Low-energy transition in spectral statistics of two-dimensional interacting fermions

Pil Hun Song
Max-Planck-Institut für Kernphysik, Postfach 103980, 69029 Heidelberg, Germany

Dima L. Shepelyansky
Laboratoire de Physique Quantique, UMR 5626 du CNRS, Université Paul Sabatier, F-31062 Toulouse Cedex 4, France
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We study the level spacing statistics $P(s)$ and eigenstate properties of spinless fermions with Coulomb interaction on a two-dimensional lattice at constant filling factor and various disorder strengths. For the noninteracting localized phase in the limit of large lattice size, Coulomb interaction induces a transition of $P(s)$ from the Poisson to the Wigner-Dyson distribution, at a critical total energy independent of the number of fermions. This implies the emergence of quantum ergodicity induced by interaction and delocalization in the Hilbert space at zero temperature inside the one-particle insulating phase.

The experimental observation of the metal-insulator transition in two dimensions (2D) by Kravchenko *et al.*[1] has attracted a great interest to interacting fermions in a disordered potential. Indeed, according to the well-established result,[2] all states are localized for noninteracting particles in 2D. Therefore, in the view of the experimental result,[1] a new theory should be developed to understand the interaction effects between the localized fermionic states. However, in spite of various theoretical attempts, a coherent theory for such systems is still not available. While for highly excited states, it has been shown that the repulsive/attractive interaction can induce a delocalization of two interacting particles,[3–6] the properties of low energy states are not understood yet. Recently, in addition to experimental and theoretical investigations, a number of attempts have been made to study these many fermionic systems through numerical simulations.[7–11] Even though several interesting features have been reported, the systems studied there are not large enough to observe interaction induced delocalization.

In this paper we use another numerical approach based on the analysis of spectral properties of multiparticle fermionic systems. Indeed, Shklovskii *et al.* have shown that the level spacing statistics is a powerful tool to analyze the Anderson transition in disordered systems.[12] When the states are localized, the levels are not correlated and the statistics is given by the Poisson distribution, $P(s)=P_P(s)=\exp(-s)$, while in the metallic phase, the states are ergodic and the statistics is close to the Wigner surmise, $P_W(s)=(\pi s/2)\exp(-\pi s^2/4)$. The critical transition point is characterized by an intermediate statistics which depends on the boundary conditions[13] and the spatial dimension of the system.[14] This approach has also been used to determine the quantum chaos border and the interaction induced thermalization in finite fermionic systems[15] and to detect Anderson transition for two electrons with the Coulomb interaction on a two-dimensional disordered lattice.[16,17] All these results demonstrate that the approach developed in Ref. 12 allows to investigate efficiently the transition from nonergodic (localized) to ergodic eigenstates. This approach applied to interacting particles in noninteracting 2D localized phase allows to establish the emergence border of quantum ergodicity induced by interaction. Indeed, in the localized phase the many-body levels are not correlated and are characterized by Poisson statistics. In contrast, the Wigner-Dyson statistics in the noninteracting localized phase can appear only if the electrons become ergodic/delocalized in space. However, strictly speaking, this ergodicity is not so directly related to the metal-insulator transition discussed in Ref. 1. Indeed, experimentally the transition is identified by the change of the resistivity dependence on temperature, i.e, resistivity grows for an insulator while it drops for a metal with the decrease of temperature. However, in both cases the resistivity is finite, i.e., the electrons are ergodic. Also in the experiment the contribution of other degrees of freedoms, e.g., phonons, is not absolutely excluded.

To identify the transition to ergodic multiparticle states we study the change of the spectral statistics, $P(s)$, with excitation energy E in a model of spinless fermions with Coulomb interaction on a two-dimensional disordered lattice. The Hamiltonian reads

$$H=V\sum_{\langle ij\rangle} a_i^\dagger a_j+\sum_i w_i n_i+U\sum_{i>j}\frac{n_i n_j}{r_{ij}}, \qquad (1)$$

where $a_i^\dagger$ (a_i) is the fermion creation (annihilation) operator at site i, V is the hopping between the nearest neighbors, the diagonal energies w_i are randomly distributed within the interval $[-W/2, W/2]$, $n_i=a_i^\dagger a_i$ is the occupation number at site i and U is the strength of the Coulomb interaction with r_{ij} the interparticle distance. The particles move in a two-dimensional cell of size $L\times L$ with the periodic boundary conditions applied. The Coulomb interaction is taken between electrons in one cell of size L and with 8 charge images in nearby 8 cells as in Ref. 16. The number of particles N_p and the cell size were varied within the intervals $2\leqslant N_p\leqslant 20$ and $8\leqslant L\leqslant 28$. With the notation in the Eq. (1), the parameter r_s, which measures the ratio of the Coulomb energy to the Fermi energy, ϵ_F, is given by $r_s=U/(2V\sqrt{\pi\nu})$, where $\nu=N_p/L^2$ is the filling factor and $\epsilon_F=4\pi\nu V$. The majority of our data have been obtained for $\nu\approx 1/32$ (nearest rational value) and for $U/V=2$, which corresponds to $r_s=3.22$.

To study the level spacing statistics $P(s)$, we generalize the approach used in Ref. 16 for many particles. First, one-particle eigenstates (orbitals) at $U=0$ are obtained and the Hamiltonian (1) is rewritten in this basis using the two-body interaction matrix elements. We consider the first M orbitals from the lowest energy and the Hamiltonian multiparticle

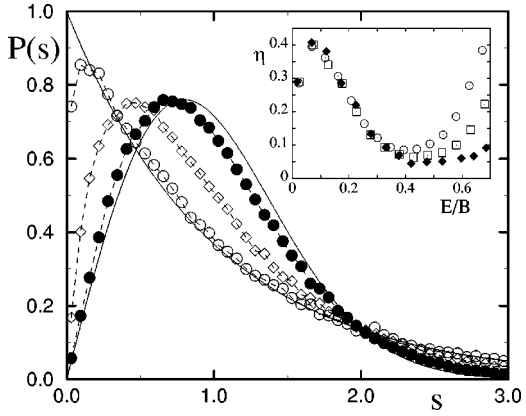

FIG. 1. Level statistics $P(s)$ for 10 particles at $L=18$, $r_s=3.22$ in the energy interval $0.25<E/B<0.3$ (with $B=4V$): $W/V=15$, $\eta=0.93$ (○); $W/V=10$, $\eta=0.51$ (◇); $W/V=7$, $\eta=0.10$ (●). Total statistics is $N_S=10^5$; 1.4×10^5; 1.8×10^5, respectively. Full lines show the Poisson distribution and the Wigner surmise. Insert, made for $N_p=6$ case of Fig. 2(b), illustrates that at low E/B the statistics $P(s)$, characterized by η, is independent of matrix size variation $N_m=797$ (○), 1231 (□), 2235 (full diamond).

matrix, constructed only from these orbitals, is diagonalized at the final stage. To the maximum, $N_p=20$ particles with up to $M=42$ orbitals has been considered and the resulting matrix size is $N_m\approx5000$. This size is significantly smaller than ${}_MC_{N_p}$ since the condition $\Sigma_{i=1}^{N_p}m_i\leqslant\Sigma_{i=1}^{N_p-1}i+M$ applies for the one-particle orbital index m_i. Such a pyramid rule allows one to use efficiently only low energy multiparticle states and to make a striking reduction of the resulting total matrix size without any serious modification of the low energy states. We checked that our level statistics results at low energy are not sensitive to the variation of M and N_m (see for example insert in Fig. 1). To compute the spectral statistics $P(s)$ at a given total excitation energy E, which is counted from the ground state energy, disorder average has been performed over $N_D=5000$ (for low energy) and $N_D=1000$ (for higher energy) configurations. In this way, the total statistics for $P(s)$ for small energy interval varies from $N_S=10^4$ at low E to $N_S=3\times10^5$ at high E.

A change of $P(s)$ with W, at a given E and all other parameters fixed, is shown in Fig. 1 for $N_p=10$. As W decreases, $P(s)$ evolves from the Poissonian to the Wigner-Dyson distribution. To measure the proximity of $P(s)$ to one of these two limiting cases, it is convenient to use the parameter η which is defined as $\eta=\int_0^{s_0}[P(s)-P_W(s)]ds/\int_0^{s_0}[P_P(s)-P_W(s)]ds$, where $s_0=0.4729\ldots$ is the smaller intersection point of $P_P(s)$ and $P_W(s)$. In this way $\eta=1$ corresponds to $P_P(s)$ and $\eta=0$ to $P_W(s)$. According to Fig. 1, η changes almost by an order of magnitude when W decreases by factor of two. This shows that for strong disorder, $W/V=15$, the multiparticle states at given E are not ergodic (localized) while for weaker disorder, $W/V=7$, they become ergodic and characterized by the random matrix spectral statistics. We note that for given values of disorder the one-particle inverse participation ratio (IPR), ξ_1 (the number of sites contributing a one-particle eigenstate) is much smaller than the total number of sites L^2 for the case of Fig. 1: $\xi_1=3.4$ and 4.2 ($W/V=15$); 5.2 and 11.6 ($W/V=10$); 8.2 and 36.7 ($W/V=7$), where the former are for the ground state

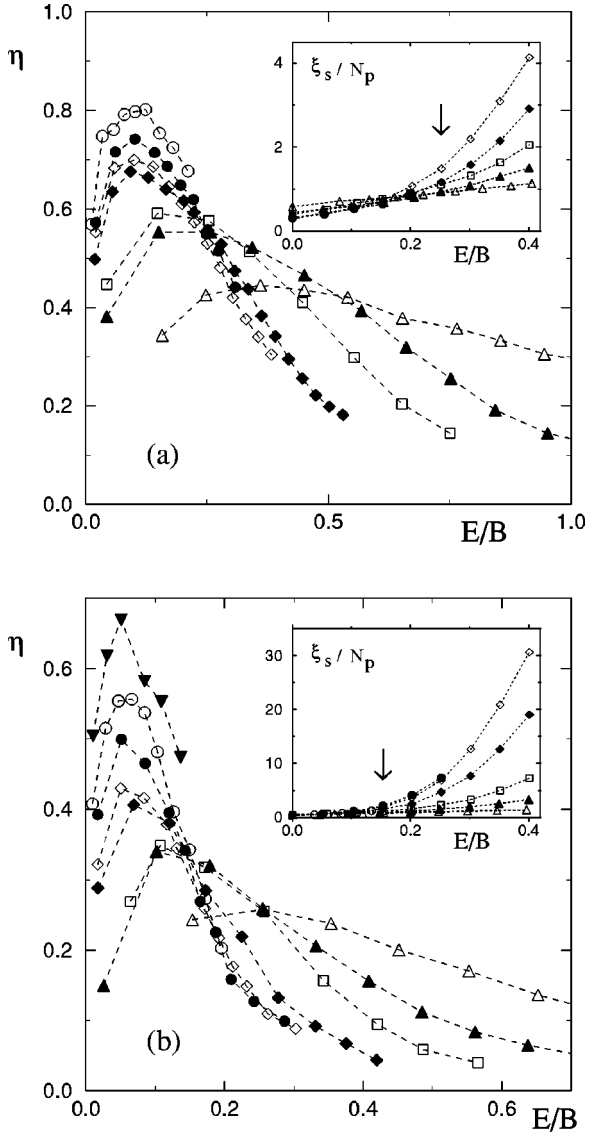

FIG. 2. Dependence of η on the rescaled total energy E/B for various number of particles $N_p=2$ (△), 3 (full triangle up), 4 (□), 6 (full diamond), 8 (◇), 10 (●), 14 (○) and 20 (full triangle down); (a) $W/V=10$ and (b) $W/V=7$; filling factor $\nu\approx1/32$ and $r_s=3.22$, $8\leqslant L\leqslant25$. Inserts show the dependence of ξ_S/N_p on E/B by same symbols; arrows mark the critical energy E_c from the main figure.

and the latter for the center of the band. This means that the transition to ergodicity is induced by the interaction which becomes effectively more strong when the one-particle localization length increases (similar to the two-interacting-particle case.[3–6]). At the same time, the comparison with the data[16] for $N_p=2$ at the same E and U/V as in Fig. 1 ($\eta\approx0.92$, 0.82, and 0.51 for $W/V=15$, 10, and 7, respectively) shows that the delocalization effect due to multiparticle interaction is stronger for weak disorder while for strong disorder it does not affect localization. At the same time the transition of $P(s)$ close to $P_W(s)$ is definitely induced by interaction. Indeed, at weak interaction the statistics is very close to $P_P(s)$ for the energy interval of Fig. 1, e.g., at $N_p=2$, $L=16$, $U/V=0.1$, $W/V=7$, $\eta\approx0.8$ (according to the data in Fig. 1d in Ref. 16).

In Fig. 2 we present the variation of η with the total

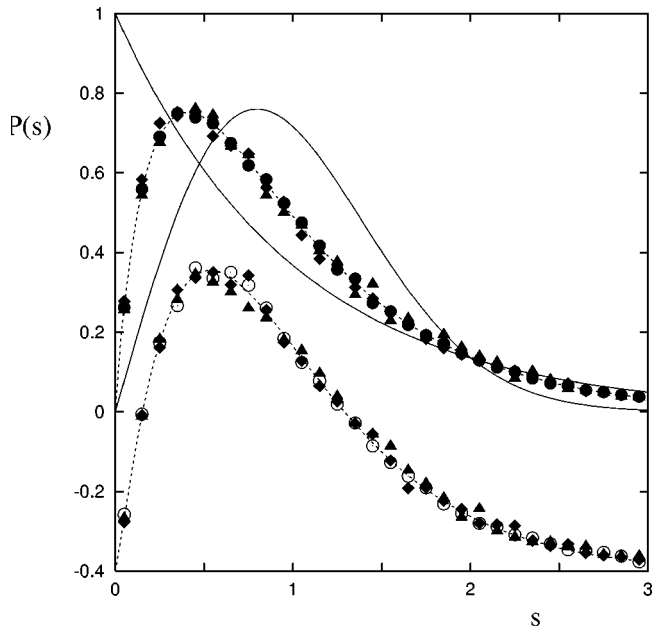

FIG. 3. Level statistics $P(s)$ at the crossing points of Fig. 2 (with the same symbols). The upper data are for $W/V=10$, $0.225<E/B<0.275$, $\eta_c\approx 0.56$, $N_S=10^4$, 4×10^4 and 10^5 for $N_p=3$, 6 and 10, respectively. The lower data (shifted down by 0.4) are for $W/V=7$, $0.125<E/B<0.175$, $\eta_c\approx 0.33$, $N_S=10^4$, 1.4×10^4, and 5×10^4 for $N_p=3$, 6, and 14, respectively. The full lines are the Poisson distribution and the Wigner surmise.

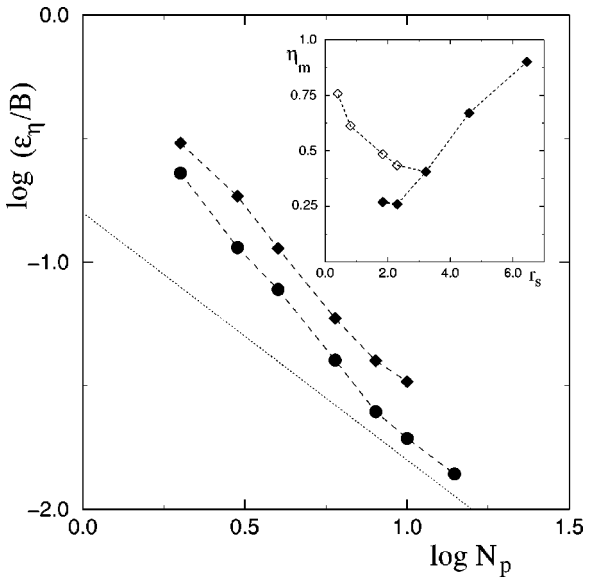

FIG. 4. Dependence of ϵ_η/B on the number of particles N_p, obtained from Fig. 2: $W/V=10$ with $\eta(E_\eta)=0.4$ (full diamond) and $W/V=7$ with $\eta(E_\eta)=0.2$ (●), where $\epsilon_\eta=E_\eta/N_p$. The straight line shows the slope when $E_\eta=$const. The inset gives the dependence of maximal η on r_s for $W/V=7$ and $N_p=6$: $U/V=2$, $8\leqslant L\leqslant 28$ (full diamond), and $L=14$, $0.25\leqslant U/V\leqslant 2$ (◇).

energy E for various number of particles at fixed filling factor and two values of disorder. For $N_p>2$, all curves have approximately the same crossing point: $E_c\approx 0.25B$ ($W/V=10$), $E_c\approx 0.15B$ ($W/V=7$), and $E_c\approx 0.1B$ ($\eta_c\approx 0.19$, $W/V=5$, not shown). As N_p increases, the statistics approaches the Poissonian case for $E<E_c$, while it goes to the Wigner case for $E>E_c$. The transition point, E_c, is characterized by an intermediate statistics, which is independent of the system size as shown in Fig. 3. The energy E_c grows with the increase of W so that for $W/V=15$ the crossing point is not found clearly within our computations (in this case $\eta\approx 0.8$–1.0 for $0<E/B\leqslant 1$ and $N_p>2$). The above data give strong evidence that the ground state and the excited states with $E<E_c$ are localized in the limit of the infinite system size. However, for excitations with $E>E_c$ the statistics is close to $P_W(s)$, which implies the ergodicity of eigenstates and their space delocalization.

To show that the eigenstate properties are qualitatively different below and above E_c we determined the variation of IPR ξ_S with N_p and E as presented in the inserts of Fig. 2. This IPR ξ_S is computed in the basis of noninteracting Slater determinants and determines how many of such states are required to construct an eigenstate. The results are obtained by averaging over 100 disorder realizations so that the statistical error is less than 5%. For $E<E_c$ the ratio ξ_S/N_p, which gives the number of noninteracting states per particle, remains constant while for $E>E_c$ it grows rapidly with N_p. For example, the ground state average value is $\xi_S/N_p=0.35;0.40;0.46$ for $W/V=10;7;5$ (the latter is not shown) with only $\pm 21;13;18\%$ variation when N_p changes from 3 to 10; 14; 20, respectively, for each W/V. In contrast, for $E=0.4B>E_c$ the ratio ξ_S/N_p grows in 2.7;9;20 times when N_p changes from 3 to 8 for $W/V=10;7;5$ respectively. These results show that the delocalization in the Hilbert space of Slater determinants takes place for $E>E_c$ with E_c independent of number of particles. A crossover from localized to extended states in the Hilbert space was extensively discussed recently for the metallic phase with extended one-particle states.[15,18] Our data in Fig. 2 represent the first evidence for a delocalization transition in the Hilbert space for the insulating phase with localized one-particle states.

An unusual property of the above transition is that it takes place at the finite total energy. This means that the energy per particle $\epsilon=E/N_p$, or the temperature, is equal to zero at the transition point as $N_p\to\infty$. In this sense this transition can be considered as a quantum zero temperature transition. To ensure that in our numerical computations ϵ was sufficiently low, we present variation of ϵ_η, defined at the fixed η level, with the number of particles in Fig. 4. According to these data ϵ_η drops more than by an order of magnitude and becomes approximately by an order of magnitude smaller than the Fermi energy $\epsilon_F\approx 0.1B$.

Dependence of η_m on r_s is found in the insert of Fig. 4. The value of r_s is varied in two different ways: (i) by changing the cell size L or (ii) by changing U/V. The first case (i) allows to obtain large r_s value and shows that the statistics approaches the Poissonian case. This result is in a qualitative agreement with the experimental data,[1] where the localized (insulating) phase appears for large r_s, and with the theoretical argument given in Ref. 16, according to which the two-body interaction drops as the filling factor decreases. The same arguments explain why η goes to 1 when the interaction strength U decreases as in the second case (ii). One can expect that the variation of η_c with r_s at large N_p and fixed ν will be qualitatively similar to that one of η_m in Fig. 4.

The comparison of the energy E_c, at which the transition at fixed ν takes place (see Fig. 2), with the transition energy

for two particles [$E_{2c}/B=1.2$ for $W/V=10$ and $E_{2c}/B=0.56$ for $W/V=7$ (Ref. 16)] shows that E_c is significantly smaller than E_{2c} ($E_c/E_{2c}\approx 0.2$). This signifies that the multiparticle interaction is more efficient for delocalization than the two-body interaction for only two particles. Our qualitative understanding for the transition to the quantum ergodicity at fixed ν and fixed total energy E_c is based on the following scenario. In the analogy with the approach developed in Ref. 16 for $N_p=2$, the dynamics of three or four particles at finite energy E can be assumed to be equivalent to one-particle dynamics in a disordered system with effective spatial dimension $3<d_{eff}<2N_p$. With the growth of E, the rate of this one-particle hopping increases[16] and the delocalization transition finally takes place. As a result, in the original system, the energy can be transferred by this small group of particles from one place to another that allows to establish the ergodicity in the system of large size at fixed filling factor. The values of η_c found here (Fig. 2) are larger than that for three-dimensional Anderson model ($\eta_c\approx 0.2$), which makes the above scenario consistent with the result of Ref. 14 according to which η_c grows with the increase of the spatial dimension d. We also note that the approach of η to 1 near the ground state is not so surprising. Indeed, even in the metallic quantum dots one has $\eta\approx 1$ near the ground state since the interaction between particles is effectively weak[15,18] and the transition to ergodicity with $\eta=0$ takes place only at higher energy.[19]

In conclusion, our results show that 2D fermions, which are strongly localized by disorder without interaction, become ergodic due to Coulomb interaction. The transition to ergodicity, in lattice and Hilbert spaces, and the random matrix statistics takes place at the finite total energy or zero temperature. These results give an indication in favor of the experimentally observed metal-insulator transition,[1] even if our approach does not allow us to investigate the conductance dependence on temperature, which is the main experimental method to detect the transition as it was discussed above. It is possible that the ergodic phase we found will be characterized by a resistivity growth with the decrease of temperature that is generally interpreted as an insulator. However, it is important to stress that in our case the ergodicity in the lattice and Hilbert spaces is induced only by electron-electron interaction. This ergodicity can generate the electron-electron induced (phononless) variable range hopping as it was discussed long ago in Refs. 20 and 21. Indeed, the recent experiments[22] on the variable range hopping in 2D samples show that the transport is phonon independent and is generated by the electron-electron interaction. This is in a qualitative agreement with the low energy ergodicity induced by Coulomb interaction found in this paper.

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[19] We can try to adopt the approach of Ref. 15 to the present case. According to Ref. 16 the two-body matrix element between particles at a distance L is $U_s\sim U/L^3$ and the number of such effectively interacting particles at temperature T is $n_{eff}\sim \nu L^2 T/\epsilon_F$. Then the spacing between directly coupled multiparticle states is $\Delta_c\sim\Delta_2/n_{eff}^2$, where at low energy the two-particle spacing is $\Delta_2\sim V/\xi_1$, since each particle can jump only inside ξ_1 sites (Refs. 4 and 16). The transition to $\eta=0$ takes place for $U_s>\Delta_c$ (Ref. 15) that gives the critical energy $E_c\sim Tn_{eff}\sim (N_p/\nu)^{1/2}V^2/U\xi_1$. However, this border is higher than the numerically found one. This indicates that the delocalization of a small group of particles is dominating.

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