Chaos in a quasiclassical hadronic atom

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We investigate the classical dynamics of the helium atom in which one electron is replaced by a heavy particle with a negative charge, such as an antiproton. The general properties of motion and the conditions for chaotic dynamics are studied via the derivation of the planetary map. The regime of strongly correlated motion of two particles is also analyzed. The properties of quantum motion are briefly discussed.

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I. INTRODUCTION

Recent experiments with anomalous metastable hadronic helium atoms in liquid helium originated interest in the properties of atoms with nuclear charge $Z=2$ in which one electron is replaced by a heavy negatively charged particle [1]. Even when this particle replaces an electron in the $1s$ state, its corresponding principal quantum number is as high as $n^*\approx\sqrt{M}$, where M is the heavy particle mass (atomic units are used), which can vary from 200 (for muons) up to 2000 (for antiprotons). This means that the motion of heavy particle is quasiclassical. The typical frequency of the Kepler motion of this particle is $\omega^*=(a^3M)^{-1/2}$, where $a=n^{*2}/M$ is the orbit size. This frequency is small in comparison to that of electron motion in $1s$ state. Such a situation corresponds to the usual Born-Oppenheimer approximation in molecular physics where the motion of nuclear core in the molecule is much slower than the electron motion. In this respect the hadronic atom shares features with both atoms and molecules [1].

Due to the above properties, the hadronic atom lies at the intersection of two interesting fields of active research in atomic and molecular physics. In atomic physics several laboratory experiments were recently performed on doubly excited Rydberg atoms [2]. In molecular physics laboratory and theoretical investigations were devoted to the analysis of energy exchange between the Rydberg electron and rotational and vibrational degrees of freedom of molecular core [3]. Recently light has been shed on the autoionization process caused by the interaction between rotational and electronic degrees of freedom [4], while for doubly excited electrons in atoms the existing analytical theories (see, e.g., [5]) need further developments.

The investigation of doubly excited states in the hadronic atom can provide the opportunity for a theoretical understanding of both the above problems. In addition, chaotic motion can take place in this doubly excited atom and therefore the problem at hand also provides a good opportunity to study the properties of quantum chaos in the laboratory.

Until now, the investigations on the hadronic atom [1,6] were mainly carried out in regions where the frequency of the heavy particle motion is much less than that of the electron motion. In this case the electron adiabatically follows the slow motion of the heavy particle and the dynamics is qualitatively the same as for the molecule in the Born-Oppenheimer approximation. A more interesting situation arises when the electron is in an excited state with relatively high principal quantum number. Indeed, in this case the frequency of its motion $\omega_e=n^{-3}$ can become comparable with the frequency of the heavy particle and therefore the Born-Oppenheimer approximation becomes invalid. However, the case in which the electron orbit remains sufficiently far from the heavy particle can be analytically treated in the same way as for molecular Rydberg states [4]. In particular, in the latter case the energy exchange between the heavy particle and the electron can be sufficiently large to produce a chaotic dynamics.

In this paper we analyze the classical dynamics of this three-body problem in the general case. This problem, while similar to the traditional three-body gravitational problem, has, however, an interesting peculiarity. Indeed, it effectively corresponds to the case with strongly different inertial and gravitational masses. Because of this the dynamics of the system has many unusual features that are absent in the standard three-body problem; it is therefore interesting to study this situation in detail.

The analysis of classical motion allows us to understand the conditions for the appearance of chaos in this nonstandard object, which contains the features of an atom and a molecule at the same time. In the chaotic regime we analyze the conditions under which chaotic ionization of the hadronic atom can take place via diffusive interchange of energy between the electron and the heavy particle. We first consider the situation in which the light electron is outside the heavy particle's orbit (Sec. II). In this case the inner particle's motion is only weakly affected by the electron and the problem can be treated following the approach developed in [4]. The opposite case, when the heavy particle is far outside the inner electron, is characterized by a more complicated correlated motion of the two particles and is analyzed in Sec. III. The

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intermediate case corresponding to the experimental situation of Ref. [1], when the sizes of both particles' orbits become comparable, is investigated in Sec. IV. Here we have found an interesting regime of strongly correlated motion of both particles in which the ionization time becomes enormously large. Finally, the investigation of the classical dynamics allows us to make simple estimates for the quantum case where effects of quantum suppression of chaotic diffusion can play an important role, leading to a sharp increase of the lifetime of the hadronic atom. This opens the possibility of real laboratory experiments.

One of the most interesting properties of the hadronic atom is that the ground state of the heavy particle lies at a very low energy $E_0 \sim -MZ^2/2$. Because of this, radiative transitions from excited states to the ground state may lead to emission of very high frequency photons, thus opening interesting perspectives for γ lasers.

In the analysis below we neglect the effects of the center of mass motion since, for helium, the mass of the core is significantly larger than that of the heavy particle. If the ratio of these masses becomes comparable to one, then the rotational motion of the core can lead to energy exchange with the excited electron and this can be treated on the basis of the approach developed in [4]. However, this regime is outside the purpose of the present paper.

II. INNER HEAVY PARTICLE: THE "PLANETARY" MAP

The classical dynamics of the hadronic atom is described by the Hamiltonian

$$H=\frac{p_h^2}{2M}+\frac{p_e^2}{2}-\frac{Z}{r_h}-\frac{Z}{r_e}+\frac{1}{|\vec{r}_h-\vec{r}_e|}, \tag{1}$$

where the index h refers to the heavy particle of mass M and the index e refers to the electron; Z represents the nuclear charge (for helium $Z=2$). The total linear momentum of the two particles is an integral of motion.

Let us start from the case in which the electron orbit is much larger than the heavy particle orbit. In this case, as a first approximation, we can neglect the effects of the electron on the heavy particle, so that the latter moves on a given Kepler orbit with fixed frequency $\omega_h=(Z/M)^{1/2}a_h^{-3/2}$, its energy being $E_{n_h}\approx-(MZ^2)/(2n_h^2)$, and the radius of the orbit is $a_h=n_h^2/(ZM)$.

We will restrict ourselves to the case in which both particles move in a plane. Explicitly the Kepler motion of the inner heavy particle is given by [7]

$$x_h(t)=a_h\left(\tfrac{3}{2}e_h-2\sum_s x_s\cos(s\omega_h t)\right),$$
$$y_h(t)=-2a_h\sum_s y_s\sin(s\omega_h t), \tag{2}$$

where the Fourier components are

$$x_s=s^{-1}J_s'(se_h),$$
$$y_s=\left[\frac{(1-e_h^2)^{1/2}}{se_h}\right]J_s(se_h), \tag{3}$$

the eccentricity is $e_h=(1-l_h^2/n_h^2)^{1/2}$, and l_h is the orbital momentum.

Since the outer electron remains sufficiently far from the inner particle we can expand the interaction term in (1), following the approach used in [4]. Then the effective potential for the electron can be written as (for the helium atom with $Z=2$)

$$V(r_e)=-\frac{1}{r_e}+\frac{x_e x_h(t)+y_e y_h(t)}{r_e^3}\approx-\frac{1}{|\vec{r}_h(t)+\vec{r}_e|}. \tag{4}$$

By means of a Kramers-Henneberger [8] transformation the motion can also be described by the Hamiltonian

$$H=\frac{p_e^2}{2}-\frac{1}{r_e}+\vec{\epsilon}(t)\vec{r}_e, \tag{5}$$

where the effective electric field is

$$\vec{\epsilon}(t)=(\ddot{x}_h(t),\ddot{y}_h(t))$$
$$=2a_h\omega_h^2\sum_s s^2(x_s\cos(s\omega_h t),y_s\sin(s\omega_h t)). \tag{6}$$

If the inner particle performs a circular motion the problem is equivalent to a hydrogen atom in a circularly polarized monochromatic field of amplitude $\epsilon=a_h\omega_h^2$. The latter problem was shown to describe also the energy exchange in a Rydberg molecule between the rotating core and the Rydberg electron [4]. When the eccentricity of the inner heavy particle is not small, the high harmonics in the sum (6) will be relevant. In particular, in the limit of a small orbital momentum of the inner particle the harmonics x_s, y_s are large up to values of $s\approx\bar{s}\approx3(l_h/n_h)^{-3}$, while for $s>\bar{s}$ they are exponentially small. This estimate follows from asymptotics of Bessel functions as discussed in [7]. It is interesting to remark that in the limit case $l_h=0$ (when the particle moves on a line) the effective electric field becomes infinite at perihelia.

The requirement that the outer electron does not touch the inner orbit leads to certain restrictions. Indeed, as we know, from the solution of the hydrogen problem in a microwave field, the energy exchange with the electron (the kick amplitude in the Kepler map description) is not exponentially small only if $l_e<(3/\omega_h)^{1/3}$. On the other hand, the minimal distance between the electron and the center is approximately $l_e^2/2$, which should be larger than the size $a_h=(Z/M)^{1/3}(\omega_h^{-2/3})$ of the inner orbit. These two conditions lead to $(3^{2/3}/2)(M/Z)^{1/3}>1$, which is always satisfied provided that the inner particle is sufficiently heavy.

We will for simplicity consider orbits extended along the x direction, the main contribution to the energy change during one orbital period of the electron therefore coming from the x component of the motion. In this case, in analogy with [7], the energy change produced by one harmonic $\epsilon_s\cos(s\omega_h t)$ of effective electric field $\epsilon_s=2a_h\omega_h^2x_s s^2$ is given by

$$2\pi\beta\epsilon_s n_e^2 A'_{s\kappa}(s\kappa e_e)\sin(s\phi), \tag{7}$$

where $\kappa=\omega_h/\omega_e$, $\omega_e=n_e^{-3}$, and ϕ is the phase of the heavy particle $\omega_h t_p$ at the moment t_p when the electron is at a perihelion. The numerical factor β takes into account that the y_s harmonics also contribute to the energy change. For the circular case $x_s=y_s$, this factor is $[1+l_e^2/(2n_e^2)+1.09(2\omega_h)^{1/3}l_e]$ [4,7]. For linear polarization, $y_s=0$ and $\beta=1$, while in the general case $\beta\approx 1.5-2$.

The contribution of all harmonics to the energy change is then given by $2\pi\beta\Sigma_s\epsilon_s n_e^2 A'_{s\kappa}(s\kappa e_e)\sin(s\phi)$, which leads to

$$\Delta E_e(\phi)=4\pi\beta a_h\omega_h^2 n_e^2\sum_{s=1}^{\infty} sA'_s(se_h)A'_{s\kappa}(s\kappa e_e)\sin(s\phi). \tag{8}$$

For a noninteger argument $s\kappa$, the function $A'_{s\kappa}(s\kappa e_e)$ is the first derivative of the Anger function

$$A'_\nu(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin(\nu\xi-x\sin\xi)\sin\xi\, d\xi; \tag{9}$$

$A_\nu(x)$ coincides with the Bessel function $J_\nu(x)$, when ν is an integer. From the asymptotic exponential decrease of Bessel functions, it follows that only a finite number of terms significantly contribute to the sum (8), namely, those with $s\leqslant s_{\min}$ where $s_{\min}$ is equal to the minimum of the two values $s\approx 3(n_h/l_h)^3$ and $s\approx 3[(\kappa n_e)/l_e]^3$. The latter cutoff makes the sum effectively finite even in the case $l_h=0$ (linear motion), which corresponds to an infinitely strong electric field. Due to that, even an infinitely strong electric field will not lead to immediate ionization.

As we have seen, the energy change (8) depends on the phase ϕ of the inner particle at the moment when the electron passes near a perihelion. The phase change between two consecutive passages is given by the Kepler motion and the electron's dynamics can be described by the planetary map

$$\bar{N}=N+kg(\phi),$$
$$\bar{\phi}=\phi+\omega_h(-2\pi\omega_h\bar{N})^{-3/2}, \tag{10}$$

where $N=E_e/\omega_h=-(2n_e^2\omega_h)^{-1}$, $k=2.6\epsilon\omega_h^{-5/3}=2.6a_h\omega_h^{1/3}$ and $g(\phi)=\Delta E_e(\phi)/(k\omega_h)$. Here k has been defined in analogy with the problem of the hydrogen atom in a linearly polarized microwave field with amplitude $\epsilon=a_h\omega_h^2$ [7]. In order to check the validity of the planetary map (10) we numerically integrated the Hamiltonian (1) in which we fixed the motion of the inner heavy particle on the Kepler orbit with a given $a_h\omega_h$. By computing the electron energy change ΔE_e as a function of the different phases of the heavy particle, taken at moments when the electron is at the perihelion, we determined the function $g(\phi)$, which is presented in Fig. 1. It would be difficult to give a closed analytical expression for the function $g(\phi)$. However, the theoretical expression (8) with $\beta=1.8$, given by the full line, is in fairly good agreement with numerical data for $\kappa\gtrsim 1$. The important feature is that the function $g(\phi)$ is quite different from the sine function and has a pronounced spiked shape. We would also like to mention that this shape is similar to the kick function found in [9] for the dynamics of the Halley comet under the influence of Jupiter.

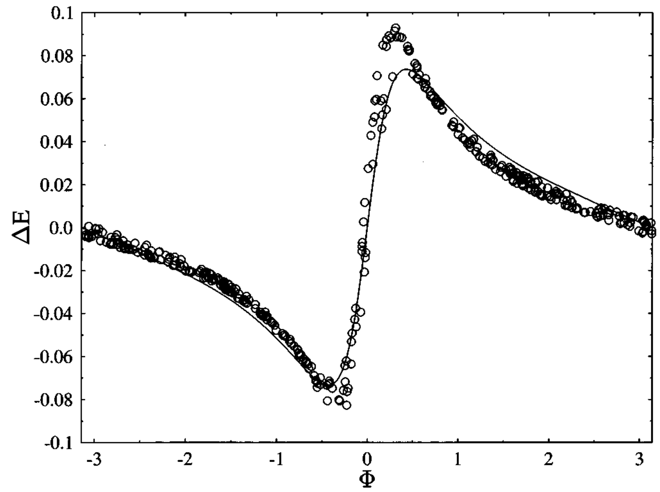

FIG. 1. Plot of the function $g(\phi)$ in (10). The inner heavy particle is bound to a Kepler orbit with parameters $a_h=0.002$, $\omega_h=4$, and $l_h=0.7$. The initial conditions for the electron are $n_e=1$ and $l_e=0.5$. Dots are numerical data and the full line is the theoretical curve (8) with $\beta=1.8$.

Another interesting point is that, even when the frequency ratio $\kappa\ll 1$, the function $g(\phi)$ approximately retains the same form. The physical reason might be that the inner particle generates high harmonics with $s_h\approx 3(l_h/n_h)^{-3}$ so that $s_h\omega_h$ can be comparable to or even larger than ω_e. In this sense the situation is different from the case of monochromatic electric field where the condition $\kappa=\omega n^3\ll 1$ implies regular motion. In the present case the motion of the outer electron can be chaotic even if $\kappa\ll 1$. In other words, due to the generation of high harmonics, the Born-Oppenheimer approximation may be invalid even when the frequency of the heavy particle motion is much less than the electron frequency. An example of the kick function for $\kappa\ll 1$ is shown in Fig. 2. While the shape of theoretical curve given by (8)-(10) is qualitatively similar to the numerically found $g(\phi)$, further analytical analysis should be done to understand the quantitative difference between theory and numerics in the regime $\kappa\ll 1$.

For the particular case of circular motion of the inner particle the function g has a sine shape $[g(\phi)=\beta\sin\phi]$. In this case the situation is similar to the microwave ionization

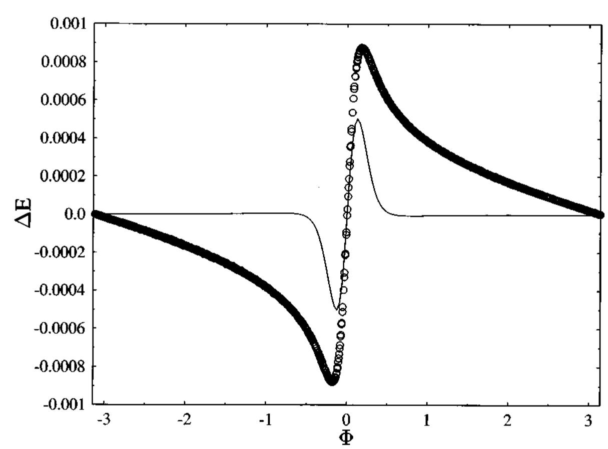

FIG. 2. Same as Fig. 1, with $\omega_h=0.1$ and $\beta=1.8$.

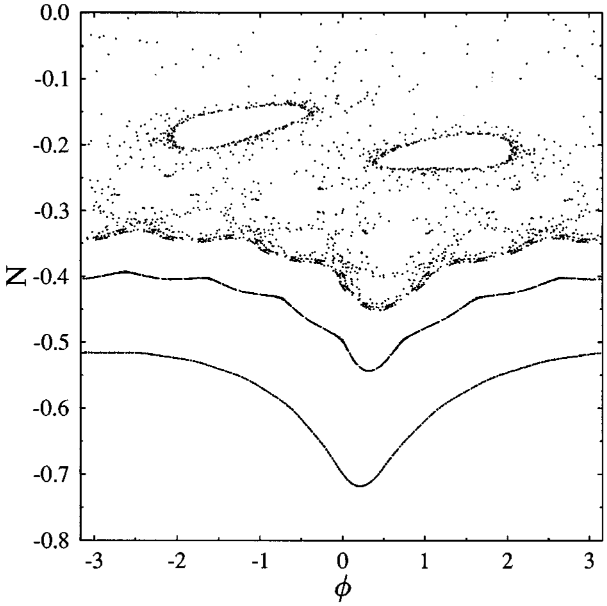

FIG. 3. Snapshot of the phase space for the planetary map (10) with the same parameters as in Fig. 1. Six orbits with different initial energies are shown.

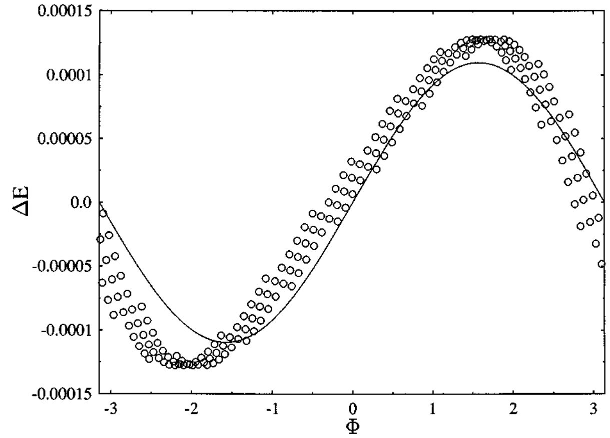

FIG. 4. Plot of the function $g(\phi)$ for the full motion in the helium atom (1). Initial conditions for the heavy particle of mass $M=2000$ are $n_h=1$ and $l_h/n_h=0.975$. The initial conditions for the electron are $n_e=6$ and $l_e/n_e=0.75$. Dots are numerical data and the full line is the theoretical curve (8) with $\beta=1.8$.

of excited states of hydrogen atom [7] and to the autoionization of molecular Rydberg states due to coupling between the rotational and the electron's motion [4]. Indeed, in this case the energy of the electron changes only when the electron passes near the perihelion and this change is given by the function $g(\phi)$. As is well known [4,7], the border for the transition to chaotic motion is determined by the condition $K=6\pi\beta k\omega_h^2 n_e^5>1$, that is,

$$a_h\omega_h^2>\frac{1}{50\beta a_e^{5/2}\omega_h^{1/3}}, \tag{11}$$

where $a_e=n_e^2$. Above the chaos border (11) the phase ϕ becomes random and diffusive ionization of the electron takes place with a diffusion rate $D=\Delta N^2/\Delta t=k^2\langle g^2(\phi)\rangle$. The ionization time can be estimated as N_I^2/D, where $N_I=E_I/\omega_h$ is determined by the energy $[E_I=(2n_e^2)^{-1}]$ required to ionize the electron from its initial state n_e. When eccentricity e_h is not small, the chaos border can be significantly decreased by the generation of high harmonics and by the form of the function $g(\phi)$. A more detailed analysis should be carried out to derive analytical estimates for the chaos border in such a case. An example of the phase plane corresponding to the planetary map is shown in Fig. 3.

So far, the derivation of the planetary map has been obtained by assigning the motion of the inner particle. The physical ground for this approximation lies in the fact that when the mass of the inner particle is very large, its interaction with the light one can only lead to small changes in the parameters of the inner orbit. The frequency of these variations will be small in comparison to the frequency of the heavy particle motion and therefore there will be no qualitative changes on the electron dynamics. However, an explicit numerical derivation of the map for two interacting particles is quite difficult, the reason being that the aforementioned slow variation of the parameters of inner motion, for example, eccentricity e_h, leads to a slow variation of the kick function $g(\phi)$. On the other hand, since these variations are slow compared to the frequency of the inner particle motion, they will not qualitatively modify the description of energy excitation given by the planetary map. A realistic example for two-particle motion in the Hamiltonian (1), which can be described by the map (10), is shown in Fig. 4. Here only the lower harmonics s [see (8)] contribute to the kick function $g(\phi)$ and the map is similar to the Kepler map. The absence of high harmonics in the function $g(\phi)$ is due to the relatively high value of the ratio l_h/n_h. For lower values of this ratio higher harmonics become more important, leading to a spiked shape for $g(\phi)$. In this case, however, a numerical evaluation of the function g is more difficult to obtain due to the precession of the orbit of the heavy particle produced by the interaction with the electron [this effect can be seen even in Fig. 4 as a slow phase shift of the kick function $g(\phi)$]. In this sense the one-dimensional planetary map gives an approximate description of the energy exchange between the particles and neglects the slow phase shift, which does not, however, qualitatively change the dynamics of the electron excitation in energy.

III. OUTER HEAVY PARTICLE

Let us now consider the case in which the heavy particle is the outermost one and stays sufficiently far from the inner electron. In such a situation $r_{\min}=l_h^2/(2M)\gg n_e^2$ and $\kappa=\omega_e/\omega_h\gg 1$. Due to these two conditions $(l_h/n_h)\gg(3/\kappa)^{1/3}$, so that the energy change of the heavy particle after one orbital period is exponentially small. Therefore, it may seem, at first glance, that no energy exchange should take place between the heavy particle and the electron. The possibility arises, however, for an interesting phenomenon; indeed the heavy particle induces an approximately static, slowly varying field, near the center of the atom, where the electron moves. This quasistatic field will lead to a precession of the electron orbit to a Stark frequency much smaller than that of electron motion, but comparable to the frequency of the heavy particle. As a consequence, this slow precession of the electron orbit can lead to an effective exchange of energy among the inner and the outer particle. The conditions under which this exchange can take place can be derived as follows. The field strength induced by the heavy particle at the center of the atom is approximately $\epsilon_{\text{Stark}} \approx a_h^{-2}$ and the Stark frequency of electron's orbit precession is $\omega_{\text{Stark}} \approx \epsilon_{\text{Stark}} \sqrt{a_e}$; in order to have an effective energy exchange between the two particles it is necessary that $\omega_{\text{Stark}} \gtrsim \omega_h \sim (a_h^3 M)^{-1/2}$. This leads to

$$\frac{a_h}{a_e} < M. \tag{12}$$

Under this condition the energy of the heavy particle only changes near the perihelion and the value of the change can be estimated as if an effective external monochromatic field was applied with frequency $\omega \approx \omega_{\text{Stark}}$ and intensity $\epsilon = a_e \omega_{\text{Stark}}^2$. The latter estimate is due to the precession of the dipole moment of the inner particle $d \approx a_e$ with frequency ω_{Stark}.

In analogy with the hydrogen atom in a microwave field it is convenient to introduce the rescaled frequency and field intensity $\omega_0 = \omega_{\text{Stark}} / \omega_h \approx \sqrt{a_e / a_h M}$ and $\epsilon_0 = a_e \omega_{\text{Stark}}^2 / (1/a_h^2) \approx (a_e/a_h)^2$. Here we implicitly assumed that the eccentricity of heavy particle e_h is small so that both axes of the elliptic orbit have approximately the same size a_h. Then, according to [7], the relative energy variation of the heavy particle is

$$\frac{\Delta E_h}{E_h} \sim a_h \Delta E_h \sim 2.6 \frac{\epsilon_0}{\omega_0^{2/3}} \sim \left(\frac{a_e}{a_h}\right)^{5/3} \frac{1}{M^{1/3}}. \tag{13}$$

The estimate for the chaos border can be obtained from the standard condition $\epsilon_0 > 1/50 \omega_0^{1/3}$ and gives

$$\frac{a_e}{a_h} > \frac{1}{6M^{1/3}}. \tag{14}$$

The derivation of the explicit map in this case is quite difficult. However, the energy change of the heavy particle after one orbital period can be expressed by some function $f(\varphi)$ of the angle $\varphi = \lambda_h - \lambda_e$ between the main ellipse axis of the heavy and light particles. An example of this function, obtained by numerically solving system (1), is shown in Fig. 5. The amplitude ΔE_h of the energy variation is approximately 10^{-4}, in satisfactory agreement with the estimate (13), which gives an amplitude approximately equal to 5×10^{-5}. The variation of angle φ is finite due to the finite cone of permitted angle of precession of the electron. In Fig. 6 we illustrate the precession of the electron orbit during a period of the outer heavy particle.

In deducting the estimate (14) we assumed that the precession frequency remains constant during an orbital period of the heavy particle; this is approximately true only when the heavy particle eccentricity is sufficiently small. Indeed, when the orbit is noncircular, the induced static field in the center decreases with the increasing of the particle distance r_h and reaches its maximal value at the perihelion. Therefore the precession frequency also changes in a complicated way. Because of this we believe that the real chaos border lies well below the estimated border (14). The estimate (14) was obtained under the additional assumption that the two axes of the ellipse are of comparable size. In the general case the value r_h at the perihelion should appear in it rather than a_h, hence the minimal distance of the heavy particle from the center cannot indefinitely grow without eventually leading the system below the chaos border. Therefore, only one possible way for ionization is left, consisting in an indefinite increase of the maximal distance a_h while keeping the distance at the perihelion approximately constant, so that $e_h \to 1$.

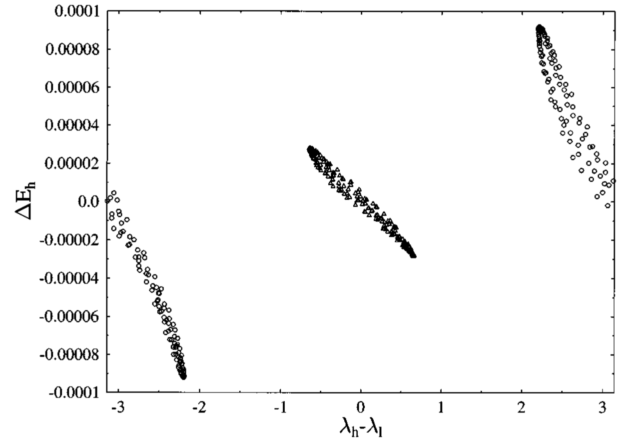

FIG. 5. Plot of ΔE_h versus the relative angle $\varphi = \lambda_h - \lambda_e$. The initial parameters for the outermost heavy particle (whose mass is $M = 2000$) are $n_h = 4\sqrt{M}$ and $l_h / n_h = -0.95$. The innermost electron starts with $n_e = 1$ and $l_e / n_e = 0.7$. Two sets of data are shown, one for the initial angle $\varphi = 0$ (triangles), the other for $\varphi = \pi$ (circles).

IV. ORBITS OF COMPARABLE SIZE

Finally, we will consider the case when the sizes of both orbits are comparable. Two different kinds of motion may occur in this case. The first possibility is that the motion of the two particles is correlated in such a way that even if the sizes of the orbits are approximately the same, close collisions between them nevertheless never occur. An example of

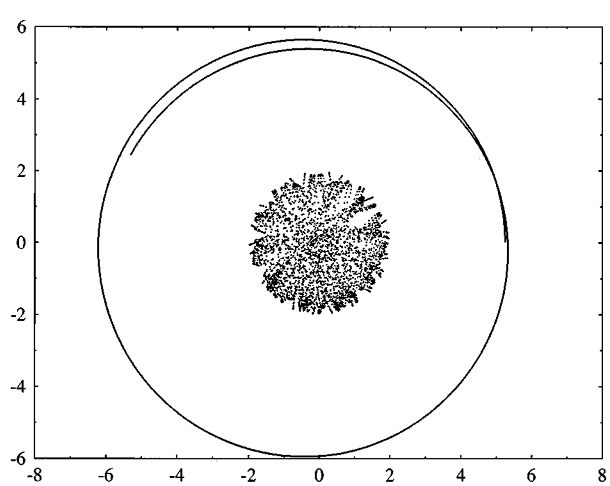

FIG. 6. Precession of the inner electron during a period of the outermost heavy particle. Here $n_h / \sqrt{M} = 2$, $l_h / n_h = 0.95$, $n_e = 1$, $l_e / n_e = 0.75$, and $M = 2000$.

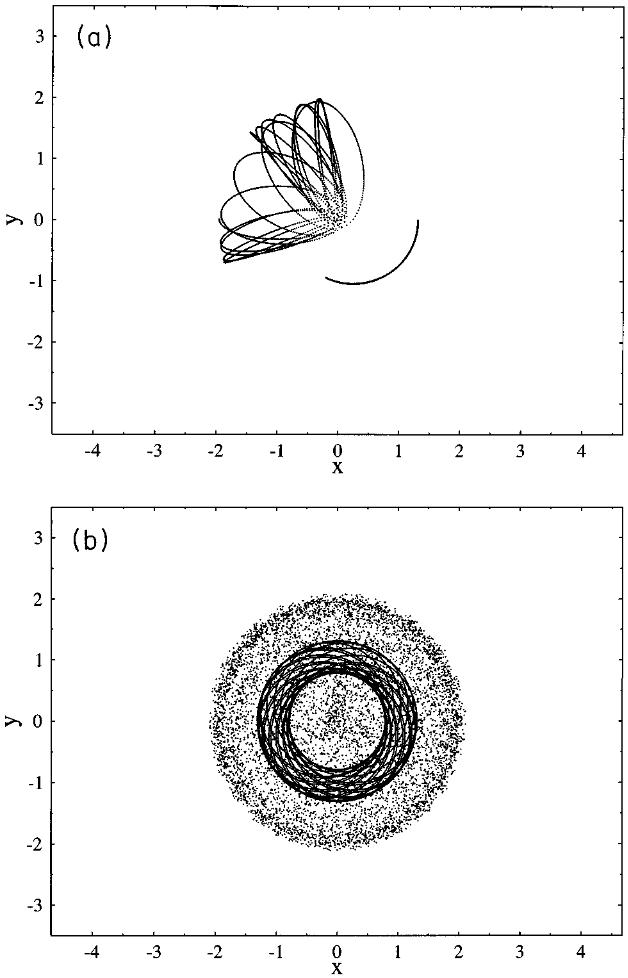

FIG. 7. (a) Stable and correlated motion of the heavy and light particles. In this case particles never collide. Here $n_h/\sqrt{M}=n_e=1$, $l_h/n_h=-0.95$, $l_e=0.30$, and $M=2000$. (b) Same as (a), but for a longer time.

this kind of motion is shown in Fig. 7: the electron is repelled by the heavy particle and electronic orbits precess following the slow motion of heavy particle thus avoiding collisions. Such a configuration remains stable during many orbital periods of the heavy particle and it is quite possible that both particles will never ionize. For certain initial conditions this type of motion looks integrable, while in other cases a more complicated motion without ionization is seen, which is probably chaotic, even in the absence of direct collisions (Fig. 8). When collisions become closer the electron transfers energy to the heavy particle, leading to its ionization (Fig. 9).

We found numerically that the stable configurations described above exist only when the orbital momentum of the heavy particle is close to its maximal value ($l_h \approx n_h$). As soon as the ratio l_h/n_h decreases (less than approximately 0.8) this synchronized motion does not occur any more and one of the particles is eventually ionized (Fig. 9). It is quite remarkable that in real laboratory experiments stable configurations of the hadronic atom were indeed observed only with $l_h \approx n_h$ [1].

Another configuration with correlated motion corresponds to the case (also considered in the helium atom [10]) when

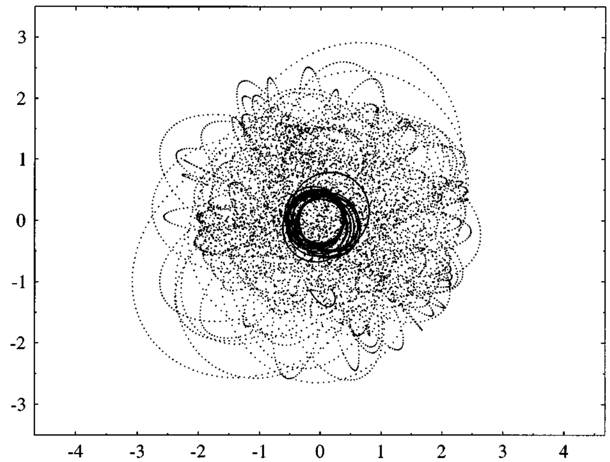

FIG. 8. Irregular, nonionizing, motion of both particles. Here $n_h/\sqrt{M}=0.7$ and $l_h/n_h=-0.95$, $n_e=1$, $l_e=0.10$, and $M=2000$.

both particles with negative charge are located on the same side of the nucleus. The simplest orbit in this kind of configuration corresponds to one-dimensional motion of both particles on a line. In this case the fast moving electron repels the heavy particle and creates a classically stable situation [11]. Other stable orbits exist in which small frequency oscillations take place in the direction perpendicular to the line. These oscillations share similar properties with the one-dimensional case, and since in this case the outer particle is heavy, such quasi-one-dimensional configurations are even more stable than in the helium atom. In the opposite case when the heavy particle is inside, the ratio of two frequencies $\omega_l/\omega_h \approx \sqrt{M}/14$ is comparable to one and the question of stability of this configuration requires further investigations.

When the motion is not correlated and collisions between particles can take place, then the heavy particle intersects the electronic "cloud" and after each collision the change in velocity of the heavy particle is $\Delta v_h \sim v_e/M$, so that the relative energy change is $\Delta E_h/E_h \sim \Delta v_h/v_h$. Since $Mv_h^2 \sim v_e^2$ it follows that after each collision $\Delta E_h/E_h \sim 1/\sqrt{M}$. The electron motion is much faster than the heavy particle motion and therefore the number of colli-

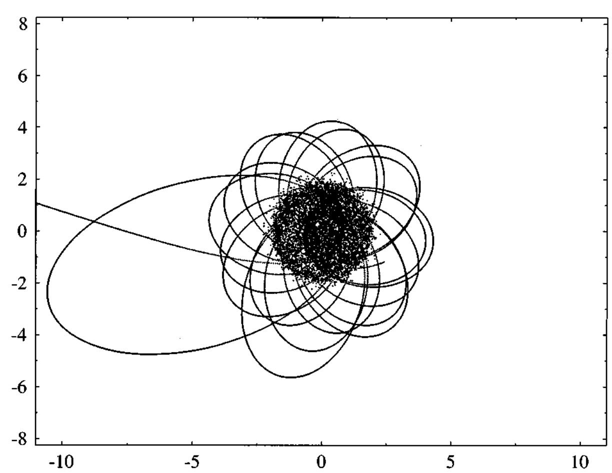

FIG. 9. Example of unstable motion. Here $n_h/\sqrt{M}=1.4$, $l_h/n_h=-0.71$, $n_e=1$, $l_e=0.7$, and $M=2000$.

sions $\mathcal{N}$ at each passage is approximately equal to $\sqrt{M}$. As a consequence, the relative energy change for the heavy particle after one orbital period is $\sqrt{\mathcal{N}}/M$. Ionization will occur when the relative energy change is order of one, which requires $\sqrt{M}$ orbital periods of the heavy particle. Therefore long-lived states can exist even when direct collisions between the light and the heavy particle take place. In the above estimate we refer to the case in which the electron and the heavy particle orbits have comparable size and $l_h \sim n_h$ and $l_e \sim n_e$. In this case the ratio between the velocities of the two particles v_h/v_e is of the order of $1/\sqrt{M}$ and therefore the slow motion of the heavy particle does not strongly effects the dynamics of the light electron.

V. CONCLUSION

The above picture is related to the classical dynamics of the system. The analysis of the dynamics in the classical case allows us to understand also the basic features of quantum motion. Indeed the planetary map description can be quantized in the same way as for the Kepler map [7]. As a consequence, quantum effects can lead to localization of classical diffusion, with a localization length $\ell_\phi \sim k^2 \langle g^2 \rangle$. When this localization length is smaller than the number of "photons" $N_I = (2n_e^2 \omega_h)^{-1}$ required for ionization, then classical ionization is suppressed and the quantum atom has a very long lifetime. In the opposite case, when $\ell_\phi > N_I$, diffusive ionization takes place and approximately follows the classical description.

A more unusual situation arises when the heavy particle is the outermost one. Here the frequency of precession of the inner electron, which produces the kick function $f(\varphi)$ for the heavy particle, is not constant and depends on the motion of the heavy particle itself. It is therefore likely that the sequence of kicks will be irregular so that quantum interference effects will be destroyed and quantum diffusion will be close to classical. It would be interesting to carry out explicit quantum numerical computations for a better understanding of the properties of quantum motion in the different regions discussed above and for a deeper understanding of manifestations of classical chaos in quantum dynamics.

In closing this paper we would like to mention another intriguing question related to the electron scattering on the hadronic ion (a similar problem arises for the scattering of an electron on a rotating molecular ion). Such a process is in some sense the time reversed version of the ionization process discussed above and is described by the planetary map. The estimate for the absorption cross section of the electron on such an ion can be obtained in a way similar to the scattering process of electrons on protons in the presence of a microwave field [12]. Indeed, in this case the electron can be captured only if its orbital momentum $l_e < (3/\omega_h)^{1/3}$ and its energy is $E_e < k\omega_h \approx a_h \omega_h^{4/3}$. Since $l_e \sim \rho v_e$, where ρ is the impact parameter of the electron and v_e its asymptotic velocity at infinite distance, we obtain for the absorption cross section

$$\sigma_a \sim \pi\rho^2 \sim \frac{l^2}{v_e} \sim \frac{1}{\omega_h^{2/3} E_e} > \frac{1}{a_h \omega_h^2} \sim M a_h^2. \tag{15}$$

As can be seen the cross section (15) is much larger than the mere geometrical cross section $\sigma \sim a_h^2$. Therefore the process of creation of neutral excited atoms can be very effective. It is possible that such an increase of the cross section can be interesting for the process of muon catalysis. Finally, our analysis of different dynamical regimes in hadronic atom can be useful for a better understanding of recent experimental investigations [13].

[1] T. Yamazaki, in *Atomic Physics 13*, edited by T.W. Hänsch, H. Walther, and B. Neizert, AIP Conf. Proc. No. 275 (AIP, New York, 1993), p. 325.
[2] W. Sandner, in *Atomic Physics 13* (Ref. [1]), p. 363.
[3] P. Labastie, M.C. Bordas, B. Tribollet, and M. Broyer, Phys. Rev. Lett. **52**, 1681 (1984); M. Lombardi and T.H. Seligman, Phys. Rev. A **47**, 3571 (1993).
[4] F. Benvenuto, G. Casati, and D.L. Shepelyansky, Phys. Rev. Lett. **72**, 1818 (1994).
[5] H. Fano, Phys. Rep. **46**, 97 (1982); J.M. Rost and J.S. Briggs, J. Phys. B **24**, 4293 (1991).
[6] T. Yamazaki *et al.*, Phys. Rev. Lett. **63**, 1590 (1989); M. Iwasaki *et al.*, *ibid.* **67**, 1246 (1991); T. Yamazaki and K. Ohtsuki, Phys. Rev. A **45**, 7782 (1992).
[7] G. Casati, I. Guarneri, and D.L. Shepelyansky, IEEE J. Quantum Electron. **QE-24**, 1420 (1988).
[8] H.A. Kramers, *Collected Scientific Papers* (North-Holland, Amsterdam, 1956), p. 866; W.C. Henneberger, Phys. Rev. Lett. **21**, 838 (1968).
[9] B.V. Chirikov and V.V. Vecheslavov, Astron. Astrophys. **221**, 146 (1989).
[10] K. Richter, G. Tanner, and D. Wintgen, Phys. Rev. A **48**, 4182 (1993).
[11] D.L. Shepelyansky, J. Phys. B **27**, L235 (1994).
[12] D.L. Shepelyansky, in *Quantum Chaos*, edited by G. Casati, I. Guarneri, and U. Smilansky (North-Holland, Amsterdam, 1993), p. 221.
[13] D. Horvath, Europhys. News **26**, 57 (1995).