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CAPTURE OF DARK MATTER BY THE SOLAR SYSTEM

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We study the capture of galactic dark matter by the solar system. The effect is due to the gravitational three-body interaction between the sun, one of the planets, and a dark matter particle. The analytical estimate for the capture cross-section is derived and the upper and lower bounds for the total mass of the captured dark matter particles are found. The estimates for their density are less reliable. The most optimistic of them gives an enhancement of dark matter density by about three orders of magnitudes compared to its value in our galaxy. However, even this optimistic value remains below the best present observational upper limits by about two orders of magnitude.

Keywords: Dark matter; solar system; chaotic dynamics; Halley's comet; Kepler map.

1. Introduction

The dark matter (DM) density in our galaxy is about (see e.g. Ref. 1)

$$\rho_g \simeq 4 \cdot 10^{-25}\,\text{g/cm}^3. \tag{1}$$

However, only upper limits on the level of $10^{-19}\,\text{g/cm}^3$ (see below) are known for the density of dark matter particles (DMPs) in the solar system (SS). Meanwhile, information on their density is of great importance for the experiments aimed at the detection of DM.

The question of capture of weakly interacting massive particles (WIMPs) or DMP by the SS was initiated in Refs. 2 and 3. Very recently, these studies have been pushed further by extensive numerical simulations performed in Refs. 4 and 5. Our interest in this problem was aroused by a recent paper,[6] where an estimate is given for the DM density in the SS, as resulting from the gravitational capture of galactic DMPs. According to the conclusions of Ref. 6, the density of the captured DM, for instance at the earth orbit, is about 10^{-20} g/cm^3, only about an order of magnitude below the best upper limits on it.

In the present paper we perform a new analytical analysis of the gravitational capture of galactic DMPs by the SS. According to our results, the increase in the DM density in the SS due to this capture is small, certainly well below 10^{-20} g/cm^3.

2. Dimensional Estimate for the Mass of Captured Dark Matter

The SS is immersed in the halo of DM and moves together with it around the center of our galaxy. To simplify the estimates, we assume that the sun is at rest with respect to the halo. The DM particles in the halo are assumed to have in the reference frame, comoving with the halo, the Maxwell distribution[7]

$$f(v)dv = \sqrt{\frac{54}{\pi}}\frac{v^2dv}{u^3}\exp\left(-\frac{3}{2}\frac{v^2}{u^2}\right), \tag{2}$$

with the local rms velocity $u \simeq 220\,\mathrm{km/s}$.

Let us elucidate what looks to be the most efficient mechanism of the DMP capture. It was pointed out and partly analyzed (though for the capture of comets, and not of DMPs) by Petrosky in Ref. 8 and Chirikov and Vecheslavov in Ref. 9. Of course, a particle cannot be captured by the sun alone. The interaction with a planet is necessary for that; this is essentially a three-body problem of the sun, planet and DMPs. Obviously, the capture is dominated by the particles with orbits close to parabolic ones with respect to the sun, and with the distances between their perihelia and the sun comparable with the radius of planet orbit r_p.

The capture can be effectively described by the so-called restricted three-body problem (see for instance Refs. 10 and 11). In this approach the interaction between two heavy bodies (the sun and a planet in our case) is treated exactly. Also treated exactly is the motion of the third, light body (a DMP in our case) in the gravitational field of the two heavy ones. One neglects, however, the back reaction of a light particle upon the motion of the two heavy bodies. Obviously, this approximation is fully legitimate for our purpose. Still, the restricted three-body problem is rather complicated, and requires in the present case both subtle analytical treatment and serious numerical calculations.[8] Under certain conditions the dynamics of the light particle (e.g. DMP) becomes chaotic.

However, the amount of DM captured by the SS can be found by means of simple estimates. The total mass captured by the sun (its mass is M) together with a planet with mass m_p, during the lifetime

$$T \simeq 4.5 \cdot 10^9\,\text{years} \simeq 10^{17}\,\mathrm{s} \tag{3}$$

of the SS, can be written as follows:

$$\Delta m_p = \rho_g T \langle \sigma v \rangle. \tag{4}$$

Here σ is the capture cross-section. The product σv is averaged over the distribution (2); with all typical velocities in the SS much smaller than u, this distribution simplifies to

$$f(v)dv = \sqrt{\frac{54}{\pi}} \frac{v^2 dv}{u^3}. \tag{5}$$

To estimate the average value $\langle \sigma v \rangle$, we resort to dimensional arguments, supplemented by two rather obvious physical requirements: the masses m_p and M of the two heavy components of our restricted three-body problem should enter the result symmetrically, and the mass m_d of the light component (DMP) should not enter the result at all. Thus, we arrive at

$$\langle \sigma v \rangle \sim \sqrt{54\pi} \frac{k^2 m_p M}{u^3}. \tag{6}$$

Here k is the Newton gravitation constant; an extra power of π, inserted into this expression, is perhaps inherent in σ. The final estimate for the captured mass is

$$\Delta m_p \sim \rho_g T \sqrt{54\pi} \frac{k^2 m_p M}{u^3}. \tag{7}$$

Since the capture would be impossible if the planet were not bound to the sun, it is only natural that the result is proportional to the corresponding effective "coupling constant" $k m_p M$.

Thus obtained values for the masses of DM captured due to the planets of the SS are presented in Table 1. We also quote therein the corresponding results of Ref. 6 for these masses. The disagreement is huge for all planets, especially for the light ones, where it exceeds two orders of magnitude. We cannot spot exactly its origin, since the calculations of Ref. 6 involve rather complex numerical simulations (it is possible that their assumption of capture radius $r_b \sim r_p (m_p/M)^{1/3}$ does not correspond to reality). On the other hand, we cannot see any reasonable possibility for a serious increase of our results. Moreover, in a sense they can be considered as upper limits for the amount of the captured DM, at least because we have neglected here the inverse process, that of the ejection of a captured DMP due to the same three-body gravitational interaction. The result (7) is given for the three-body problem. The dynamical mechanism of capture is described below, in the next section.

The total mass Δm_T of the DM captured by the planets is strongly dominated by the heavy Jovian planets — Jupiter, Saturn, Uranus and Neptune — and constitutes, according to Table 1, about $\Delta m_T \sim 1.5 \cdot 10^{21}$ g. This value is small compared

Table 1. DM mass captured by planets (in g).

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
this work	$0.22 \cdot 10^{18}$	$3.2 \cdot 10^{18}$	$3.9 \cdot 10^{18}$	$0.42 \cdot 10^{18}$	$1239 \cdot 10^{18}$	$372 \cdot 10^{18}$	$57 \cdot 10^{18}$	$67 \cdot 10^{18}$
Ref. 6	$0.42 \cdot 10^{20}$	$3.5 \cdot 10^{20}$	$3.8 \cdot 10^{20}$	$1.2 \cdot 10^{20}$	$49 \cdot 10^{20}$	$28 \cdot 10^{20}$	$12 \cdot 10^{20}$	$16 \cdot 10^{20}$

to the total mass, $\sim 10^{33}$ g, of the common matter in the SS. It is small even when compared to the total noncaptured mass of the DM in the SS: this total mass, calculated with the value (1) for the DM density, constitutes $\sim 10^{31}$ g (we assume here that the effective radius of the SS is about 10^5 a.u.). However, it is an order of magnitude larger than the DM mass of the density (1) inside the radius of the Neptune orbit $r_N \approx 30$ a.u.

The contribution to the discussed effect of the diffuse (nondark) matter in the SS should be significantly smaller, since in a homogeneous dust the gravitational forces acting on DMPs are compensated for.

The dynamical mechanism of capture is described in the next section.

3. Dynamical Approach

For the restricted three-body problem, in a close similarity to the dynamics of comets,[8,9] the DMP dynamics can be described by a symplectic area-preserving map:

$$\bar{w} = w + F(\phi), \quad \bar{\phi} = \phi + 2\pi \bar{w}^{-3/2}. \tag{8}$$

Here $w = -2Er_p/km_dM$ is the energy of the DMP with mass m_d rescaled by its gravitational energy at distance r_p from the sun, ϕ is the phase of the planet on its circular orbit at the moment when the DMP is at the perihelion, and $F(\phi)$ is a certain periodic function of ϕ. Bars denote the new values of variables after one rotation around the sun. The physical meaning of this dynamical map is rather simple: the first equation gives the change of DMP energy after one passage near the sun; the second equation gives the change of the planetary phase between two passages of DMP, and is essentially determined by the Kepler law. The first equation is valid also for scattering particles with positive energy ($w < 0$). Thus, the DMP can be captured by the sun and the planet only if its rescaled energy $|w| < F_{\max}$. After the capture, the DMP dynamics is described by the map (8) until ejection. To compute the captured DM mass Δm_p we assume that, once captured, the DMP remains captured for the whole lifetime T of the SS. In this way we obtain the maximum bound for Δm_p.

The kick function $F(\phi)$ was computed in Ref. 8 for the case where a comet (or DMP) and a planet move in one plane and where the perihelion distance $q > r_p$. In this case $F(\phi) = (m_p/M)\beta(q/r_p)\sin\phi$ and the function $\beta(x) \approx 26\exp(-4x^{3/2}/3\sqrt{2})/x^{1/4}$ so that $\beta(1) \approx 10$. Effectively, the function F is determined by the frequency Fourier component of the force between the planet and the DMP; since the rotation of the planet is rapid compared to the rotation of the DMP, the amplitude of the component is exponentially small for $q \gg r_p$ when the DMP motion is smooth and analytical. In this case β is exponentially small and there is practically no trapping of the DMP. For $q \sim r_p$ the motion is not analytic due to the close passage between the planet and the DMP, and β is relatively large. In this case a DMP with rescaled energies $-w < \beta m_p/M$ can be captured by the planet. It is interesting to note that the map (8) with $F(\phi) \sim \sin\phi$ is known as the Kepler map. It describes the process of microwave ionization of Rydberg atoms and chaotic autoionization of molecular Rydberg states (see Ref. 12 and references therein).

We note that the energy change of the DMP given by $F(\phi)$ results from the integration over the whole orbit rotation of the DMP around the sun, which includes many orbital periods of the planet. Thus, this energy change appears from long-range interaction and has a qualitatively different origin compared to local close collisions between DMP and planet, which were assumed to give the main contribution for DMP energy change in Refs. 2, 3 and 6.

Let us now estimate the capture cross-section σ assuming that for all DMPs the dynamics is described by the Kepler map with fixed $\beta \sim 1$. Then only DMPs with energies $|w| = v^2 r_p / k m_p M = v^2/v_p^2 < \beta m_p / M$ are captured under the condition that $q < r_p$ (here v_p is the velocity of the planet). The value of q can be expressed via the DMP parameters at infinity, where its velocity is v and its impact parameter is r_d, and hence $q = (v r_d)^2 / 2kM$.[13] Since $q \sim r_p$ we obtain the cross-section

$$\sigma \sim \pi r_d^2 \sim \frac{2\pi k M r_p}{v^2} \sim 2\pi r_p^2 \left(\frac{v_p}{v}\right)^2 \sim \frac{2\pi r_p^2 M}{\beta m_p}, \tag{9}$$

where the last relation is taken for those typical velocities, $v^2 \sim \beta v_p^2 m_p / M$, at which the capture of DMPs takes place (for $q \approx 1.4 r_p$ we have $\beta \approx 5$). Then Eqs. (4) and (9) give the captured mass Δm_p of (7) with an additional numerical factor $\beta \sim 1$.

According to the above estimates, DMPs captured by Jupiter have typical velocities at infinity $v \sim (\beta m_p / M)^{1/2} v_p \sim 1\,\mathrm{km/s}$ for typical $\beta \sim 5$ and $m_p/M \approx 10^{-3}$, $v_p \approx 13\,\mathrm{km/s}$. This value of v is in good agreement with the numerical simulations of Ref. 5, which give typical captured DMP velocities for Jupiter of $1\,\mathrm{km/s}$.

Another interesting feature of the analytical expression for the cross-section of captured particles σ (9) is that it is much larger than the area of the planet orbit. In fact, σ diverges at small velocities as $\sigma \sim 1/v^2$ but this divergence is weaker than that of the Rutherford cross-section. In our case of the restricted three-body problem the divergence appears due to the property of the Kepler motion where the DMP distance at perihelion is proportional to the square of the orbital momentum, which in turn is proportional to the product of the velocity v and the impact parameter r_d at infinity. In addition, it is important to use the value of the typical DMP velocity captured by the planet for a perihelion distance of the order of r_p. This leads to the analytical equation (9) for the capture cross-section in the restricted three-body problem.

4. Density of Dark Matter

While the total masses Δm_p of the captured DM can be (hopefully) described by the simple dimensional estimate (7), the situation for the corresponding DM densities $\Delta \rho_p$ is more subtle. The reason is as follows. The captured DMPs had initial trajectories predominantly close to parabolas with respect to the sun, and the velocities of these DMPs change only slightly as a result of scattering. Therefore, it is quite natural that the final, elliptical trajectories of these DMPs have large semimajor axes.

Indeed, DMPs captured into an elliptic trajectory had initially a hyperbolic trajectory, focused at the sun and close to a parabolic one. As a result of the capture, the eccentricity e of the trajectory changes from $e = 1 + \epsilon_1$ to $e = 1 - \epsilon_2$, with $\epsilon = \epsilon_1 + \epsilon_2 \ll 1$. It is quite natural that the final, elliptical trajectories of the captured DMPs have large semimajor axes.

To estimate their typical values, we recall[13] that the radius-vector r of a captured DMP (counted off the sun) is related to the azimuthal angle ϕ as follows:

$$r = \frac{p}{1 + e\cos\phi}, \tag{10}$$

where p is the so-called orbit parameter (its value is irrelevant to our line of reasoning). Obviously, the maximal $r_{\max}$ and minimal distance $r_{\min}$ from the sun correspond to $\cos\phi = \pm 1$, so that their ratio is

$$\frac{r_{\max}}{r_{\min}} = \frac{1+e}{1-e}. \tag{11}$$

In the numerator of this ratio, we can safely put with our accuracy $1 + e \simeq 2$, as it was done previously in Ref. 14 for the problem of accretion on massive black holes. For the denominator of the ratio, we recall that the difference $1 - e$ is related to the gravitational perturbation for planet, and therefore is proportional to m_p. Thus, for dimensional reasons,

$$\frac{r_{\max}}{r_{\min}} \sim \frac{M}{m_p}. \tag{12}$$

The minimal distance between DMPs and the Sun, $r_{\min}$, should be on the same order of magnitude as the radius r_p of the planet orbit. Therefore, the semimajor axis a of the resulting ellipse is huge:

$$r_{\max} \sim r_p \left(\frac{M}{m_p}\right). \tag{13}$$

In particular, in the case of Jupiter our estimate gives $r_a \sim 10^3 r_p$. A similar numerical factor appears in Ref. 8.

According to the numerical calculations of Ref. 8, in the case of Jupiter the values of the semimajor axes r_a for the resulting trajectories belong to the interval 10^3–10^4 au for $q/r_p = 4$–6. The fact that it is comets that are considered in Ref. 8, and not DMPs, is obviously of no importance to this conclusion. The minimum value of r_a is defined by the maximum $w_{\rm ch} \sim r_p/r_a$ value which can be reached by an injected DMP during its chaotic motion. In fact, $w_{\rm ch}$ is the chaos border and according to the Chirikov resonance overlap criterion[15] we have $w_{\rm ch} \approx (3\pi\beta(m_p/M))^{2/5}$, as was shown in Ref. 8. For Jupiter $m_p/M \approx 10^{-3}$ and at $\beta \approx 5$ corresponding to $q \approx 1.4 r_p$ we have $r_a/r_p \approx 1/w_{\rm ch} \approx 3$. We note that this value of β gives the maximum $F_{\max} \approx 0.005$, corresponding to the similar value found for Halley's comet.[9] In fact, the data presented in Ref. 9 show that Halley's comet has the chaos border around $w_{\rm ch} \approx r_p/r_a \approx 0.3$ (see Fig. 3 of Ref. 9).

Of course, the values of r_a linked to the chaos border in w are the minimum ones since during its chaotic dynamics DMPs have also $0 < w \ll w_{\rm ch}$ with larger r_a. However, we are interested in orbits captured for very large times T (3). Such times are two orders of magnitude larger than the typical diffusive lifetime of Halley's comet, found to be of the order of 10^7 years.[9] It is known that chaotic trajectories may be sticking to boundaries of integrable islands for very long times (see Ref. 16 and references therein) and hence we can expect that those orbits will be somewhere in the vicinity of the chaos border around $w_{\rm ch} \sim 1$ with $r_a \sim r_p$. In fact, for the case where the inclination angle between the planes of DMP and planet $\theta_i > 0$ and where $q < r_p$, the function $F(\phi)$ contains higher harmonics of ϕ (see the case of Halley's comet in Ref. 9). This leads to easier emergence of chaos so that even for light planets one may have the chaos border $w_{\rm ch} \sim 1$.

Therefore, we can make an assumption resulting in the most optimistic prediction for the "partial" DM densities $\Delta\rho_p$. We assume that each of the total masses Δm_p of the captured DM occupies the volume $(4\pi/3)r_p^3$, where r_p is the orbit radius of the corresponding planet. We do not claim that this assumption is correct, but believe that comparison of its (almost certainly overoptimistic) results with the observational limits will be instructive. The corresponding values of the "partial" DM densities $\Delta\rho_p = \Delta m_p/(4\pi r_p^2/3)$ (in g/cm^3) are presented in Table 2. We omit from it the densities due to Uranus and Neptune, tiny even at the discussed scale. Then, in accordance with the accepted model, the total DM density $\rho_{\rm DM}$ at a given radius does not coincide with the corresponding $\Delta\rho_p$. It includes, in line with it, the sum of the contributions to the density due to all the planets, outer with respect to the given one.

5. Ergodic Time Scale

The estimates given above neglect ejection of DMPs from the SS. Such an assumption is not justified if the DMP dynamics in the SS becomes completely ergodic on a time scale $T_e \ll T$. Then after the time T_e the detailed balance principle becomes valid and the density of the captured DMP becomes its galactic density, as was argued in Ref. 2 (see also the discussion in Refs. 3–5). However, the estimate of T_e has certain subtle points. In the frame of the map (8) it is given by the diffusion time from $w = 0$ to $w = w_{\rm ch}$. For the Kepler map the diffusion coefficient is $D \approx \beta^2(m_p/M)^2/2$ and hence $T_e \sim 2(Mw_{\rm ch}/\beta m_p)^2 T_d$, where T_d is an average

Table 2. Density of DM for planets (in g/cm^3).

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
$\Delta\rho_p$	$2.7\cdot10^{-22}$	$6.0\cdot10^{-22}$	$2.7\cdot10^{-22}$	$8.4\cdot10^{-24}$	$6.2\cdot10^{-22}$	$3.0\cdot10^{-23}$
$\rho_{\rm DM}$	$1.8\cdot10^{-21}$	$1.5\cdot10^{-21}$	$9.3\cdot10^{-22}$	$6.6\cdot10^{-22}$	$6.5\cdot10^{-22}$	$3.0\cdot10^{-23}$

period of the DMP. For the case of Jupiter such an estimate gives a satisfactory value of $T_e \sim 10^7$ years for the case of Halley's comet, as is discussed in Ref. 9. However, β sharply decreases with the increase in the perihelion distance $q \propto \ell^2$, where ℓ is the orbital momentum of the DMP. As a result, a growth of ℓ can give a sharp increase of T_e which can become comparable with T. The effects linked to variations of ℓ were not considered in Ref. 9. They are properly treated in the numerical simulations of Refs. 4 and 5 but there only the effect of Jupiter is considered. Other planets and fluctuating galactic fields can give stronger growth of ℓ with a significant increase of T_e. Indeed, from the studies of Rydberg atoms in a microwave field it is known that time-oscillating space homogeneous fields can produce strong variation of the eccentricity of orbits (see Ref. 17 and Fig. 11 there). Also, it is known that noise generates penetration of chaotic trajectories inside integrable islands and very slow decay of Poincaré recurrences with diverging trapping time.[18] In addition to that, recent large scale numerical simulations of Ref. 19 show significant changes in the eccentricity of planets on a time scale of the order of T. Therefore, the question of the T_e value for DMPs captured by the SS requires further study. One does not rule out the possibility that it is comparable to or even larger than T. In such a situation the upper bound (7) will be close to the real value of the total captured mass.

In any case it is clear that there are practically no ejections of captured particles on a time scale of the DMP orbital period T_c. A typical captured DMP rescaled energy is $w_c \sim \beta m_p/M$, corresponding to one iteration of the map (8), which gives a change of w from negative to positive values. The rotation period of such DMPs is rather large compared to the period T_p of the planet: $T_c/T_p \sim (\beta m_p/M)^{-3/2}$. For the case of Jupiter $T_p \approx 11$ years and at $\beta \sim 1$ we have $T_c \sim 3 \cdot 10^4 T_p \sim 3 \cdot 10^5$ years. This is a factor of 10^4 shorter than the SS lifetime T. This gives the lower bound of the captured DM mass, which is obtained with replacing T with T_c in Eq. (7).

6. Observational Upper Limit on the Density of Dark Matter

Lastly, let us consider the observational data on the DM in the SS. The most reliable and accurate information on it follows from the studies of the perihelion precession of Venus, Earth and Mars. Under the assumptions that the DM density ρ_{DM} is distributed in a spherically symmetric manner with respect to the sun and that the eccentricity of the planetary orbit is small, the corresponding relative shift of the perihelion per period is (see Ref. 20 and references therein)

$$\frac{\delta\phi}{2\pi} = -\frac{2\pi\rho_{\mathrm{DM}} r^3}{M}, \tag{14}$$

where r is the radius of the orbit. This relation becomes almost obvious (up to an overall numerical factor) if one recalls that, in virtue of the Gauss theorem, for a spherically symmetric density $\rho(r)$ the action of the DM inside the orbit reduces to that of a pointlike mass, and therefore does not induce the perihelion precession.

Table 3. Angle of perihelion precession (in seconds per century).

Planet	Venus	Earth	Mars
$\delta\phi_{\rm th}$	8.6248	3.8388	1.3510
$\delta\phi_{\rm obs}$	8.6247 ± 0.0002	3.8390 ± 0.0003	1.3512 ± 0.0003

On the other hand, for such a density $\rho(r)$, the DM outside the orbit does not influence at all the motion of a planet.

The recent, most precise observational data[21] on the precession of perihelia are presented in Table 3 (therein the theoretical values $\delta\phi_{\rm th}$ of the perihelion rotation and the results of observations $\delta\phi_{\rm obs}$ are given in angular seconds per century). With these data, one arrives at the upper limits on the DM density at the distances from the sun, corresponding to the orbit radii of Venus, Earth and Mars, on the level of

$$\rho_{\rm DM} < 2 \cdot 10^{-19}\,{\rm g/cm}^3. \tag{15}$$

This observational upper limit exceeds by about two orders of magnitude the results (almost certainly overestimated) presented in Table 2.

7. Summary

Our results do not mean, however, that the searches for the dark matter in the solar system are senseless. Of course, the capture of the galactic DM analyzed here is not the only conceivable source of the DM in the SS. It is quite possible in particular that the SS itself has arisen due to a local high-density fluctuation of the DM.

Now, on the related theoretical problems. To obtain more firm results for the captured DM mass and density, one needs to take into account the fact that the kick function $F(\phi)$ in the map (8) depends on an inclination angle between planes of DMPs and planet orbits and also on the DMP perihelion distance. However, a typical case of Halley's comet analyzed by Chirikov and Vecheslavov in Ref. 9 gives the map function of a form similar to that discussed here, so that the estimates presented should also be applicable for such more general DMP orbits. Further analytical and numerical studies are required for a better understanding of DMP dynamics inside the solar system.

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