Effects of decoherence and imperfections for quantum algorithms

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Abstract. We study effects of static inter-qubit interactions and random errors in quantum gates on the stability of various quantum algorithms including the Grover quantum search algorithm and the quantum chaos maps. For the Grover algorithm our numerical and analytical results show existence of regular and chaotic phases depending on the static imperfection strength ε. The critical border ε_c between two phases drops polynomially with the number of qubits n_q as $\varepsilon_c \sim n_q^{-3/2}$. In the regular phase ($\varepsilon < \varepsilon_c$) the algorithm remains robust against imperfections showing the efficiency gain $\varepsilon_c/\varepsilon$ for $\varepsilon > 2^{-n_q/2}$. In the chaotic phase ($\varepsilon > \varepsilon_c$) the algorithm is completely destroyed. The results for the Grover algorithm are compared with the imperfection effects for quantum algorithms of quantum chaos maps where the universal law for the fidelity decay is given by the Random Matrix Theory (RMT). We also discuss a new gyroscopic quantum error correction method which allows to reduce the effect of static imperfections. In spite of this decay GYQEC allows to obtain a significant gain in the accuracy of quantum computations.

Keywords: Imperfections, quantum chaos, random matrix theory, quantum error correction

In realistic quantum computations [1] the elementary gates are never perfect and therefore it is very important to analyze the effects of imperfections and quantum errors on the algorithm accuracy. A usual model of quantum errors assumes that angles of unitary rotations fluctuates randomly in time for any qubit in some small interval ε near the exact angle values determined by the ideal algorithm. In this case a realistic quantum computation remains close to the ideal one up to a number of performed gates $N_g \sim 1/\varepsilon^2$. For example, the fidelity f of computation, defined as a square of scalar product of quantum wavefunctions of ideal and perturbed algorithms, remains close to unity if a number of performed gates is smaller than N_g. This result has been established analytically and numerically in extensive studies of various quantum algorithms [2, 3].

Another source of quantum errors comes from internal imperfections generated by residual static couplings between qubits and one-qubit energy level shifts which fluctuate from one qubit to another but remain static in time. These static imperfections may lead to appearance of many-body quantum chaos, which modifies strongly the hardware properties of realistic quantum computer [4]. The effects of static imperfections on the accuracy of quantum computation have been investigated on the examples of quantum algorithms for the models of complex quantum dynamics (see e.g. [3] and Refs. therein) and a universal law for fidelity decay induced by static imperfections has been established [3] for quantum algorithms simulating dynamics in the regime of quantum chaos. This law is based on the RMT treatment of imperfections. At the same time it has been realized that the effects of static imperfections for dynamics in an integrable regime are not universal and more complicated. Therefore it is important to investigate the effects of static imperfections on an example of the well known Grover algorithm (GA) [5]. The results of these investigations [6] show that a quantum phase transition to quantum chaos takes place for the imperfection strength

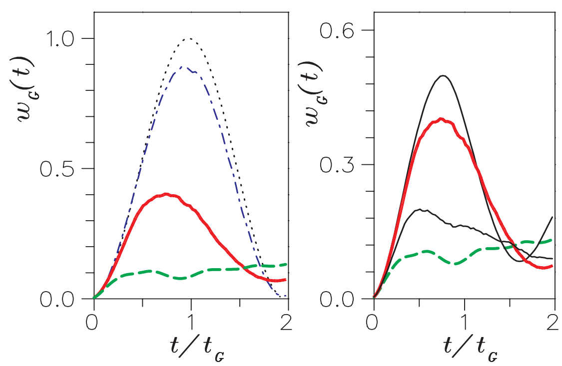

Figure 1: Probability $w_G(t)$ of a searched state in GA as a function of the Grover iteration t at $n_{tot} = 12, \varepsilon = 0.002, t_G = 34.5$. Left: curves show data for ideal GA, GA with gate to gate randomly fluctuating coefficients a_i, b_{ij} (see Eq. (2) in [6]), GYQEC with $l_g = 10$, GA with static imperfections (from top to bottom at $t/t_g = 1$). Right: curves show data for GYQEC at $l_g = 1, 10, 20$ and GA with static imperfections (also from top to bottom).

$\varepsilon > \varepsilon_c \approx 1.7/(n_g\sqrt{n_{tot}})$ where $n_g = O(n_{tot})$ is the total number of quantum gates for one Grover iteration and $n_{tot} = n_q + 1$ is the total number of qubits. Notations and detailed explanations are given in [6]. Here we give the description of the gyroscopic quantum error correction (GYQEC) method allowing significantly suppress static imperfections in GA. For the first time this method is discussed in [7] for the quantum tent map.

GYQEC is based on a random change of numeration of qubits after l_g quantum gates. Namely, after l_g gates about $n_{tot}/2$ swap operations are performed between random pairs of qubits so that the initial numeration of qubits is replaced by completely random one. However, in the quantum computer code this change is taken into account and the algorithm continues to run with new qubit numeration. In a sense the method uses a freedom of numeration of qubits in the program code and makes gyroscopic random rotations between all possibilities.

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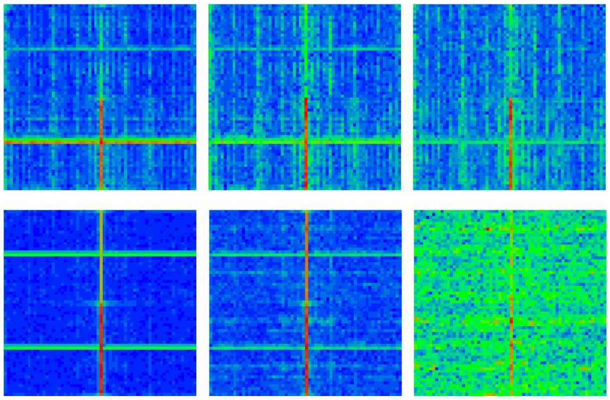

Figure 2: Husimi function in GA (compare with Fig.2 in [6]), shown at moment $t \sim t_G$ when $w_G(t)$ has maximum , for $\varepsilon = 0.002, 0.004, 0.008$ (left to right respectively); $n_{tot} = 12$. Top (bottom) row corresponds to the computation with (without) GYQEC at $l_g = 1$. Density is proportional to color changing from blue/zero to red/maximum.

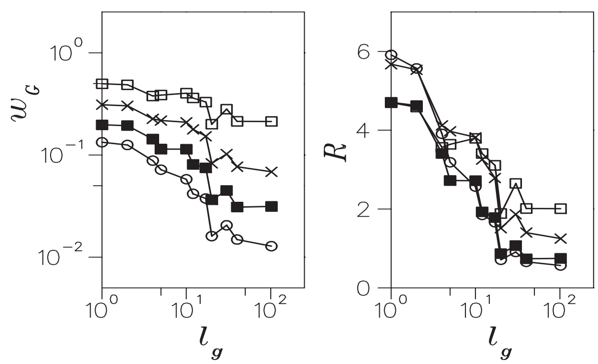

Figure 3: Left: probability w_G, averaged over 10 disorder realizations of static imperfections and taken at maximum, for $\varepsilon = 0.002, 0.003, 0.004, 0.005$, computation with GYQEC at different l_g (top to bottom). Right: the gain factor R given by the ratio of w_G (from left) to its maximum value obtained in computations without GYQEC (same symbols). Here $n_{tot} = 12$.

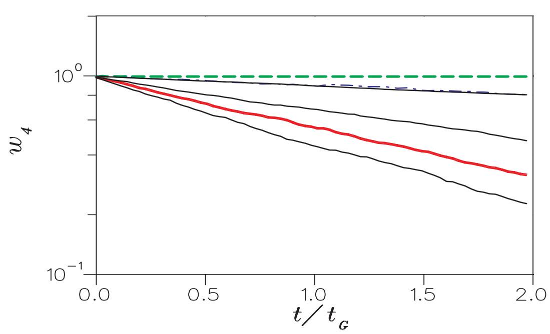

Figure 4: Probability $w_4(t)$ in the four states (see Eq. (3) in [6]) as a function of iteration time t for $n_{tot} = 12, \varepsilon = 0.002, t_G = 34.5$. From top to bottom, curves show data without GYQEC, with time fluctuating couplings, with GYQEC at $l_g = 1$ (practically coincides with the previous curve) and at $l_g = 5, 10, 20$.

These rotations suppress the effects of static imperfections (see Fig. 1). The GA accuracy is impoved with a decrease of l_g even if at $l_g = 1$ the GYQEC is not able to reach the case with randomly time fluctuating couplings between qubits [8]. A pictorial image of the accuracy improvement for the Husimi function in shown in Fig. 2. Indeed, GYQEC gives a significant increase of the probability of searched state corresponding to lower horizontal line in a phase space square in Fig. 2. The variation of the searched probability w_G with l_g is shown in Fig. 3 for various values of ε. GYQEC gives a maximal improvement of accuracy at minimal $l_g = 1$ when the effect of randomization of static imperfections becomes maximal. For $l_g = 1$ and $n_{tot} = 12$ we reach the maximal accuracy gain factor $R \approx 6$ which it is not very sensitive to ε in a certain range. We expect that this R value will grow with the number of qubits n_{tot} since in this case random rotations of computational basis will give stronger randomization of static imperfections. We note that the static imperfections preserve the total probability w_4 in 4-states (see [6]) until $\varepsilon < \varepsilon_c$ while time fluctuations of couplings a_i, b_{ij} and GYQEC method give an exponential time decay of w_4 with a rate $\Gamma \propto \varepsilon^2$ (see Fig. 4). In spite of this decay we obtain the accuracy gain.

In summary, we discussed here a new GYQEC method which performs random rotations in the computational basis of quantum computation code keeping track of these rotations in the quantum algorithm. This method uses a generic property of numeration freedom in the computational basis and allows to suppress significantly the effects of static imperfections. The GYQEC is rather general and can be applied to any quantum algorithm.

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[8] For static imperfections w_G value is by factor 9 smaller compared to the case of time fluctuating couplings (Fig. 1), that's why it's important to suppress static imperfections.