystal a and the velocity of the electron are equal to

$$a = \frac{\epsilon}{\omega^2}, \quad v = \frac{\epsilon}{\omega}. \tag{17}$$

The frequency of perturbation is $\omega = v/a$ so that $\epsilon = v^2/a$. Since in the crystal the distance between the atoms is approximately the same in all directions, the analogy is valid for $\epsilon/\omega^2 > n_0^2$. The necessary condition of channeling is that the critical injection angle θ must be much less than 1, which implies $\theta \approx v_\perp/v \approx 1/(v\sqrt{a}) \approx (\omega^{4/3}/\epsilon)^{3/2} \ll 1$. This is the condition of inapplicability of the Kepler map (3). From the stabilization condition (8), it follows that channeling takes place for electrons with momentum $m > 10/v$. This is always satisfied for fast electrons with $v \sim 137$. The existence of channeling for very energetic electrons (which corresponds to strong fields for the stabilization problem) gives one more piece of evidence for the existence of stabilization of the atom in strong fields in the regime where one-photon frequency is larger than the ionization energy.

VI. CONCLUSION

Based on the Kramers map (11), and using an analogy with the Kepler map, we obtained the estimate for the one-photon ionization rate (16). This ionization rate sharply decreases with the stabilization parameter $S = \epsilon m/\omega$. Such stabilization for excited states has some interesting advantages compared to the stabilization of atoms in the ground state. Indeed, in this case stabilization can take place with $\epsilon \ll 1$ and $\omega \ll 1$. This leads to a large energy difference δE between the excited states and the ground state. Consequently, the energy in an excited state is approximately $(\epsilon/\omega)^2/2 \gg 1$, while the energy of the ground state remains as in the unperturbed atom (it is not the case for $\epsilon, \omega \gg 1$, when the ground state is also stabilized, since there the electron has the same energy of free oscillations). Owing to this, in the case of Rydberg stabilization it is possible to have radiative transitions to the ground state with the radiation of x-ray photons. For the frequency of a CO_2 laser $\omega \approx 1/300$ (0.1 eV) and $m = 0$ (or 1), the stabilization will take place for $\epsilon \approx \frac{1}{30}$ (1.6×10^8 V/cm). The size of the atom will be larger than the size of the field oscillations $\alpha = \epsilon/\omega^2$ for $n > 40$. According to (13) and (16) for the field $\epsilon = 5 \times 10^8$ V/cm and $n = 60$, the lifetime of the atom will be about 5×10^5 orbital periods or 10^{-6} s (we take for the estimate the case with $\gamma = 0.6$). Of course, to obtain such states the time of field switching must be less than the time of orbital periodic, as we discussed above. Since recently it was predicted that the Rydberg atoms can form long living states (bands) in the solid state [23] (giving very high density of excited atoms), it will be interesting to consider the possibility of stabilization not only for a separate atom but also for such Rydberg solid states.

Recently, after this work had been finished, the papers in Ref. [21] were published. The numerical data presented there indicate the existence of long living states in strong fields. However, the important difference of the approach developed there is that the main priority was given to the motion in the z direction, while the dynamics in ρ had been in fact eliminated from consideration. On the basis of the arguments developed above, I think that such elimination is not correct and that it is responsible for another incorrect analytic stabilization border obtained in [21]. Moreover, the z-stabilization border in [21] for small m is higher than (8). This means complete ionization above the border (8), which is in contradiction with the more detailed results presented here and in [3,4]. Another difference is connected to the fact that the switching process of the field was not taken into account and, due to this, the destabilization border (9) was missed. A more detailed discussion of differences between [21] and the approach developed here and in [3,4] will be given elsewhere [24].

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