Benvenuto, Casati, and Shepelyansky Reply: In our Letter [1] we had shown that chaotic diffusive ionization of molecular Rydberg states can take place for relatively high orbital momentum \( l \) when the quantum defect is negligibly small [namely, \( 4 < l < l_c = (3/\omega)^{1/3} \). Also in our diffusive ionization it was assumed that the minimal distance between electron and core \( r_{\text{min}} = l^2/2 \) always remains much larger than the core size \( d \ll (3/\omega)^{2/3} \).

Farrelly, Bellamo, and Uzer (FBU) [2] claim that the main mechanism of ionization is due to the change of orbital momentum which will lead to direct collisions with the core when \( r_{\text{min}} \) becomes comparable with \( d \). While from [1] it is clear that this direct collision mechanism is not important for ionization [see discussion after Eq. (7) in [1]], here we give more detailed analysis to confirm this statement.

First, let us stress that FBU make their computations in the frame with circular polarized effective electric field [Eq. (7) in [1] and Eq. (1) in [2]]. This frame (KHF) is obtained, via the Kramers-Henneberger transformation, from the original frame (OF) with rotating core [Eqs. (1)–(4)]. According to this transformation the quantity \( \ell \) computed by FBU is related to the angular momentum \( l \) of the electron in OF by \( \ell = l + \delta \ell \) with \( \delta \ell = d \omega r \cos(\omega t) \), where \( r \) denotes the distance between the electron and the center. The same estimate for the variation \( \delta \ell \) also directly follows from (7). For typical initial conditions with the orbit size \( r_0 \approx n_0^2 \approx 1 \) the difference between \( \ell \) and \( l \) is small since \( d \omega \ll 1 \). In this case both \( l \) and \( \ell \) can be used to estimate the minimal distance between electron and core \( r_{\text{min}} \approx l^2/2 \). However, for large orbit size (or small initial Keplerian energy \( E_0 = -1/r_0 \)) the value of \( \delta \ell \) can become comparable with the initial value of \( l \) leading to large oscillations of \( \ell \). Such oscillations are seen in Fig. 1(b) of the Comment. However, the electron angular momentum \( l \) remains approximately constant during ionization so that the condition \( r_{\text{min}} \approx l^2/2 \gg d \) is preserved and ionization proceeds without direct collisions in agreement with the statement of [1]. This is clearly evident from Fig. 1, in which we plot \( r_{\text{min}}/d \) for two typical chaotic ionizing orbits. The reason for large oscillations of \( \ell \) is due to growth of interaction in KHF (7) with \( r_0 \) while in OF interaction decreases with \( r_0 \) and the variation of \( l \) remains small. We cannot reproduce Fig. 1(b), since FBU do not provide the absolute scaling of the figure, and therefore we do not know the initial value of \( r_0 \). However, according to their Fig. 1(b) ionization occurs only after four orbital periods. After each period the maximal energy change is \( \Delta E \approx 0.015 \)

(see Fig. 1 in [1]), and therefore we can estimate that the initial size of the orbit is \( r_0 \approx 1/\Delta E \approx 100 \). The choice of such large \( r_0 \approx 100 \) gives large \( \ell \) fluctuations, and this led FBU to incorrect conclusions about the importance of direct collisions.

Let us also mention that for the system studied in [2] the energy change is always given by the Kepler map (6) even if the direct collision takes place. Indeed, the Kepler map nicely describes the one-dimensional hydrogen atom where each collision is a direct collision (see Ref. [6] in [1]) and an electron is ionized after many such collisions.

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