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CAPTURE OF DARK MATTER BY THE SOLAR SYSTEM

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We study the capture of galactic dark matter by the Solar System. The effect is due to the gravitational three-body interaction between the Sun, one of the planets, and a dark matter particle. The analytical estimate for the capture cross-section is derived and the upper and lower bounds for the total mass of the captured dark matter particles are found. The estimates for their density are less reliable. The most optimistic of them give an enhancement of dark matter density by about three orders of magnitudes compared to its value in our Galaxy. However, even this optimistic value remains below the best present observational upper limits by about two orders of magnitude.

Keywords: dark matter, Solar System, chaotic dynamics, comet Halley, Kepler map

1. Introduction

The Dark Matter (DM) density in our Galaxy is about (see e.g. Ref. 1):

$$\rho_g \simeq 4 \cdot 10^{-25} \text{ g/cm}^3. \quad (1)$$

However, only upper limits on the level of 10^{-19} g/cm^3 (see below) are known for the density of Dark Matter Particles (DMP) in the Solar System (SS). Meanwhile, information on their density is of great importance for the experiments aimed at the detection of DM.

The question of capture of Weakly Interacting Massive Particles (WIMP) or DMP by the SS was pioneered in Refs. 2, 3. Very recently these studies are pushed further by extensive numerical simulations performed in Refs. 4, 5. Our interest to this problem was attracted by a recent Ref. 6 where an estimate is given for the DM density in SS, as resulting from the gravitational capture of galactic DMP's. According to the conclusions of Ref. 6, the density of the captured DM, for instance,

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at the Earth orbit is about 10^{-20} g/cm³, only about an order of magnitude below the best upper limits on it.

In the present note we perform new analytical analysis of the gravitational capture of galactic DMP's by the SS. According to our results, the increase of the DM density in the SS due to this capture is small, certainly well below 10^{-20} g/cm³.

2. Dimensional estimate for mass of captured dark matter

The Solar System is immersed in the halo of dark matter and moves together with it around the center of our Galaxy. To simplify the estimates, we assume that the Sun is at rest with respect to the halo. The dark matter particles in the halo are assumed to have in the reference frame, co-moving with the halo, the Maxwell distribution (see Ref. 7):

$$f(v) dv = \sqrt{\frac{54}{\pi}} \frac{v^2 dv}{u^3} \exp\left(-\frac{3}{2} \frac{v^2}{u^2}\right), \quad (2)$$

with the local rms velocity $u \simeq 220$ km/s.

Let us elucidate what looks to be the most efficient mechanism of the DMP capture. It was pointed out and partly analyzed (though for the capture of comets, but not of DMP's), by Petrosky in Ref. 8 and Chirikov and Vecheslavov in Ref. 9. Of course, a particle cannot be captured by the Sun alone. The interaction with a planet is necessary for it, this is essentially a three-body problem of the Sun, planet and DMP. Obviously, the capture is dominated by the particles with orbits close to parabolic ones with respect to the Sun, and with the distances between their perihelia and the Sun comparable with the radius of planet orbit r_p .

The capture can be effectively described by the so-called restricted three-body problem (see for instance Refs. 10, 11). In this approach the interaction between two heavy bodies (the Sun and a planet in our case) is treated exactly. As exactly is treated the motion of the third, light body (a DMP in our case) in the gravitational field of the two heavy ones. One neglects however the back reaction of a light particle upon the motion of the two heavy bodies. Obviously, this approximation is fully legitimate for our purpose. Still, the restricted three-body problem is rather complicated, and requires in the present case both subtle analytical treatment and serious numerical calculations (see Ref. 8). Under certain conditions the dynamics of light particle (e.g. DPM) becomes chaotic.

However, the amount of DM captured by the SS can be found by means of simple estimates. The total mass captured by the Sun (its mass is M) together with a planet with mass m_p , during the lifetime

$$T \simeq 4.5 \cdot 10^9 \text{ years} \simeq 10^{17} \text{ s} \quad (3)$$

of the SS, can be written as follows:

$$\Delta m_p = \rho_g T \langle \sigma v \rangle; \quad (4)$$

here σ is the capture cross-section. The product σv is averaged over distribution (2); with all typical velocities in the SS much smaller than u , this distribution simplifies to

$$f(v) dv = \sqrt{\frac{54}{\pi}} \frac{v^2 dv}{u^3}. \quad (5)$$

To estimate the average value $\langle \sigma v \rangle$, we resort to dimensional arguments, supplemented by two rather obvious physical requirements: the masses m_p and M of the two heavy components of our restricted three-body problem should enter the result symmetrically, and the mass m_d of the light component (DMP) should not enter the result at all. Thus, we arrive at

$$\langle \sigma v \rangle \sim \sqrt{54\pi} \frac{k^2 m_p M}{u^3}; \quad (6)$$

here k is the Newton gravitation constant; an extra power of π , inserted into this expression, is perhaps inherent in σ . The final estimate for the captured mass is

$$\Delta m_p \sim \rho_g T \sqrt{54\pi} \frac{k^2 m_p M}{u^3}. \quad (7)$$

Since the capture would be impossible if the planet were not bound to the Sun, it is only natural that the result is proportional to the corresponding effective "coupling constant" $km_p M$.

Thus obtained values for the masses of DM captured due to the planets of the SS, are presented in Table 1. We quote also therein the corresponding results of Ref. 6 for these masses. The disagreement is huge for all planets, and especially for the light ones where it exceeds two orders of magnitude. We cannot spot exactly its origin since the calculations of Ref. 6 involve rather complex numerical simulations (it is possible that their assumption of capture radius $r_b \sim r_p (m_p/M)^{1/3}$ does not correspond to reality). On the other hand, however, we cannot see any reasonable possibility for a serious increase of our results. Moreover, in a sense they can be considered as upper limits for the amount of the captured DM, at least because we have neglected here the inverse process, that of the ejection of a captured DMP due to the same three-body gravitational interaction. The result (7) is given for the three body problem. The dynamical mechanism of capture is described below in next Section. The contribution of the diffusive (non-dark) matter in the SS should be significantly smaller since a homogeneously distributed dust gives compensation of gravitational forces acting on DMP.

The total mass Δm_T of the DM captured by the planets is strongly dominated by the heavy Jovian planets, Jupiter, Saturn, Uranus, Neptune, and constitutes according to Table 1 about $\Delta m_T \sim 1.5 \cdot 10^{21}$ g. This value is small as compared to the total mass $\sim 10^{33}$ g of the common matter in the SS. It is small even as compared to the total non-captured mass of the DM in the SS: this total mass, calculated with value (1) for the DM density, constitutes $\sim 10^{31}$ g (we assume here that the effective radius of the SS is about 10^5 au). However, it is an order

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of magnitude larger than the DM mass of density (1) inside the radius of Neptune orbit $r_N \approx 30$ au.

The contribution to the discussed effect of the diffuse (non-dark) matter in the SS should be significantly smaller since in a homogeneous dust the gravitational forces acting on DMP are compensated.

The dynamical mechanism of capture is described in the next Section.

Table 1. DM mass captured by planets (in g)

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
this work	$0.22 \cdot 10^{18}$	$3.2 \cdot 10^{18}$	$3.9 \cdot 10^{18}$	$0.42 \cdot 10^{18}$	$1239 \cdot 10^{18}$	$372 \cdot 10^{18}$	$57 \cdot 10^{18}$	$67 \cdot 10^{18}$
Xu, Siegel Ref. 6	$0.42 \cdot 10^{20}$	$3.5 \cdot 10^{20}$	$3.8 \cdot 10^{20}$	$1.2 \cdot 10^{20}$	$49 \cdot 10^{20}$	$28 \cdot 10^{20}$	$12 \cdot 10^{20}$	$16 \cdot 10^{20}$

3. Dynamical approach

For the restricted three-body problem, in a close similarity to dynamics of comets (see Refs. 8, 9), the DMP dynamics can be described by a symplectic area-preserving map

$$\bar{w} = w + F(\phi) \quad , \quad \bar{\phi} = \phi + 2\pi\bar{w}^{-3/2} \quad . \quad (8)$$

Here $w = -2Er_p/km_dM$ is the energy of DMP with mass m_d rescaled by its gravitational energy on distance r_p from the Sun; ϕ is the phase of the planet on its circular orbit at the moment when the DMP is at the perihelion and $F(\phi)$ is a certain periodic function of ϕ . Bars denote the new values of variables after one rotation around the Sun. The physical meaning of this dynamical map is rather simple: the first equation gives the change of DMP energy after one passage near the Sun, the second equation gives the change of the planetary phase between two passages of DMP and is essentially determined by the Kepler law. The first equation is valid also for scattering particles with positive energy ($w < 0$). Thus DMP can be captured by the Sun and the planet only if its rescaled energy $|w| < F_{max}$. After the capture, the DMP dynamics is described by map (8) until ejection. To compute the captured DM mass Δm_p we assume that, after being once captured, the DMP remains captured for the whole life time T of SS. In this way we obtain the maximum bound for Δm_p .

The kick function $F(\phi)$ was computed in Ref. 8 for the case when a comet (or DMP) and a planet move in one plane and when the perihelion distance $q > r_p$. In this case $F(\phi) = (m_p/M)\beta(q/r_p)\sin\phi$ and the function $\beta(x) \approx 26 \exp(-4x^{3/2}/3\sqrt{2})/x^{1/4}$ so that $\beta(1) \approx 10$. Effectively the function F is determined by the frequency Fourier component of the force between the planet and DMP, since the rotation of the planet is rapid compared to the rotation of DMP the amplitude of the component is exponentially small for $q \gg r_p$ when the DMP motion is smooth and analytical. In this case β is exponentially small and there is

practically no trapping of DMP. For $q \sim r_p$ the motion is not analytic due to close passage between the planet and DMP and β is relatively large. In this case a DMP with rescaled energies $-w < \beta m_p/M$ can be captured by the planet. It is interesting to note that the map (8) with $F(\phi) \sim \sin \phi$ is known as the Kepler map. It describes the process of microwave ionization of Rydberg atoms and chaotic autoionization of molecular Rydberg states (see Ref. 12 and Refs. therein).

We note that the energy change of DMP given by $F(\phi)$ results from the integration over the whole orbit rotation of DMP around the Sun which includes many orbital periods of the planet. Thus this energy change appears from long-range interaction and has qualitatively different origin compared to local close collisions between DMP and planet which were assumed to give the main contribution for DMP energy change in Refs. 2, 3, 6.

Let us now estimate the capture cross-section σ assuming that for all DMP the dynamics is described by the Kepler map with fixed $\beta \sim 1$. Then only DMP with energies $|w| = v^2 r_p / k m_p M = v^2 / v_p^2 < \beta m_p / M$ are captured under the condition that $q < r_p$ (here v_p is the velocity of the planet). The value of q can be expressed via the DMP parameters at infinity where its velocity is v and its impact parameter is r_d and hence $q = (v r_d)^2 / 2kM$ (see Ref. 13). Since $q \sim r_p$ we obtain the cross-section

$$\sigma \sim \pi r_d^2 \sim 2\pi k M r_p / v^2 \sim 2\pi r_p^2 (v_p / v)^2 \sim 2\pi r_p^2 M / (\beta m_p), \quad (9)$$

where the last relation is taken for those typical velocities $v^2 \sim \beta v_p^2 m_p / M$ at which the capture of DMP takes place (for $q \approx 1.4 r_p$ we have $\beta \approx 5$). Then Eqs. (4), (9) give the captured mass Δm_p of (7) with an additional numerical factor $\beta \sim 1$.

According the above estimates DPM captured by Jupiter have typical velocities at infinity $v \sim (\beta m_p / M)^{1/2} v_p \sim 1 \text{ km/s}$ for typical $\beta \sim 5$ and $m_p / M \approx 10^{-3}$, $v_p \approx 13 \text{ km/s}$. This value of v is in a good agreement with the numerical simulations of Peter Ref. 5 which give typical captured DPM velocities for Jupiter of 1 km/s .

 Table 2. Density of DM for planets (in g/cm^3)

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
$\Delta \rho_p$	$2.7 \cdot 10^{-22}$	$6.0 \cdot 10^{-22}$	$2.7 \cdot 10^{-22}$	$8.4 \cdot 10^{-24}$	$6.2 \cdot 10^{-22}$	$3.0 \cdot 10^{-23}$
ρ_{DM}	$1.8 \cdot 10^{-21}$	$1.5 \cdot 10^{-21}$	$9.3 \cdot 10^{-22}$	$6.6 \cdot 10^{-22}$	$6.5 \cdot 10^{-22}$	$3.0 \cdot 10^{-23}$

Table 3. Angle of perihelia precession (in seconds per century)

Planet	Venus	Earth	Mars
$\delta \phi_{th}$	8.6248	3.8388	1.3510
$\delta \phi_{obs}$	8.6247 ± 0.0002	3.8390 ± 0.0003	1.3512 ± 0.0003

Another interesting feature of the analytical expression for the cross-section

of captured particles σ (9) is that it is much larger than the area of the planet orbit. In fact, σ diverges at small velocities as $\sigma \sim 1/v^2$ but this divergence is weaker compared to the divergence of the Rutherford cross-section. In our case of the restricted three-body problem the divergence appears due to the property of the Kepler motion where the DMP distance at perihelion is proportional to square of the orbital momentum which in its turn is proportional to the product of the velocity v and impact parameter r_d at infinity. In addition it is important to use the value of typical DMP velocity captured by the planet for perihelion distance of the order of r_p . This leads to the analytical equation (9) for the capture cross-section in the restricted three-body problem.

4. Density of dark matter

While the total masses Δm_p of the captured DM can be (hopefully) described by simple dimensional estimate (7), the situation for the corresponding DM densities $\Delta \rho_p$ is more subtle. The reason is as follows. The captured DMP's had initial trajectories predominantly close to parabolas with respect to the Sun, and the velocities of these DMP's change only slightly as a result of scattering. Therefore, it is quite natural that the final, elliptical trajectories of these DMP's have large semi-major axes.

Indeed, DMP captured into an elliptic trajectory had initially a hyperbolic trajectory, focussed at the Sun and close to a parabolic one. As a result of the capture, the eccentricity e of the trajectory changes from $e = 1 + \epsilon_1$ to $e = 1 - \epsilon_2$, with $\epsilon = \epsilon_1 + \epsilon_2 \ll 1$. It is quite natural that the final, elliptical trajectories of the captured DMP's have large semi-major axes.

To estimate their typical values, we recall (see Ref. 13) that the radius-vector r of a captured DMP (counted off the Sun) is related to the azimuthal angle ϕ as follows:

$$r = p / (1 + e \cos \phi), \quad (10)$$

where p is the so-called orbit parameter (its value is irrelevant for our line of reasoning). Obviously, the maximal r_{max} and minimal distance r_{min} from the Sun correspond to $\cos \phi = \pm 1$, so that their ratio is

$$r_{max}/r_{min} = (1 + e)/(1 - e). \quad (11)$$

In the numerator of this ratio, we can safely put with our accuracy $1 + e \simeq 2$. As to its denominator, we recall that the difference $1 - e$ is related to the gravitational perturbation by planet, and therefore is proportional to m_p . Thus, by dimensional reasons,

$$r_{max}/r_{min} \sim M/m_p. \quad (12)$$

The minimal distance between DMP and the Sun r_{min} should be on the same order of magnitude as the radius r_p of the planet orbit. Therefore, the semi-major

axis a of resulting ellipse is huge:

$$r_{max} \sim r_p (M/m_p). \quad (13)$$

In particular, in the case of Jupiter our estimate gives $r_a \sim 10^3 r_p$. A similar numerical factor appears in Ref. 8.

According to the numerical calculations of Ref. 8, in the case of Jupiter the values of the semi-major axes r_a for the resulting trajectories belong to the interval $10^3 - 10^4$ au for $q/r_p = 4 - 6$. The fact that it is comets that are considered in Ref. 8, but not DMP's, is obviously of no importance for this conclusion. The minimum value of r_a is defined by the maximum $w_{ch} \sim r_p/r_a$ value which can be reached by an injected DMP during its chaotic motion. In fact w_{ch} is the chaos border and according to the Chirikov resonance-overlap criterion (see Ref. 14) we have $w_{ch} \approx (3\pi\beta(m_p/M))^{2/5}$ as it was shown in Ref. 8. For Jupiter $m_p/M \approx 10^{-3}$ and at $\beta \approx 5$ corresponding to $q \approx 1.4r_p$ we have $r_a/r_p \approx 1/w_{ch} \approx 3$. We note that this value of β gives the maximum $F_{max} \approx 0.005$ corresponding to the similar value found for the comet Halley (see Ref. 9). In fact, the data presented in Ref. 9 show that the comet Halley have the chaos border around $w_{ch} \approx r_p/r_a \approx 0.3$ (see Fig.3 in Ref. 9).

Of course the values of r_a linked to the chaos border in w are the minimum ones since during its chaotic dynamics DMP have also $0 < w \ll w_{ch}$ with larger r_a . However, we are interested in orbits captured for very large times T (3). Such times are by two orders of magnitude larger than a typical diffusive life time of comet Halley found to be of the order of 10^7 years (see Ref. 9). It is known that chaotic trajectories may be sticking to boundaries of integrable islands for very long times (see Ref. 15 and Refs. therein) and hence we can expect that those orbits will be somewhere in vicinity of the chaos border around $w_{ch} \sim 1$ with $r_a \sim r_p$. In fact, for the case when the inclination angle between the planes of DMP and planet $\theta_i > 0$ and when $q < r_p$ the function $F(\phi)$ contains higher harmonics of ϕ (see the case of the comet Halley in Ref. 9). This leads to easier emergence of chaos so that even for light planets one may have the chaos border $w_{ch} \sim 1$.

Therefore we can make an assumption resulting in the most optimistic prediction for the "partial" dark matter densities $\Delta\rho_p$. We assume that each of the total masses Δm_p of the captured DM occupies the volume $(4\pi/3)r_p^3$ where r_p is the orbit radius of the corresponding planet. We do not claim that this assumption is correct, but believe, however, that the comparison of its (almost certainly, overoptimistic) results with the observational limits will be instructive. The corresponding values of the "partial" dark matter densities $\Delta\rho_p = \Delta m_p/(4\pi r_p^3/3)$ (in g/cm^3) are presented in Table 2. We omit in it the densities due to Uranus and Neptune, tiny even at the discussed scale. Then, in accordance with the accepted model, the total dark matter density ρ_{DM} at a given radius does not coincide with the corresponding $\Delta\rho_p$. It includes, in line with it, the sum of the contributions to the density due to all the planets, outer with respect to the given one.

5. Ergodic time scale

The estimates given above neglect ejection of DMP from the SS. Such an assumption is not justified if the DMP dynamics in SS becomes completely ergodic on a time scale $T_e \ll T$. Then after the time T_e the detailed balance principle becomes valid and the density of captured DMP becomes as its galactic density as it was argued in Ref. 2 (see also discussion in Refs. 3, 4, 5). However, the estimate of T_e have certain subtle points. In the frame of the map (8) it is given by the diffusion time from $w = 0$ to $w = w_{ch}$. For the Kepler map the diffusion coefficient is $D \approx \beta^2(m_p/M)^2/2$ and hence $T_e \sim 2(Mw_{ch}/\beta m_p)^2 T_d$ where T_d is an average period of DMP. For the case of Jupiter such an estimate gives a satisfactory value of $T_e \sim 10^7$ years for the case of the comet Halley as it is discussed in Ref. 9. However, β sharply decreases with the increase of perihelion distance $q \propto \ell^2$, where ℓ is the orbital momentum of DMP. As a result a growth of ℓ can give sharp increase of T_e which can become comparable with T . The effects linked to variations of ℓ were not considered in Ref. 9. They are properly treated in the numerical simulations of Refs. 4, 5 but there only the effect of Jupiter is considered. Other planets and fluctuating galactic fields can give stronger growth of ℓ with a significant increase of T_e . Indeed, from the studies of Rydberg atoms in a microwave field it is known that time-oscillating space homogeneous fields can produce strong variation of eccentricity of orbits (see Ref. 16 and Fig. 11 there). Also it is known that noise generates penetration of chaotic trajectories inside integrable islands and very slow decay of Poincaré recurrences with diverging trapping time (see Ref. 17). In addition to that recent large scale numerical simulations of Ref. 18 show significant changes of eccentricity of planets on a time scale of the order of T . Therefore the question about T_e value for DMP captured by the SS requires further studies. It is not excluded that it is comparable or even larger than T . In such a situation the upper bound (7) will be close to the real value of total captured mass.

In any case it is clear that there is practically no ejections of captured particles on a time scale of DMP orbital period T_c . A typical captured DMP rescaled energy is $w_c \sim \beta m_p/M$ corresponding to one iteration of the map (8) which gives a change of w from negative to positive values. The rotation period of such DMP is rather large compared to the period T_p of the planet: $T_c/T_p \sim (\beta m_p/M)^{-3/2}$. For the case of Jupiter $T_p \approx 11$ years and at $\beta \sim 1$ we have $T_c \sim 3 \cdot 10^4 T_p \sim 3 \cdot 10^5$ years. This is by a factor 10^4 shorter than the SS life time T . This gives the lower bound of the captured DM mass which is obtained by replacing T by T_c in the equation (7).

6. Observational upper limit on the density of dark matter

At last, let us consider the observational data on the DM in our SS. The most reliable and accurate information on it follows from the studies of the perihelion precession of Venus, Earth, and Mars. Under the assumptions that the DM density ρ_{DM} is distributed spherically symmetric with respect to the Sun and that the eccentricity of the planetary orbit is small, the corresponding relative shift of the perihelion per

period is (see Ref. 19 and Refs. therein):

$$\frac{\delta\phi}{2\pi} = -\frac{2\pi\rho_{\text{DM}}r^3}{M}, \quad (14)$$

where r is the radius of the orbit. This relation gets almost obvious (up to an overall numerical factor) if one recalls that, in virtue of the Gauss theorem, for a spherically symmetric density $\rho(r)$ the action of the DM inside the orbit reduces to that of a point-like mass, and therefore does not induce the perihelion precession. On the other hand, for such density $\rho(r)$, the DM outside the orbit does not influence at all the motion of a planet.

The recent, most precise observational data (see Ref. 20) on the precession of perihelia are presented in Table 3 (therein the theoretical values $\delta\phi_{\text{th}}$ of the perihelion rotation and the results of observations $\delta\phi_{\text{obs}}$ are given in angular seconds per century). With these data, one arrives at the upper limits on the DM density at the distances from the Sun, corresponding to the orbit radii of Venus, Earth, and Mars, on the level of

$$\rho_{\text{DM}} < 2 \cdot 10^{-19} \text{ g/cm}^3. \quad (15)$$

This observational upper limit exceeds by about two orders of magnitude the results (almost certainly overestimated) presented in Table 2.

7. Summary

Our results do not mean, however, that the searches for the dark matter in the SS are senseless. Of course, the capture of the Galactic DM analyzed here is not the only conceivable source of the dark matter in the Solar System. It is quite possible in particular that the Solar System itself has arisen due to a local high-density fluctuation of the dark matter.

Now on the related theoretical problems. To obtain more firm results for the captured DM mass and density one needs to take into account that the kick function $F(\phi)$ in the map (8) depends on an inclination angle between planes of DMP and planet orbits and also on the DMP perihelion distance. However, a typical case of the comet Halley analyzed by Chirikov and Vechev in Ref. 9 gives the map function of a form similar to one discussed here so that the estimates presented should be also applicable for such more general DMP orbits. Further analytical and numerical studies are required for a better understanding of DMP dynamics inside the Solar System.

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