Georgeot and Shepelyansky Reply: In our Letter [1], we showed on the example of the Arnold cat map that classical chaotic dynamics of exponentially many orbits can be simulated in polynomial time on a quantum computer. The Liouville density distribution $P(x_i, y_j)$ is encoded on a discretized lattice ($2^{n_q} \times 2^{n_q}$) using $3n_q - 1$ qubits organized in three registers. After each map iteration, the distribution is coded in the quantum state $\sum_{i,j} a_{ij}|x_i\rangle\,|y_j\rangle\,|0\rangle$ with $P(x_i, y_j) = a_{ij}$ and $x_i = i/N$, $y_j = j/N$, $N = 2^{n_q}$. One measurement of qubits in this basis gives one point in the phase space, and therefore the distribution $P(x_i, y_j)$ can be obtained approximately in polynomial number of measurements. However, the same information can be obtained via classical Monte Carlo simulation with a polynomial number of orbits, as it was discussed by us in [2] and later repeated in [3]. Based on this observation, on which we agree, the Comment [3] makes a general claim that no new information can be extracted efficiently from quantum computation of such classical maps (paragraph 3 in [3]). Here we show that this statement is incorrect. Indeed, the quantum Fourier transform (QFT) of a_{ij} provides *nondiagonal observables* [1], namely, the Fourier components $\tilde{P}(k_x, k_y) = \sum_{i,j} \exp[i2\pi(k_x x_i + k_y y_j)]a_{ij}/N$. They obviously contain important *additional* information relevant for the classical dynamics, and require $O(n_q^2)$ operations including measurement. In contrast, all known classical algorithms will require an exponential number of operations to obtain correct probabilities at high harmonics $k_{x,y} \sim N$. Such harmonics are important since due to chaos a significant part of total probability is transferred to wave vectors with $k \sim \exp(ht)$, where h is the Kolmogorov-Sinai entropy, and t is the number of iterations (see [13] in [1]). We note that the claim of [3] applies equally to the Shor algorithm, where all information is also encoded only in a squared moduli of amplitudes, but where the QFT gives classically inaccessible information.

In Fig. 1 we present the probability $\tilde{P}(k_x, k_y)$ in Fourier space for different times t. We note that a two-dimensional (2D) QFT can be efficiently implemented by application of usual QFT to each register consecutively. The results show that $\tilde{P}$ is composed of well-pronounced peaks, most of which move with time to high wave vectors k. They remain stable in the presence of noise in the quantum gates (e.g., top right panel in Fig. 1 is unchanged if 1% noise is added in each gate). The location and amplitude of these peaks can be extracted from a polynomial number of measurements of qubits after the 2D QFT.

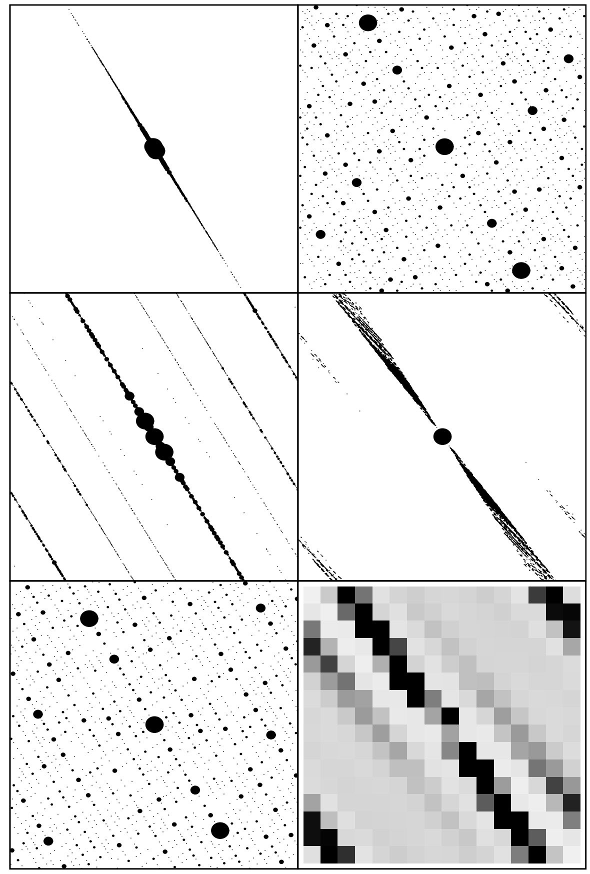

FIG. 1. Fourier coefficients $|\tilde{P}(k_x, k_y)|^2$ of Liouville distribution for $-N/2 \le k_{x,y} \le N/2$, initial state as in Fig. 1 of [1]. Left column: cat map at $t = 3$, 5, and 7 from top to bottom for $n_q = 10$. Top right: same at $t = 5$, $n_q = 7$. Middle right: $|\tilde{P}(k_x, k_y)|^2$ for perturbed cat map (see text) at $t = 5$, $n_q = 10$. Peaks are shown by circles; maximal circle size marks peaks with $1 > |\tilde{P}(k_x, k_y)|^2 > 0.1$, circles twice smaller those with $0.1 > |\tilde{P}(k_x, k_y)|^2 > 0.01$, etc. Bottom right: coarse-grained image of $|\tilde{P}(k_x, k_y)|^2$ (proportional to grayness) for the data of middle right panel, $n_f = 4$.

For the Arnold cat map, the dynamics in (k_x, k_y) space is especially simple, given by $\bar{k}_x = k_x - k_y$, $\bar{k}_y = 2k_y - k_x (\mathrm{mod} N)$. However, generally this dynamics is very complicated. To exemplify this, we simulated the perturbed cat map $\bar{y} = y + x + x^2$, $\bar{x} = x + \bar{y} (\mathrm{mod} 1)$ (Fig. 1). It can be iterated in $O(n_q^2)$ operations on a quantum computer using modular multiplication. In this case, main peaks can be seen directly for short times, while for larger times a polynomial number of measurements of the first n_f qubits [2] gives a coarse-grained image of $|\tilde{P}(k_x, k_y)|^2$, including very high harmonics, which are inaccessible to classical computation.

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[1] B. Georgeot and D. L. Shepelyansky, Phys. Rev. Lett. **86**, 5393 (2001).
[2] B. Georgeot and D. L. Shepelyansky, quant-ph/0102082v2.
[3] L. Diosi, preceding Comment, Phys. Rev. Lett. **88**, 219801 (2002).

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