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Google matrix analysis of the multiproduct world trade network

Leonardo Ermann[1] and Dima L. Shepelyansky[2,a]

[1] Departamento de Física Teórica, GIyA, CNEA, Av. Libertador 8250, C1429BNP Buenos Aires, Argentina
[2] Laboratoire de Physique Théorique du CNRS, IRSAMC, Université de Toulouse, UPS, 31062 Toulouse, France

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Abstract. Using the United Nations COMTRADE database [United Nations Commodity Trade Statistics Database, available at: `http://comtrade.un.org/db/`. Accessed November (2014)] we construct the Google matrix G of multiproduct world trade between the UN countries and analyze the properties of trade flows on this network for years 1962−2010. This construction, based on Markov chains, treats all countries on equal democratic grounds independently of their richness and at the same time it considers the contributions of trade products proportionally to their trade volume. We consider the trade with 61 products for up to 227 countries. The obtained results show that the trade contribution of products is asymmetric: some of them are export oriented while others are import oriented even if the ranking by their trade volume is symmetric in respect to export and import after averaging over all world countries. The construction of the Google matrix allows to investigate the sensitivity of trade balance in respect to price variations of products, e.g. petroleum and gas, taking into account the world connectivity of trade links. The trade balance based on PageRank and CheiRank probabilities highlights the leading role of China and other BRICS countries in the world trade in recent years. We also show that the eigenstates of G with large eigenvalues select specific trade communities.

1 Introduction

According to the data of UN COMTRADE [1] and the international trade statistics 2014 of the World Trade Organization (WTO) [2] the international world trade between world countries demonstrates a spectacular growth with an increasing trade volume and number of trade products. It is well clear that the world trade plays the fundamental role in the development of world economy [3]. According to the WTO Chief Statistician Hubert Escaith "In recent years we have seen growing demand for data on the world economy and on international trade in particular. This demand has grown in particular since the 2008−2009 crisis, whose depth and breadth surprised many experts" [2]. In global the data of the world trade exchange can be viewed as a large multi-functional directed World Trade Network (WTN) which provides important information about multiproduct commercial flows between countries for a given year. At present the COMTRADE database contains data for $N_c = 227$ UN countries with up to $N_p \approx 10^4$ trade products. Thus the whole matrix of these directed trade flows has a rather large size $N = N_pN_c \sim 10^6$. A usual approach is to consider the export and import volumes, expressed in US dollars (USD). An example of the world map of countries characterized by their import and export trade volume for year 2008 is shown in Figure 1. However, such an approach gives only an approximate description of trade where hidden links and interactions between certain countries and products are not taken into account since only a country global import or export are considered. Thus the statistical analysis of these multiproduct trade data requires a utilization of more advanced mathematical and numerical methods.

In fact, in the last decade, modern societies developed enormous communication and social networks including the World Wide Web (WWW), Wikipedia, Twitter, etc. (see e.g. [4]). A necessity of information retrieval from such networks led to a development of efficient algorithms for information analysis on such networks appeared in computer science. One of the most spectacular tools is the PageRank algorithm developed by Brin and Page in 1998 [5], which became a mathematical foundation of the Google search engine (see e.g. [6]). This algorithm is based on the concept of Markov chains and a construction of the Google matrix G of Markov transitions between network nodes. The right eigenvector of this matrix G, known as PageRank vector, allows to rank all nodes according to their importance and influence on the network. The studies of various directed networks showed that it is useful to analyze also the matrix G^* constructed for the same network but with an inverted direction of links [7,8]. The PageRank vector of G^* is known as the CheiRank vector. The spectral properties of Google matrix for various networks are described in reference [9].

[a] e-mail: `dima@irsamc.ups-tlse.fr`

Table 1. Codes and names of the 61 products from COMTRADE Standard International Trade Classification (SITC) Rev. 1.

Code	Name	Code	Name
00	Live animals	54	Medicinal and pharmaceutical products
01	Meat and meat preparations	55	Perfume materials, toilet & cleansing preptions
02	Dairy products and eggs	56	Fertilizers, manufactured
03	Fish and fish preparations	57	Explosives and pyrotechnic products
04	Cereals and cereal preparations	58	Plastic materials, etc.
05	Fruit and vegetables	59	Chemical materials and products, nes
06	Sugar, sugar preparations and honey	61	Leather, lthr. Manufs., nes & dressed fur skins
07	Coffee, tea, cocoa, spices & manufacs. Thereof	62	Rubber manufactures, nes
08	Feed. Stuff for animals excl. Unmilled cereals	63	Wood and cork manufactures excluding furniture
09	Miscellaneous food preparations	64	Paper, paperboard and manufactures thereof
11	Beverages	65	Textile yarn, fabrics, made up articles, etc.
12	Tobacco and tobacco manufactures	66	Non metallic mineral manufactures, nes
21	Hides, skins and fur skins, undressed	67	Iron and steel
22	Oil seeds, oil nuts and oil kernels	68	Non ferrous metals
23	Crude rubber including synthetic and reclaimed	69	Manufactures of metal, nes
24	Wood, lumber and cork	71	Machinery, other than electric
25	Pulp and paper	72	Electrical machinery, apparatus and appliances
26	Textile fibres, not manufactured, and waste	73	Transport equipment
27	Crude fertilizers and crude minerals, nes	81	Sanitary, plumbing, heating and lighting fixt.
28	Metalliferous ores and metal scrap	82	Furniture
29	Crude animal and vegetable materials, nes	83	Travel goods, handbags and similar articles
32	Coal, coke and briquettes	84	Clothing
33	Petroleum and petroleum products	85	Footwear
34	Gas, natural and manufactured	86	Scientif & control instrum, photogr gds, clocks
35	Electric energy	89	Miscellaneous manufactured articles, nes
41	Animal oils and fats	91	Postal packages not class. According to kind
42	Fixed vegetable oils and fats	93	Special transact. Not class. According to kind
43	Animal and vegetable oils and fats, processed	94	Animals, nes, incl. Zoo animals, dogs and cats
51	Chemical elements and compounds	95	Firearms of war and ammunition therefor
52	Crude chemicals from coal, petroleum and gas	96	Coin, other than gold coin, not legal tender
53	Dyeing, tanning and colouring materials		

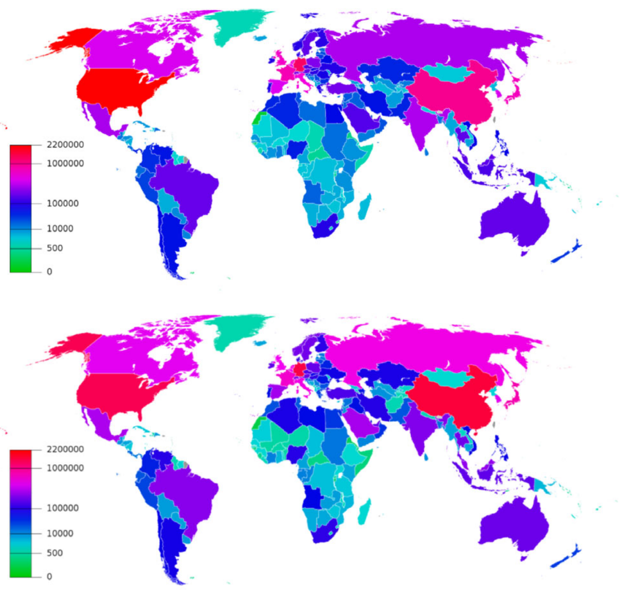

Fig. 1. World map of countries with color showing country import (top panel) and export (bottom panel) trade volume expressed in millions of USD given by numbers of the color bars. The data are shown for year 2008 with $N_c = 227$ countries for trade in all $N_p = 61$ products (from UN COMTRADE [1]). Names of countries can be found at [10].

The approach of Google matrix to the analysis of WTN was started in [11]. The striking feature of this approach is that it treats all UN countries on equal democratic grounds, independently of richness of a given country, in agreement with the principles of UN where all countries are equal. This property of G matrix is based on the property of Markov chains where the total probability is conserved to be unity since the sum of elements for each column of G is equal to unity. Even if in this approach all countries are treated on equal grounds still the PageRank and CheiRank analysis recovers about 75% of industrially developed countries of $G20$. However, now these countries appear at the top ranking positions not due to their richness but due to the efficiency of their trade network. Another important aspect found in [11] is that both PageRank and CheiRank vectors appear very naturally in the WTN corresponding to import and export flows.

In this work we extend the Google matrix analysis for the multiproduct WTN obtained from COMTRADE [1] with up to $N_p = 61$ trade products for up to $N_c = 227$ countries. The global G matrix of such trade flows has a size up to $N = N_pN_c = 13\,847$ nodes. The names and codes of products are given in Table 1 and their trade volumes, expressed in percent of the whole world trade volume, are given in Table 2 for years 1998 and 2008.

Table 2. Columns represent data: codes of 61 products of COMTRADE SITC Rev. 1, ImportRank and ExportRank $\hat{K} = \hat{K}^*$ in year 2008, product fraction in global trade volume in 2008, $\hat{K} = \hat{K}^*$ in 1998, product fraction in 1998.

Code	$\hat{K}(08)$	% Vol(08)	$\hat{K}(98)$	% Vol (98)	Code	$\hat{K}(08)$	% Vol(08)	$\hat{K}(98)$	% Vol (98)
00	53	0.10	51	0.17	54	9	2.89	16	1.88
01	27	0.69	26	0.83	55	25	0.76	28	0.79
02	34	0.44	34	0.56	56	30	0.55	43	0.36
03	28	0.63	22	0.99	57	58	0.03	57	0.04
04	21	1.07	19	1.13	58	15	1.95	13	2.07
05	19	1.16	18	1.50	59	22	1.04	20	1.13
06	49	0.23	44	0.36	61	51	0.19	42	0.37
07	33	0.47	29	0.73	62	26	0.73	24	0.85
08	38	0.39	36	0.45	63	35	0.43	32	0.61
09	40	0.34	41	0.39	64	20	1.14	17	1.79
11	31	0.54	31	0.65	65	18	1.40	11	2.46
12	47	0.24	35	0.49	66	17	1.71	14	2.01
21	56	0.05	53	0.11	67	7	3.63	10	2.74
22	37	0.39	48	0.32	68	11	2.27	15	1.95
23	44	0.26	50	0.22	69	13	2.04	12	2.12
24	39	0.35	30	0.65	71	2	11.82	1	15.03
25	43	0.29	45	0.34	72	3	10.42	3	12.26
26	50	0.22	37	0.45	73	4	10.06	2	12.38
27	41	0.33	47	0.33	81	42	0.31	46	0.34
28	16	1.92	25	0.84	82	23	0.93	21	1.03
29	48	0.24	40	0.39	83	45	0.26	49	0.26
32	24	0.82	39	0.42	84	10	2.42	6	3.44
33	1	14.88	4	5.02	85	29	0.59	27	0.79
34	14	2.04	23	0.99	86	12	2.25	8	2.95
35	46	0.26	52	0.17	89	6	3.72	5	4.54
41	57	0.03	58	0.04	91	61	0.00	61	0.00
42	32	0.49	38	0.44	93	5	3.92	9	2.92
43	54	0.08	55	0.08	94	59	0.01	59	0.01
51	8	3.01	7	3.07	95	55	0.08	54	0.11
52	52	0.11	56	0.05	96	60	0.00	60	0.00
53	36	0.39	33	0.60					

The main problem of construction of such a matrix is not its size, which is rather modest compared to those studied in [9], but the necessity to treat all countries on democratic grounds and at the same time to treat trade products on the basis of their trade volume. Indeed, the products cannot be considered on democratic grounds since their contributions to economy are linked with their trade volume. Thus, according to Table 2, in year 2008 the trade volume of *Petroleum and petroleum products* (code 33 in Tab. 1) is by a factor 300 larger than those of *Hides, skins and fur skins* (undressed) (code 21 in Tab. 1). To incorporate these features in our mathematical analysis of multiproduct WTN we developed in this work the google personalized vector method (GPVM) which allows to keep a democratic treatment of countries and at the same time to consider products proportionally to their trade volume. As a result we are able to perform analysis of the global multiproduct WTN keeping all interactions between all countries and all products. This is a new step in the WTN analysis since in our previous studies [11] it was possible to consider a trade between countries only in one product or only in all products summed together (all commodities). The new finding of such global WTN analysis is an asymmetric ranking of products: some of them are more oriented to import and others are oriented to export while the ranking of products by the trade volume is always symmetric after summation over all countries. This result with asymmetric ranking of products confirms the indications obtained on the basis of ecological ranking [12], which also gives an asymmetry of products in respect to import and export. Our approach also allows to analyze the sensitivity of trade network to price variations of a certain product.

We think that the GPVM approach allows to perform a most advanced analysis of multiproduct world trade. The previous studies have been restricted to studies of statistical characteristics of WTN links, patterns and their topology (see e.g. [13–19]). The applications of PageRank algorithm to the WTN was discussed in [20] but effects of export had been not analyzed there, the approach based on HITS algorithm was used in [21]. In comparison to the above studies, the approach developed here for the multiproduct WTN has an advantage of analysis of ingoing and outgoing flows, related to PageRank and CheiRank, and of taking into account of multiproduct aspects of the WTN. Even if the importance to multiproduct WTN analysis is clearly understood by researchers (see e.g. [22]) the Google matrix methods have not been efficiently used up to now. We also note that the matrix methods are extensively used for analysis of correlations of trade indexes (see e.g. [23,24]) but these matrices are Hermitian being qualitatively different from those appearing in the frame of Markov chains. Here we make the steps in multi-functional or multiproduct Google matrix analysis of the WTN extending the approach used in [11].

2 Methods

Below we give the mathematical definition for the construction of the Google matrix G, which belongs to the class of Perron-Frobenius operators and Markov chains. The matrix G is constructed for the import (ingoing) trade flows. We also use the matrix G^* built from the export (outgoing) trade flows. The matrix size N is given by the product of number of countries N_c by the number of products N_p. The main features of matrices G and G^* are: all elements are real positive numbers or zeros; the sum of elements in each matrix column is equal to unity, that gives the probability conservation required for Markov chains. We use the right eigenvectors of PageRank P and CheiRank P^* respectively for the matrices G, G^* with the largest eigenvalue $\lambda = 1$ ($GP = P$, $G^*P^* = P^*$). These vectors give the stationary distribution of probability over the nodes. The important element of G and G^* is their democratic (equal grounds) treatment of all world countries independently of their richness. This results from the construction rules of G, G^* where for each country the sum of elements in each column, corresponding to any product of given country, is equal to unity. At the same time we keep the contribution of products to be proportional to their trade volume since their effect on the trade is indeed related with their volume contribution in the world trade. Thus, the important new element of this work is the new proposed method which uses a certain personalized vector in construction of G, G^* and satisfies the above requirements.

At the same time we note that it is preferable to work in a certain fixed class of operators, e.g. Google matrix and Markov chains. Already only this requirement implies that we need to treat countries in a democratic manner since by the construction sum of elements in each column should be unity. For one product, or for a sum of all products, the construction of G, G^* is relatively straightforward as described in [11]. The most tricky part is the case of many products which contribution should be treated proportionally to their fraction in the world trade. We describe the construction method of G, G^* which takes into account both these features of the world trade. We note that, as discussed in [11], the obtained results have no significant dependence on the damping factor α, which we keep below at the fixed value $\alpha = 0.5$. The simple examples of constructions of matrices G, G^* for directed networks are illustrated in Figures 3 and 4 in [9]. Below we present all mathematical definitions and describe the main features of these matrices and eigenvectors.

2.1 Google matrix construction for the WTN

For a given year, we build N_p money matrices $M^p_{c,c'}$ of the WTN from the COMTRADE database [1] (see [11]).

$$M^p_{c,c'} = \text{product}\, p\, \text{transfer (in USD) from country}\, c'\, \text{to}\, c. \quad (1)$$

Here the country indexes are $c, c' = 1, \ldots, N_c$ and a product index is $p = 1, \ldots, N_p$. According to the COMTRADE database the number of UN registered countries is $N_c = 227$ (in recent years) and the number of products is $N_p = 10$ and $N_p = 61$ for 1 and 2 digits respectively from the Standard International Trade Classification (SITC) Rev. 1. For convenience of future notation we also define the volume of imports and exports for a given country and product respectively as:

$$V^p_c = \sum_{c'} M^p_{c,c'}, V^{*p}_c = \sum_{c'} M^p_{c',c}. \quad (2)$$

The import and export volumes $V_c = \sum_p V^p_c$ and $V^*_c = \sum_p V^{*p}_c$ are shown for the world map of countries in Figure 1 for year 2008.

In order to compare later with PageRank and CheiRank probabilities we define volume trade ranks in the whole trade space of dimension $N = N_p \times N_c$. Thus the ImportRank ($\hat{P}$) and ExportRank ($\hat{P}^*$) probabilities are given by the normalized import and export volumes

$$\hat{P}_i = V^p_c/V, \hat{P}^*_i = V^{*p}_c/V, \quad (3)$$

where $i = p + (c-1)N_p$, $i = 1, \ldots, N$ and the total trade volume is:

$$V = \sum_{p,c,c'} M^p_{c,c'} = \sum_{p,c} V^p_c = \sum_{p,c} V^{*p}_c.$$

The Google matrices G and G^* are defined as $N \times N$ real matrices with non-negative elements:

$$G_{ij} = \alpha S_{ij} + (1-\alpha) v_i e_j,\ {G^*}_{ij} = \alpha {S^*}_{ij} + (1-\alpha) v^*_i e_j, \quad (4)$$

where $N = N_p \times N_c$, $\alpha \in (0,1]$ is the damping factor ($0 < \alpha < 1$), e_j is the row vector of unit elements ($e_j = 1$), and v_i is a positive column vector called a *personalization vector* with $\sum_i v_i = 1$ [6]. We note that the usual Google matrix is recovered for a personalization vector $v_i = e_i/N$. In this work, following [11], we fix $\alpha = 0.5$. As discussed in [6,9,11] a variation of α in a range (0.5, 0.9) does not significantly affect the probability distributions of PageRank and CheiRank vectors. We specify the choice of the personalization vector a bit below.

The matrices S and S^* are built from money matrices ${M^p}_{cc'}$ as:

$$S_{i,i'} = \begin{cases} M^p_{c,c'}\delta_{p,p'}/V^{*p}_{c'} & \text{if } V^{*p}_{c'} \neq 0 \\ 1/N \text{ if } V^{*p}_{c'} = 0 \end{cases}$$

$$S^*_{i,i'} = \begin{cases} M^p_{c',c}\delta_{p,p'}/V^p_{c'} & \text{if } V^p_{c'} \neq 0 \\ 1/N \text{ if } V^p_{c'} = 0 \end{cases} \quad (5)$$

where $c, c' = 1, \ldots, N_c$; $p, p' = 1, \ldots, N_p$; $i = p + (c-1)N_p$; $i' = p' + (c'-1)N_p$; and therefore $i, i' = 1, \ldots, N$. Note that the sum of each column of S and S^* are normalized to unity and hence the matrices G, G^*, S, S^* belong to the class of Google matrices and Markov chains. The eigenvalues and eigenstates of G, G^* are obtained by a direct numerical diagonalization using the standard numerical packages.

2.2 PageRank and CheiRank vectors from GPVM

PageRank and CheiRank (P and P^*) are defined as the right eigenvectors of G and G^* matrices respectively at eigenvalue $\lambda = 1$:

$$\sum_j G_{ij}\psi_j = \lambda\psi_i, \sum_j {G^*}_{ij}{\psi^*}_j = \lambda{\psi^*}_j. \quad (6)$$

For the eigenstate at $\lambda = 1$ we use the notation $P_i = \psi_i, P^* = {\psi^*}_i$ with the normalization $\sum P_i = \sum_i {P^*}_i = 1$. For other eigenstates we use the normalization $\sum_i |\psi_i|^2 = \sum_i |\psi_i^*|^2 = 1$. According to the Perron-Frobenius theorem the components of P_i, ${P^*}_i$ are positive and give the probabilities to find a random surfer on a given node [6]. The PageRank K and CheiRank K^* indexes are defined from the decreasing ordering of P and P^* as $P(K) \geq P(K+1)$ and $P^*(K) \geq P^*(K^*+1)$ with $K, K^* = 1, \ldots, N$.

If we want to compute the reduced PageRank and CheiRank probabilities of countries for *all commodities* (or equivalently all products) we trace over the product space getting $P_c = \sum_p P_{pc} = \sum_p P\left(p + (c-1)N_p\right)$ and $P_c^* = \sum P_{pc}^* = \sum_p P^*\left(p + (c-1)N_p\right)$ with their corresponding K_c and K_c^* indexes. In a similar way we obtain the reduced PageRank and CheiRank probabilities for products tracing over all countries and getting $P_p = \sum_c P\left(p + (c-1)N_p\right) \sum_p P_{pc}$ and $P_p^* = \sum_c P^*\left(p + (c-1)N_p\right) \sum P_{pc}^*$ with their corresponding product indexes K_p and K_p^*.

In summary we have $K_p, K_p^* = 1, \ldots, N_p$ and $K_c, K_c^* = 1, \ldots, N_c$. A similar definition of ranks from import and export trade volume can be done in a straightforward way via probabilities $\hat{P}_p, \hat{P}_p^*, \hat{P}_c, \hat{P}_c^*, \hat{P}_{pc}, \hat{P}_{pc}^*$ and corresponding indexes $\hat{K}_p, \hat{K}_p^*, \hat{K}_c, \hat{K}_c^*, \hat{K}, \hat{K}^*$.

To compute the PageRank and CheiRank probabilities from G and G^* keeping democracy in countries and proportionality of products to their trade volume we use the GPVM approach with a personalized vector in (4). At the first iteration of Google matrix we take into account the relative product volume per country using the following personalization vectors for G and G^*:

$$v_i = \frac{V_c^p}{N_c \sum_{p'} V_c^{p'}}, v_i^* = \frac{V_c^{*p}}{N_c \sum_{p'} V_c^{*p'}}, \quad (7)$$

using the definitions (2) and the relation $i = p + (c-1)N_p$. This personalized vector depends both on product and country indexes. In order to have the same value of personalization vector in countries we can define the second iteration vector proportional to the reduced PageRank and

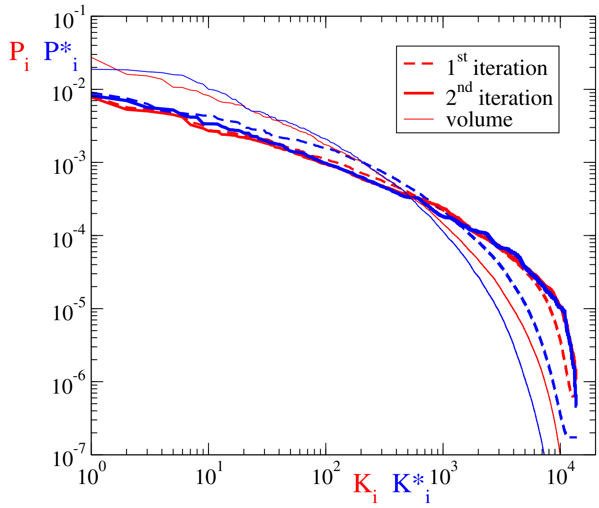

Fig. 2. Dependence of probabilities of PageRank $P(K)$, CheiRank $P^*(K^*)$, ImportRank $\hat{P}(\hat{K})$ and ExportRank $\hat{P}^*(\hat{K}^*)$ as a function of their indexes in logarithmic scale for WTN in 2008 with $\alpha = 0.5$ at $N_c = 227$, $N_p = 1$, $N = 13\,847$. Here the results for GPVM after 1st and 2nd iterations are shown for PageRank (CheiRank) in red (blue) with dashed and solid curves, respectively. ImportRank and ExportRank (trade volume) are shown by red and blue thin curves, respectively. The fit exponents for PageRank and CheiRank are $\beta = 0.61, 0.7$ for the first iteration, $\beta = 0.59, 0.65$ for the second iteration, and $\beta = 0.94, 1.04$ for ImportRank and ExportRank (for the range $K \in [10, 2000]$).

CheiRank vectors in products obtained from the GPVM Google matrix of the first iteration:

$$v'(i) = \frac{P_p}{N_c}, v'^*(i) = \frac{P_p^*}{N_c}. \quad (8)$$

In this way we keep democracy in countries but weighted products. This second iteration personalized vectors are used for the main part of computations and operations with G and G^*. This procedure with two iterations forms our GPVM approach. The difference between results obtained from the first and second iterations is not very large (see Figs. 2 and 3) but a detailed analysis of ranking of countries and products shows that the personalized vector for the second iteration improves the results making them more stable and less fluctuating. In all figures below (except Figs. 2 and 3) we show the results after the second iteration.

The obtained results show the distribution of nodes on the PageRank-CheiRank plane (K, K^*). In addition to two ranking indexes K, K^* we use also 2DRank index K_2 which combines the contribution of these indexes as described in [8]. The ranking list $K_2(i)$ is constructed by increasing $K \rightarrow K+1$ and increasing 2DRank index $K_2(i)$ by one if a new entry is present in the list of first $K^* < K$ entries of CheiRank, then the one unit step is done in K^* and K_2 is increased by one if the new entry is present in the list of first $K < K^*$ entries of CheiRank. More formally, 2DRank $K_2(i)$ gives the ordering of the sequence of sites, that appear inside the squares

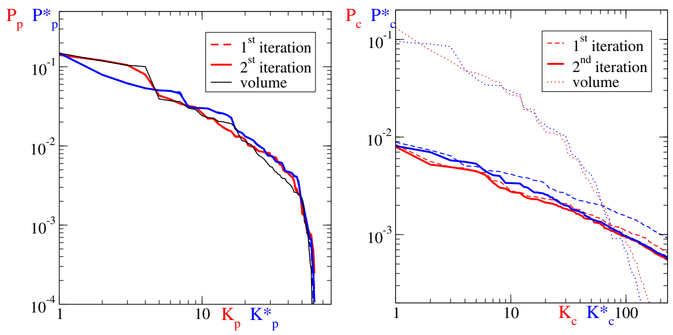

Fig. 3. Probability distributions of PageRank and CheiRank for products $P_p(K_p)$, $P_p^*(K_p^*)$ (left panel) and countries $P_c(K_c)$, $P_c^*(K_c^*)$ (right panel) in logarithmic scale for WTN from Figure 2. Here the results for the 1st and 2nd GPVM iterations are shown by red (blue) curves for PageRank (CheiRank) with dashed and solid curves, respectively. The probabilities from the trade volume ranking are shown by black curve (left) and dotted red and blue curves (right) for ImportRank and ExportRank, respectively.

$[1, 1; K = k, K^* = k; \ldots]$ when one runs progressively from $k = 1$ to N. Additionally, we analyze the distribution of nodes for reduced indexes (K_p, K_p^*), (K_c, K_c^*).

We also characterize the localization properties of eigenstates of G, G^* by the inverse participation ratio (IPR) defined as $\xi = (\sum_i |\psi_i|^2)^2 / \sum_i |\psi_i|^4$. This characteristic determines an effective number of nodes which contribute to a formation of a given eigenstate (see details in Ref. [9]).

2.3 Correlators of PageRank and CheiRank vectors

Following previous works [7,8,11] the correlator of PageRank and CheiRank vectors is defined as:

$$\kappa = N \sum_{i=1}^{N} P(i) P^*(i) - 1. \tag{9}$$

The typical values of κ are given in [9] for various networks.

For global PageRank and CheiRank the product-product correlator matrix is defined as:

$$\kappa_{pp'} = N_c \times \sum_{c=1}^{N_c} \left[\frac{P(p + (c-1)N_p) P^*(p' + (c-1)N_p)}{\sum_{c'} P(p + (c'-1)N_p) \sum_{c''} P^*(p' + (c''-1)N_p)} \right] - 1. \tag{10}$$

Then the correlator for a given product is obtained from (10) as:

$$\kappa_p = \kappa_{pp'} \delta_{p,p'}, \tag{11}$$

where $\delta_{p,p'}$ is the Kronecker delta.

We also use the correlators obtained from the probabilities traced over products ($P_c = \sum_p P_{pc}$) and over countries ($P_p = \sum_c P_{pc}$) which are defined as:

$$\kappa(c) = N_c \sum_{c=1}^{N_c} P_c P_c^* - 1,$$
$$\kappa(p) = N_p \sum_{p=1}^{N_p} P_p P_p^* - 1. \tag{12}$$

In the above equations (9)–(12) the correlators are computed for PageRank and CheiRank probabilities. We can also compute the same correlators using probabilities from the trade volume in ImportRank $\hat{P}$ and ExportRank $\hat{P}^*$ defined by (3).

We discuss the values of these correlators in Section 4.

3 Data description

All data are obtained from the COMTRADE database [1]. We used products from COMTRADE SITC Rev. 1 classification with number of products $N_p = 10$ and 61. We choose SITC Rev. 1 since it covers the longest time interval. The main results are presented for $N_p = 61$ with up to $N_c = 227$ countries. The names of products are given in Table 1, their ImportRank index K and their fraction (in percent) of global trade volume in years 1998 and 2008 are given in Table 2. The data are collected and presented for the years 1962−2010. Our data and results are available at [25], the data for the matrices $M^p_{c,c'}$ are available at COMTRADE [1] with the rules of their distribution policy. Following [11] we use for countries ISO 3166-1 alpha-3 code available at Wikipedia.

4 Results

We apply the above methods to the described data sets of COMTRADE and present the obtained results below.

4.1 PageRank and CheiRank probabilities

The dependence of probabilities of PageRank $P(K)$ and CheiRank $P^*(K^*)$ vectors on their indexes K, K^* are shown in Figure 2 for a selected year 2008. The results can be approximately described by an algebraic dependence $P \propto 1/K^\beta$ with the exponent values given in the caption. It is interesting to note that we find approximately the same $\beta \approx 0.6$ both for PageRank and CheiRank in contrast to the WWW, universities and Wikipedia networks where usually one finds $\beta \approx 1$ for PageRank and $\beta \approx 0.6$ for CheiRank [6,9]. We attribute this to an intrinsic property of WTN where the countries try to keep economy balance of their trade. The data show that the range of probability variation is reduced for the Google ranking compared to the volume ranking. This results from a democratic ranking of countries used in the Google matrix analysis that gives a reduction of richness dispersion between countries. The results also show that the variation of probabilities for 1st and 2nd GPVM results are not very large that demonstrates the convergence of this approach.

After tracing probabilities over countries we obtain probability distributions $P_p(K_p)$, $P^*_p(K^*_p)$ over products shown in Figure 3. The variation range of probabilities is the same as for the case of volume ranking. This shows that the GPVM approach correctly treats products keeping their contributions proportional to their volume. The difference between 1st and 2nd iterations is rather small and is practically not visible on this plot. The important result well visible here is a visible difference between PageRank and CheiRank probabilities while there is no difference between ImportRank and ExportRank probabilities since they are equal after tracing over countries.

After tracing over products we obtain probability distributions $P_c(K_c)$, $P^*_c(K^*_c)$ over countries shown in Figure 3. We see that the probability of volume ranking varies approximately by a factor 1000 while for PageRank and CheiRank such a factor is only approximately 10. Thus the democracy in countries induced by the Google matrix construction reduces significantly the variations of probabilities among countries and inequality between countries.

Both panels of Figure 3 show relatively small variations between 1st and 2nd GPVM iterations confirming the stability of this approach. In next sections we present the results only for 2nd GPVM iteration. This choice is confirmed by consideration of ranking positions of various nodes of global matrices G, G^* which show less fluctuations compared to the results of the 1st GPVM iteration.

From the global ranking of countries and products we can select a given product and then determine local ranking of countries in a given product to see how strong is their trade for this product. The results for three selected products are discussed below for year 2008. For comparison we also present comparison with the export-import ranking from the trade volume.

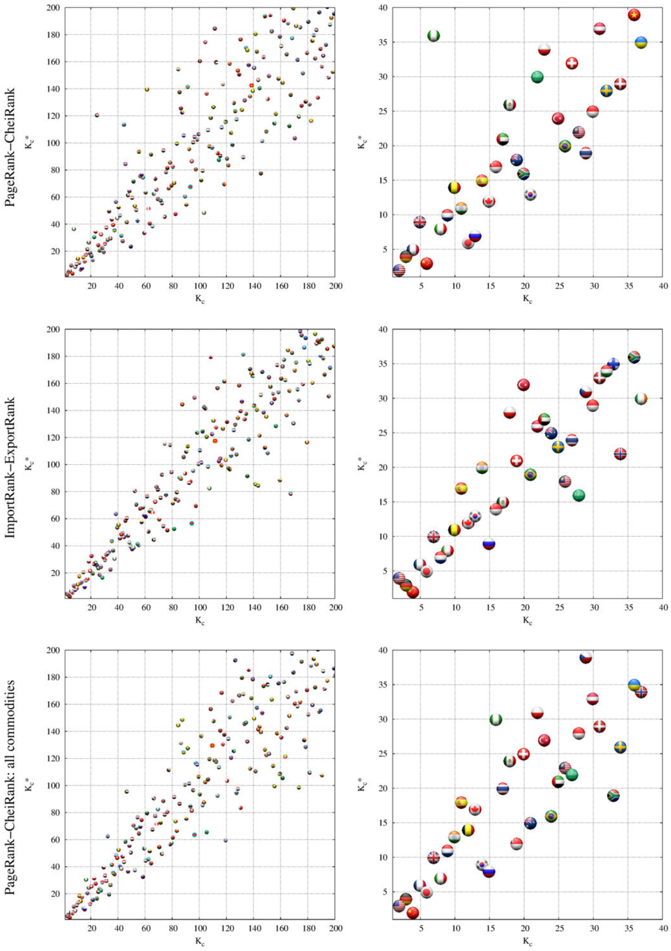

Fig. 4. Country positions on PageRank-CheiRank plane (K_c, K^*_c) obtained by the GPVM analysis (top panels), ImportRank-ExportRank of trade volume (center panels), and for PageRank-CheiRank of *all commodities* (bottom panels, data from [11]). Left panels show global scale $(K_c, K^*_c \in [1, 200])$ and right panels show zoom on top ranks $(K_c, K^*_c \in [1, 40])$. Each country is shown by circle with its own flag (for a better visibility the circle center is slightly displaced from its integer position (K_c, K^*_c) along direction angle $\pi/4$). Data are shown for year 2008.

4.2 Ranking of countries and products

After tracing the probabilities $P(K), P^*(K^*)$ over products we obtain the distribution of world countries on the PageRank-CheiRank plane (K_c, K^*_c) presented in Figure 4 for a test year 2008. In the same figure we present the rank distributions obtained from ImportRank-ExportRank probabilities of trade volume and the results obtained in [11] for trade in *all commodities*. For the GPVM data we see the global features already discussed in [11]: the countries are distributed in a vicinity of diagonal $K_c = K^*_c$ since each country aims to keep its trade balanced. The top 20 list of top K_2 countries recover 15 of 19 countries of $G20$ major world economies (EU is the number 20) thus obtaining 79% of the whole list. This is close to the percent obtained in [11] for trade in *all commodities*.

The global distributions of top countries with $K_c \leq 40$, $K^*_c \leq 40$ for the three ranking methods, shown in Figure 4, are similar on average. But some modifications introduced by the GPVM analysis are visible. Thus China (CHN) moves on 2nd position of CheiRank while it is in the 1st position for trade volume ranking and CheiRank of *all commodities*. Also e.g. Saudi Arabia (SAU) and Russia (RUS) move from the CheiRank positions $K^*_c = 21$ and $K^*_c = 7$ in *all commodities* [11] to $K^*_c = 29$ and $K^*_c = 6$ in the GPVM ranking, respectively. Other example is a significant displacement of Nigeria (NGA). We explain such differences as the result of larger connectivity required for getting high ranking in the multiproduct WTN. Indeed, China is more specialized in specific products compared to USA (e.g. no petroleum production and export) that leads to its displacement in K^*_c. We note that the ecological ranking gives also worse ranking positions for China comparing to the trade volume ranking [12]. In a similar way the trade of Saudi Arabia is strongly dominated by petroleum and moreover its petroleum trade is strongly oriented on USA that makes its trade network concentrated on a few links while Russia is improving its

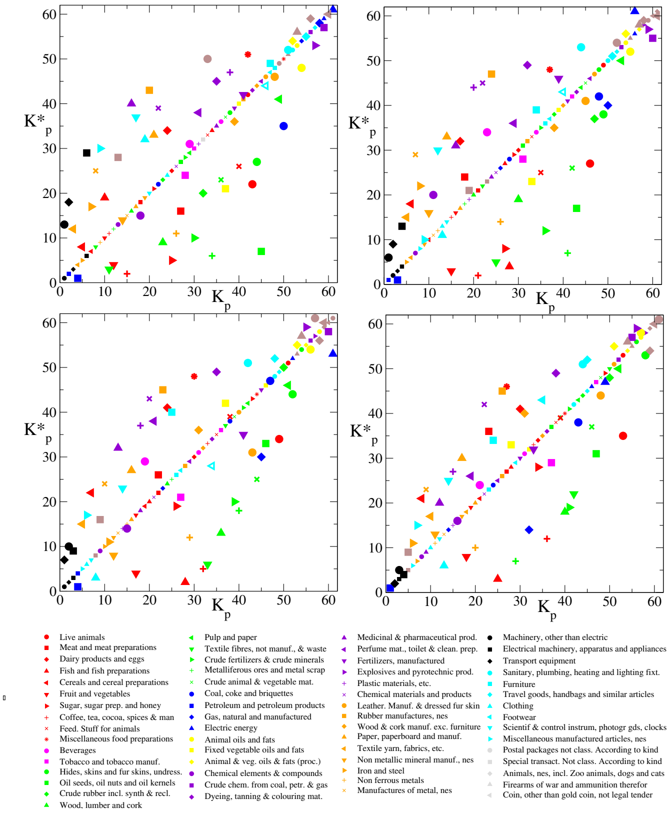

Fig. 5. Two dimensional ranking of products on the PageRank-CheiRank plane (K_p, K_p^*). Each product is represented by its specific combination of color and symbol: color illustrates the first digit of COMTRADE SITC Rev. 1 code with the corresponding name shown in the legend on the bottom; symbols correspond to product names listed in Table 1 with their code numbers; the order of names on the bottom panel of symbols of this figure is the same as in Table 1 (counting from top to bottom and left to right). The trade volume ranking via ImportRank-ExportRank is shown by small symbols at the diagonal $\hat{K}_p = \hat{K}_p^*$, after tracing over countries this ranking is symmetric in products. Top left and right panels show years 1963 and 1978, while bottom left and right panels show years 1993 and 2008, respectively.

position in K_c^* due to significant trade links with EU and Asia.

In global, the comparison of three ranks of countries shown in Figure 4 confirms that the GPVM analysis gives a reliable ranking of multiproduct WTN. Thus we now try to obtain new features of multiproduct WTN using the GPVM approach.

The main new feature obtained within the GPVM approach is shown in Figure 5 which gives the distribution of products on the PageRank-CheiRank plane (K_p, K_p^*) after tracing of global probabilities $P(K), P^*(K^*)$ over all world countries. The data clearly show that the distribution of products over this plane is asymmetric while the ranking of products from the trade volume produces the symmetric ranking of products located directly on diagonal $K_p = K_p^*$. Thus the functions of products are asymmetric: some of them are more oriented to export (e.g. *03 Fish and fish preparations, 05 Fruit and vegetables, 26 Textile fibers, not manuf. etc., 28 Metalliferous ores and metal scrap, 84 Clothing*); in last years (e.g. 2008) *34 Gas, natural and manufactured* also takes well pronounced export oriented feature characterized by location in the lower right triangle ($K_p^* < K_p$) of the square plane (K_p, K_p^*). In contrast to that the products located in the upper left triangle ($K_p^* > K_p$) represent import oriented products (e.g. *02 Dairy products and eggs, 04 Cereals and cereal preparations, 64 Paper, paperboard and manuf., 65 Textile yarn, fabrics, etc., 86 Scientific & control instrum, photogr gds, clocks*).

It is interesting to note that the machinery products 71, 72. 73 are located on leading import oriented positions in 1963, 1978, 1993 but they become more close to symmetric positions in 2008. We attribute this to development of China that makes the trade in these products more symmetric in import-export. It is interesting to note that in 1993 the product *33 Petroleum and petroleum products* loses its first trade volume position due to low petroleum prices but still it keeps the first CheiRank position showing its trade network importance for export. Each product moves on (K_p, K_p^*) with time. However, a part of the above points, we can say that the global distribution does not manifest drastic changes. Indeed, e.g. the green symbols of first digit 2 remain export oriented for the whole period 1963−2008. We note that the established asymmetry of products orientation for the world trade is in agreement with the similar indications obtained on the basis of ecological ranking in [12]. However, the GPVM approach used here have more solid mathematical and statistical foundations with a reduced significance of fluctuations comparing to the ecological ranking.

The comparison between the GPVM and trade volume ranking methods provides interesting information. Thus in petroleum code 33 we have on top positions Russia, Saudi Arabia, United Arab Emirates while from the CheiRank order of this product we find Russia, USA, India (see Fig. 6 and Tab. 3). This marks the importance of the role of USA and India played in the WTN and in the redistribution of petroleum over nearby region countries, e.g. around India. Also Singapore is on a local petroleum position just behind India and just before Saudi Arabia, see Table 3. This happens due to strong involvement of India and Singapore in the trade redistribution flows of petroleum while Saudi Arabia has rather restricted trade connections strongly oriented on USA and nearby countries.

For electrical machinery 72 there are less modifications in the top export or CheiRank positions (see Fig. 6) but we observe significant broadening of positions on PageRank-CheiRank plane comparing to ImportRank-ExportRank. Thus, Asian countries (China, Japan, S. Korea, Singapore) are located on the PageRank-CheiRank plane well below the diagonal $K = K^*$ showing a significant trade advantages of these countries in product 72 comparing to Western countries (USA, Germany, France, UK).

Another product, shown in Figure 6, is *03 Fish and fish preparations*. According to the trade volume export ranking the top three positions are attributed to China,

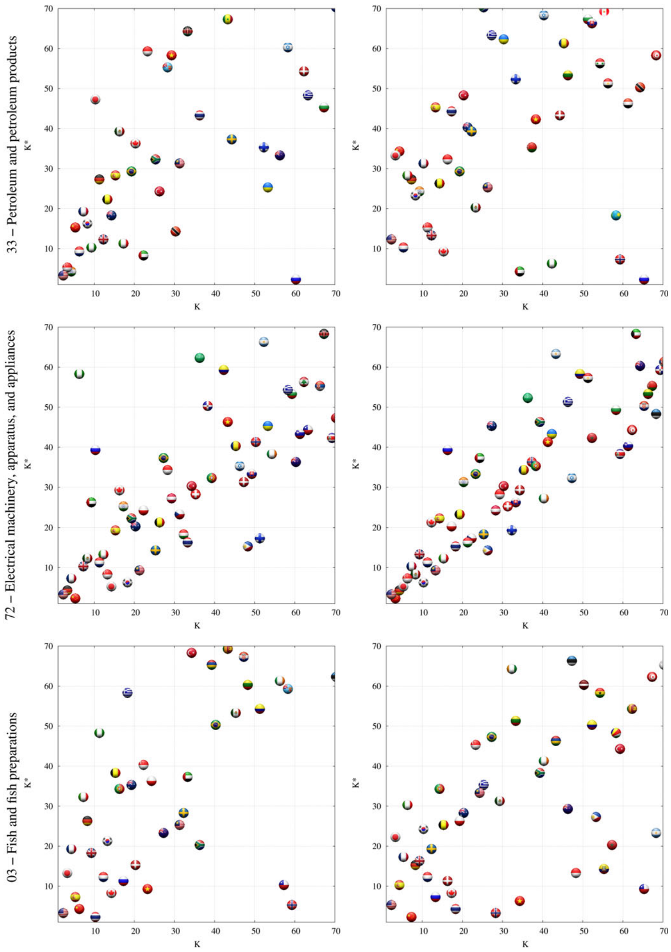

Fig. 6. Left panels show results of the GPVM data for country positions on PageRank-CheiRank plane of local rank values K, K^* ordered by (K_{cp}, K^*_{cp}) for specific products with $p = 33$ (top panel), $p = 72$ (center panel) and $p = 03$ (bottom panel). Right panels show the ImportRank-ExportRank planes respectively for comparison. Data are given for year 2008. Each country is shown by circle with its own flag as in Figure 4.

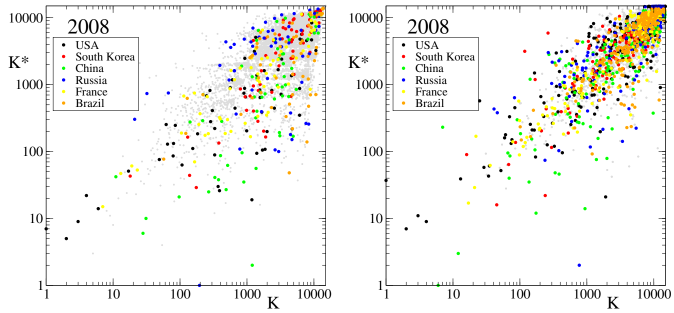

Fig. 7. Global plane of rank indexes (K, K^*) for PageRank-CheiRank (left panel) and ImportRank-ExportRank (right panel) for $N = 13\,847$ nodes in year 2008. Each country and product pair is represented by a gray circle. Some countries are highlighted in colors: USA with black, South Korea with red, China with green, Russia with red, France with yellow and Brazil with orange.

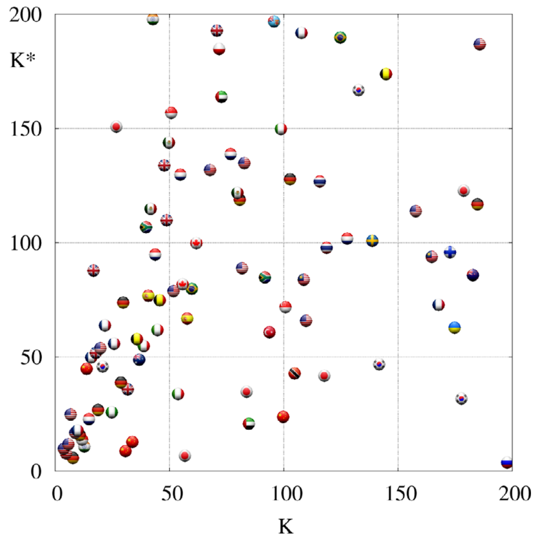

Fig. 8. Top 200 global PageRank-CheiRank indexes (K, K^*) distributions for year 2008. Each country (for different products) is represented by its flag.

Norway, Thailand. However, from CheiRank of product 03 we find another order with Thailand, USA, China. This result stresses again the broadness and robustness of the trade connections of Thailand and USA. As another example we note a significant improvement of Spain CheiRank position showing its strong commercial relations for product 03. On the other side Russia has relatively good position in the trade volume export of 03 product but its CheiRank index becomes worse due to absence of broad commercial links for this product.

The global top 20 positions of indexes K, K^*, $K_2, \hat{K}$, $\hat{K}^*$ are given in Table 3 for year 2008. We note a significant improvement of positions of Singapore and India in PageRank-CheiRank positions comparing to their positions in the trade volume ranking. This reflects their strong commercial relations in the world trade. In the trade volume ranking the top positions are taken by 33 petroleum and digit 7 of machinery products. This remains mainly true for PageRank-CheiRank positions but we see the spectacular improvement of positions of *84 Clothing* for China ($K^* = 2$) and *93 Special transact.* for USA ($K = 4$) showing thus these two products have strong commercial exchange all over the world even if their trade volume is not so dominant.

We show the plane (K, K^*) for the global world ranking in logarithmic scale in 2008 in Figure 7. The positions of trade nodes of certain selected countries are shown by color. We observe that the trade volume gives a higher concentration of nodes around diagonal comparing to the GPVM ranking. We attribute this to the symmetry of trade volume in products.

In Figure 8 we show the distributions of top 200 ranks of the PageRank-CheiRank plane (zoom of left panel of Fig. 7). Among the top 30 positions of K^* there are 8 products of USA, 6 of China, 3 of Germany and other countries with less number of products. The top position at $K^* = 1$ corresponds to product 33 of Russia while Saudi

Table 3. Top 20 ranks for global PageRank K, CheiRank K^*, 2dRank K_2, ImportRank $\hat{K}$ and ExportRank $\hat{K}^*$ for given country and product code for year 2008.

#	K country & code		K^* country & code		K_2 country & code		$\hat{K}$ country & code		$\hat{K}^*$ country & code	
1	USA	33	Russia	33	Germany	73	USA	33	China	72
2	USA	73	China	84	USA	73	USA	71	Russia	33
3	USA	71	Germany	73	USA	33	USA	72	China	71
4	USA	93	Japan	73	USA	71	USA	73	Germany	73
5	Germany	73	USA	73	India	33	Japan	33	Germany	71
6	USA	72	China	72	Singapore	33	China	72	Saudi Arabia	33
7	France	73	USA	33	Germany	71	China	33	USA	71
8	Germany	71	India	33	USA	72	Germany	71	Japan	73
9	Singapore	33	USA	71	France	73	Germany	73	USA	73
10	India	33	China	71	Netherlands	33	Netherlands	33	Japan	71
11	China	33	Singapore	33	USA	93	Germany	72	USA	72
12	Netherlands	33	Saudi Arabia	33	Nigeria	33	China	71	China	89
13	France	33	Germany	71	Germany	72	USA	89	Germany	72
14	UK	71	USA	72	China	72	Italy	33	China	84
15	UK	73	France	73	China	71	Germany	33	Japan	72
16	Germany	72	Thailand	3	UK	33	South Korea	33	South Korea	72
17	USA	89	Kazakhstan	33	Germany	93	France	73	France	73
18	South Korea	33	U. Arab Emir.	33	China	33	China	28	Italy	71
19	France	71	USA	28	South Korea	33	Germany	93	U. Arab Emir.	33
20	Sudan	73	Netherlands	33	Australia	33	India	33	Germany	93

Arabia is only at $K^* = 12$ for this product. The lists of all $N = 13\,847$ network nodes with their K, K_2, K^* values are available at [25].

4.3 Time evolution of ranking

The time evolution of indexes of products K_p, K_p^* is shown in Figure 9. To obtain these data we trace PageRank and CheiRank probabilities over countries and show the time evolution of rank indexes of products K_p, K_p^* for top 15 rank products of year 2010. The product *33 Petroleum and petroleum products* remains at the top CheiRank position $K_p^* = 1$ for the whole period while in PageRank it shows significant variations from $K_p = 1$ to 4 being at $K_p = 4$ at 1986−1999 when the petroleum had a low price. Products with first digit 7 have high ranks of K_p but especially strong variation is observed for K_p^* of *72 Electrical machinery* moving from position 26 in 1962 to 4 in 2010. Among other indexes with strong variations we note *58 Plastic materials*, *84 Clothing*, *93 Special transact.*, *34 Gas, natural and manufactured*.

The time evolution of products 33 and 72 on the global index plane (K, K^*) is shown in Figure 10 for 6 countries from Figure 7. Thus for product 72 we see a striking improvement of K^* for China and Korea that is at the origin of the global importance improvement of K_p^* in Figure 9. For the product 33 in Figure 10 Russia improves significantly its rank positions taking the top rank $K^* = 1$ (see also Tab. 3).

The variation of global ranks K, K^* with time is shown for 4 products and 10 countries in Figure 11. For products 72, 73 on a scale of 50 years we see a spectacular improvement of K^* for China, Japan, Korea. For the product 33 we see strong improvement of K^* for Russia in last 15 years. It is interesting to note that at the period 1986−1992 of cheap petroleum 33 USA takes the top position $K^* = 1$ with a significant increase of its corresponding K value. We think that this is a result of political decision to make an economical pressure on USSR since such an increase of export of cheap price petroleum is not justified from the economical view point.

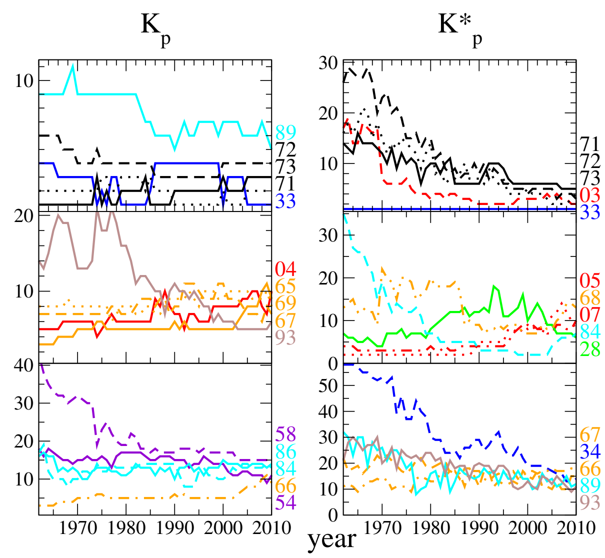

Fig. 9. Time evolution of PageRank K_p and CheiRank K_p^* indexes for years 1962 to 2010 for certain products marked on the right panel side by their codes from Table 1. Top panels show top 5 ranks of 2010, middle and bottom panels show ranks 6 to 10 and 11 to 15 for 2010, respectively. Colors of curves correspond to the colors of Figure 5 marking the first code digit.

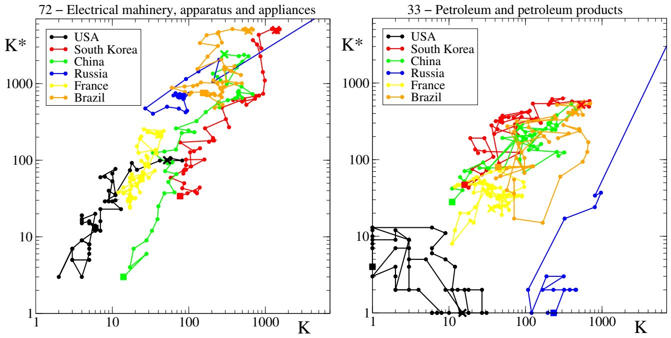

Fig. 10. Time evolution of ranking of two products 72 and 33 for 6 countries of Figure 7 shown on the global PageRank-CheiRank plane (K, K^*). Left and right panels show the cases of *72 Electrical machinery, apparatus and appliances* and *33 petroleum and petroleum products*, respectively. The evolution in time starts in 1962 (marked by cross) and ends in 2010 (marked by square).

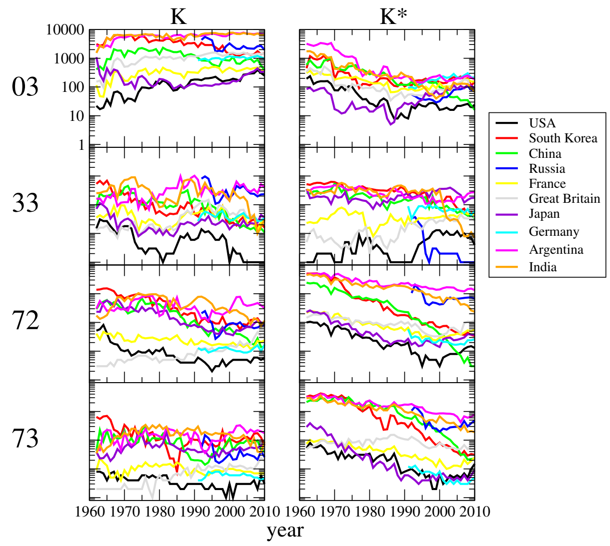

Fig. 11. Time evolution of global ranking of PageRank and CheiRank indexes K, K^* for selected 10 countries and 4 products. Left and right panels show K and K^* as a function of years for products: *03 Fish and fish preparations*; *33 Petroleum and petroleum products*; *72 Electrical machinery, apparatus and appliances*; and *73 Transport equipment* (from top to bottom). In all panels the ranks are shown in logarithmic scale for 10 given countries: USA, South Korea, China, Russia, France, Brazil, Great Britain, Japan, Germany and Argentina marked by curve colors.

For the product 33 we also note a notable improvement of K^* of India which is visible in CheiRank but not in ExportRank (see Tab. 3). We attribute this not to a large amount of trade volume but to a significant structural improvements of trade network of India in this product. We note that the strength and efficiency of trade network is also at the origin of significant improvement of PageRank and CheiRank positions of Singapore comparing to the trade volume ranking. Thus the development of trade connections of certain countries significantly improves their

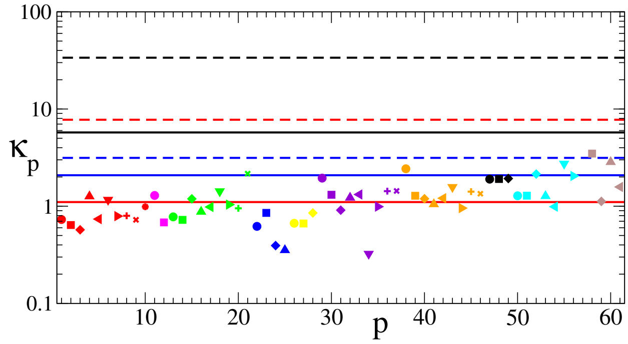

Fig. 12. PageRank-CheiRank correlators κ_p (11) from the GPVM are shown as a function of the product index p with the corresponding symbol from Figure 5. PageRank-CheiRank and ImportRank-ExportRank correlators are shown by solid and dashed lines respectively, where the global correlator κ (9) is shown in black, the correlator for countries $\kappa(c)$ (12) is shown by red lines, the correlator for products $\kappa(p)$ (12) is shown by blue lines. Here product number p is counted in order of appearance in Table 1. The data are given for year 2008 with $N_p = 61, N_c = 227, N = 13\,847$.

Google rank positions. For the product 03 we note the improvement of K^* positions of China and Argentina while Russia shows no improvements in this product trade for this time period.

4.4 Correlation properties of PageRank and CheiRank

The properties of κ correlator of PageRank and CheiRank vectors for various networks are reported in [7,9]. There are directed networks with small or even slightly negative values of κ, e.g. Linux Kernel or Physical Review citation networks, or with $\kappa \sim 4$ for Wikipedia networks and even larger values $\kappa \approx 116$ for the Twitter network.

The values of correlators defined by equations (9)–(12) are shown in Figures 12 and 13 for a typical year 2008. For the global PageRank-CheiRank correlator we find $\kappa \approx 5.7$ (9) while for Import-Export probabilities the corresponding value is significantly larger with $\kappa \approx 33.7$. Thus the trade volume ranking with its symmetry in products gives an artificial increase of κ by a significant factor. A similar enhancement factor of Import-Export remains for correlators in products $\kappa(p)$ and countries $\kappa(c)$ from equation (12) while for PageRank-CheiRank we obtain moderate correlator values around unity (see Fig. 12). The PageRank-CheiRank correlator κ_p (11) for specific products have relatively low values with $\kappa_p < 1$ for practically all products with $p \le 45$ (we remind that here p counts the products in the order of their appearance in Tab. 1, it is different from COMTRADE code number).

The correlation matrix of products $\kappa_{pp'}$ (10) is shown in Figure 13. This matrix is asymmetric and demonstrates the existence of relatively high correlations between products *73 Transport equipment*, *65 Textile yarn, fabrics, made up articles, etc.* and *83 Travel goods, handbags and similar articles* that all are related with transportation of products.

Table 4. Top 10 values of 4 different eigenvectors from Figure 16. The corresponding eigenvalues form left to right are $\lambda = 0.9548$, $\lambda = 0.9345$, $\lambda = 0.452 + i0.775$ and $\lambda = 0.424 + i0.467$. There is only one product in each of these top 10 list nodes which are: *57 Explosives and pyrotechnic products*; *06 Sugar, sugar preparations and honey*; *56 Fertilizers, manufactured*; *52 Crude chemicals from coal, petroleum and gas.*

K_i	$\lvert\psi_i\rvert$	Country	$\lvert\psi_i\rvert$	Country	$\lvert\psi_i\rvert$	Country	$\lvert\psi_i\rvert$	Country
		prod: 57		prod:06		prod:56		prod:52
1	0.052	USA	0.216	Mali	0.332	Brazil	0.288	Japan
2	0.044	Tajikistan	0.201	Guinea	0.304	Bolivia	0.279	Rep. of Korea
3	0.042	Kyrgyzstan	0.059	USA	0.274	Paraguay	0.245	China
4	0.022	France	0.023	Germany	0.031	Argentina	0.020	Australia
5	0.021	Mexico	0.021	Mexico	0.017	Uruguay	0.013	USA
6	0.018	Italy	0.021	Canada	0.009	Chile	0.012	U. Arab Em.
7	0.018	Canada	0.018	UK	0.004	Portugal	0.010	Canada
8	0.015	Germany	0.015	Israel	0.004	Angola	0.010	Singapore
9	0.013	U. Arab Em.	0.015	C. d'Ivoire	0.004	Spain	0.009	Germany
10	0.012	Qatar	0.014	Japan	0.003	France	0.008	New Zealand

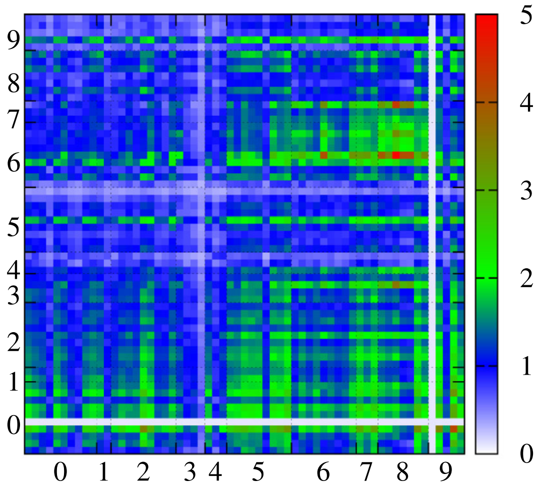

Fig. 13. Product PageRank-CheiRank correlation matrix $\kappa_{p,p'}$ (10) for year 2008 with correlator values shown by color. The code indexes p and p' of all $N_p = 61$ products are shown on x and y axes by their corresponding first digit (see Tab. 1).

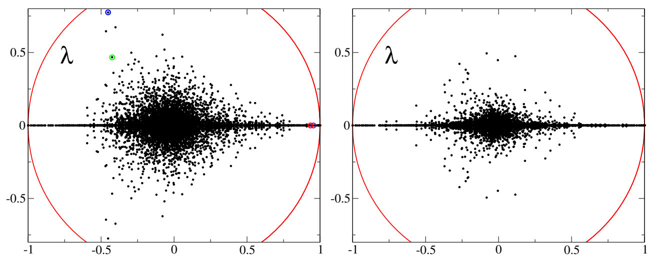

Fig. 14. Spectrum of Google matrices G (left panel) and G^* (right panel) represented in the complex plane of λ. The data are for year 2008 with $\alpha = 1$, and $N = 13\,847$, $N_c = 227$, $N_p = 61$. Four eigenvalues marked by colored circles are used for illustration of eigenstates in Figures 15 and 16.

4.5 Spectrum and eigenstates of WTN Google matrix

Above we analyzed the properties of eigenstates of G and G^* at the largest eigenvalue $\lambda = 1$. However, in total there are N eigenvalues and eigenstates. The results obtained for the Wikipedia network [26] demonstrated that eigenstates with large modulus of λ correspond to certain specific communities of the network. Thus it is interesting to study the spectral properties of G for the multiproduct WTN. The spectra of G and G^* are shown in Figure 14 for year 2008. It is interesting to note that for G the spectrum shows some similarities with those of Wikipedia (see Fig. 1 in [26]). At $\alpha = 1$ there are 12 and 7 degenerate eigenvalues $\lambda = 1$ for G and G^*, respectively. Thus the spectral gap appears only for $\alpha < 1$. The dependence of IPR ξ of eigenstates of G on $Re\lambda$ is shown in Figure 15. The results show that $\xi \ll N$ so that the eigenstates are well localized on a certain group on nodes.

The eigenstates ψ_i can be ordered by their decreasing amplitude $|\psi_i|$ giving the eigenstate index K_i with the largest amplitude at $K_i = 1$. The examples of four eigenstates are shown in Figure 16. We see that the amplitude is mainly localized on a few top nodes in agreement of small values of $\xi \sim 4$ shown in Figure 15. The top ten amplitudes of these four eigenstates are shown in Table 4 with corresponding names of countries and products. We see that for a given eigenstate these top ten nodes correspond to one product clearly indicating strong links of trade between certain countries. Thus for *06 Sugar* we see strong link between geographically close Mali and Guinea with further links to USA, Germany, etc. In a similar way for *56 Fertilizers* there is a group of Latin American countries Brazil, Bolivia, Paraguay linked to Argentina, Uruguay, etc. We see a similar situation for products 57 and 52. These results confirm the observation established in [26] for Wikipedia that the eigenstates with large modulus of λ select interesting specific network communities. We think that it would be interesting to investigate the properties of eigenstates in further studies.

4.6 Sensitivity to price variations

Above we established the global mathematical structure of multiproduct WTN and presented results on its ranking and spectral properties. Such ranking properties bring new interesting and important information about the WTN.

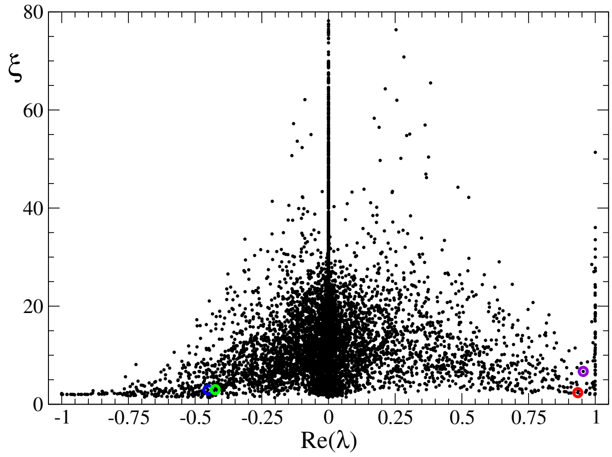

Fig. 15. Inverse participation ratio (IPR) ξ of all eigenstates of G as a function of the real part of the corresponding eigenvalue λ from the spectrum of Figure 14. The eigenvalues marked by color circles are those from Figure 14.

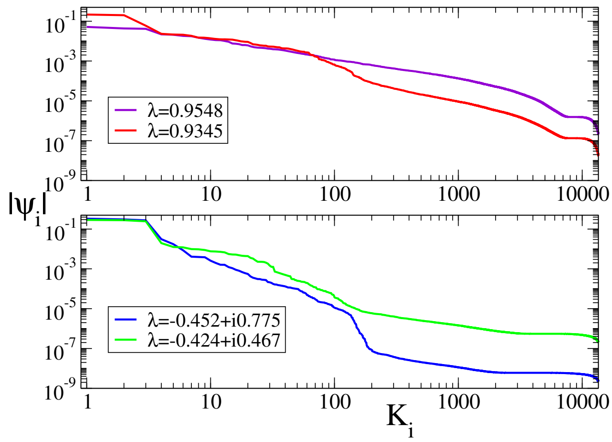

Fig. 16. Eigenstate amplitudes $|\psi_i|$ ordered by its own decreasing amplitude order with index K_i for 4 different eigenvalues of Figure 14 (states are normalized as $\sum_i |\psi_i| = 1$). Top panel shows two examples of real eigenvalues with $\lambda = 0.9548$ and $\lambda = 0.9345$ while bottom panel shows two eigenvalues with large imaginary part with $\lambda = 0.452 + i0.775$ and $\lambda = 0.424 + i0.467$. Node names (country, product) for top ten largest amplitudes of these eigenvectors are shown in Table 4.

However, from the view point of economy it is more important to analyze the effects of crisis contamination and price variations. Such an analysis represents a complex task to which we hope to return in our further investigations. However, the knowledge of the global WTN structure is an essential building block of this task and we think that the presented results demonstrate that this block is available now.

Using the knowledge of WTN structure, we illustrate here that it allows to obtain nontrivial results on sensitivity to price variations for certain products. We consider as an example year 2008 and assume that the price of product *33 Petroleum and petroleum products* is increased by a relative fraction δ going from its unit value 1 to $1+\delta$ (or $\delta = \delta_{33}$). Then we compute the derivatives of probabilities

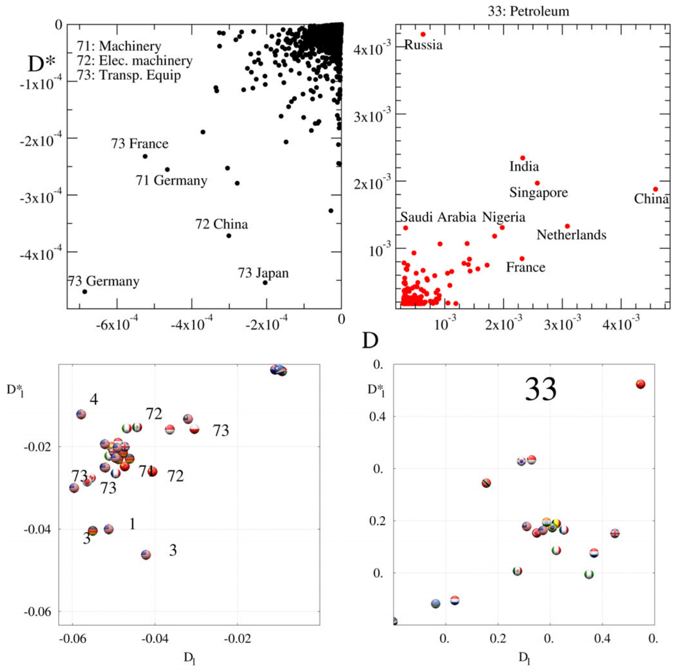

Fig. 17. Derivatives $D = dP/d\delta_{33}$ and $D^* = dP^*/d\delta_{33}$ for a price variation δ_{33} of *33 Petroleum and petroleum products* for year 2008. Top left and right panels show the cases of negative and positive D and D^* respectively, with some products and countries labeled by their 2 digit code. Bottom panels show the positive and negative cases of the logarithmic derivatives $D_l = D/P$ and $D_l^* = D^*/P^*$ for countries and products with $K_2 \leq 50$, where the flags and 2 digit codes for countries and products are shown (in right panels only product 33 is present). Codes are described in Table 1.

of PageRank $D = dP/d\delta = \Delta P/\delta$ and CheiRank $D^* = dP^*/d\delta = \Delta P^*/\delta$. The computation is done for values of $\delta = 0.01, 0.03, 0.05$ ensuring that the result is not sensitive to a specific δ value. We also compute the logarithmic derivatives $D_l = d\ln P/d\delta$, $D_l^* = d\ln P^*/d\delta$ which give us a relative changes of P, P^*.

The results for the price variation δ_{33} of *33 Petroleum and petroleum products* are shown in Figure 17. The derivatives for all WTN nodes are shown on the planes (D, D^*) and (D_l, D_l^*). For (D, D^*) the nodes are distributed in two sectors with $D > 0, D^* > 0$ and $D < 0, D^* < 0$. The largest values with $D > 0, D^* > 0$ correspond to nodes of countries of product 33 which are rich in petroleum (e.g. Russia, Saudi Arabia, Nigeria) or those which have strong trade transfer of petroleum to other countries (Singapore, India, China, etc). It is rather natural that with the growth of petroleum prices the rank probabilities P, P^* of these countries grow. A more unexpected effect is observed in the sector $D < 0, D^* < 0$. Here we see that an increase of petroleum price leads to a decrease of probabilities of nodes of countries Germany, France, China, Japan trading in machinery products $71, 72, 73$.

For comparison we also compute the derivatives D, D^*, D_l, D_l^* from the probabilities (3) defined by the trade volume of Import-Export instead of PageRank-CheiRank.

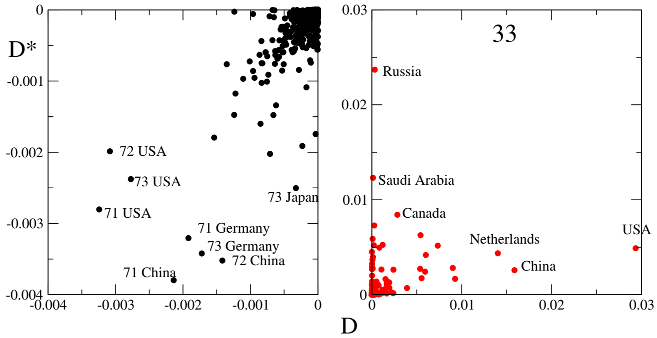

Fig. 18. Same as in top panels of Figure 17 but using probabilities from the trade volume (3).

The results are shown in Figure 18 for petroleum price variation to be compared with Figure 17. The distribution of D, D^* is rather different from those values obtained with PageRank-CheiRank probabilities. This is related to the fact that PageRank and CheiRank take into account the global network structure while the trade volume gives only local relations in trade links between countries. The difference between these two methods becomes even more striking for logarithmic derivatives D_l, D_l^*. Indeed, for the trade volume ranking the variation of probabilities P^*, P due to price variation of a given product can be computed analytically taking into account the trade volume change with δ_p. The computations give $D_{cp} = (1-f_p)P_{cp}$, $D^*{}_{cp} = (1-f_p)P^*{}_{cp}$ for a derivative of probability of product p and country c over the price of product δ_p and $D_{cp'} = -f_p P_{cp'}$, $D_{cp'} = -f_p P_{cp'}$, $D^*{}_{cp'} = -f_p P^*{}_{cp'}$ (if $p' \neq p$), where f_p is a fraction of product p in the world trade. From these expressions we see that the logarithmic derivatives are independent of country and product. Indeed, for the case of Figure 18 we obtain analytically and by direct numerical computations that $D_l = D_l^* = -0.2022$ (for all countries if $p' \neq p = 33$) and $D_l = D_l^* = 0.7916$ (for all countries if $p' = p = 33$). Due to simplicity of this case we do not show it in Figure 18.

The results for price variation of *34 Gas, natural and manufactured* are presented in Figure 19 showing derivatives of PageRank and CheiRank probabilities over δ_{34}. We see that for absolute derivatives D, D^* the mostly affected are now nodes of gas producing countries for the sector $D, D^* > 0$, while for the sector $D, D^* < 0$ the mostly affected are countries linked to petroleum production or trade, plus USA with products 71,72,73. For the sector of logarithmic derivatives $D_l, D_l^* < 0$ among top K_2 and K, K^* nodes we find nodes of countries of product 33 and also 93.

Thus the analysis of derivatives provides an interesting new information of sensitivity of world trade to price variations.

4.7 World map of CheiRank-PageRank trade balance

On the basis of the obtained WTN Google matrix we can now analyze the trade balance in various products between the world countries. Usually economists consider the export and import of a given country as it is shown in Figure 1. Then the trade balance of a given country c can be defined making summation over all products:

$$B_c = \sum_p (P^*_{cp} - P_{cp}) \,/\, \sum_p (P^*_{cp} + P_{cp}) = (P^*_c - P_c) \,/\, (P^*_c + P_c). \quad (13)$$

In economy, P_c, P^*_c are defined via the probabilities of trade volume $\hat{P}_{cp}, \hat{P}^*_{cp}$ from (3). In our approach, we define P_{cp}, P^*_{cp} as PageRank and CheiRank probabilities. In contrast to the trade volume our approach takes into account the multiple network links between nodes.

The comparison of the world trade balance obtained by these two methods is shown in Figure 20. We see that the leadership of China becomes very well visible in CheiRank-PageRank balance map while it is much less pronounced in the trade volume balance. The Google matrix analysis also highlights the dis-balance of trade network of Nigeria (strongly oriented on petroleum export and machinery import) and Sudan. It is interesting to note that the positive CheiRank-PageRank balance is mainly located in the countries of BRICS (Brazil, Russia, India, China, South Africa). In contrast to that,

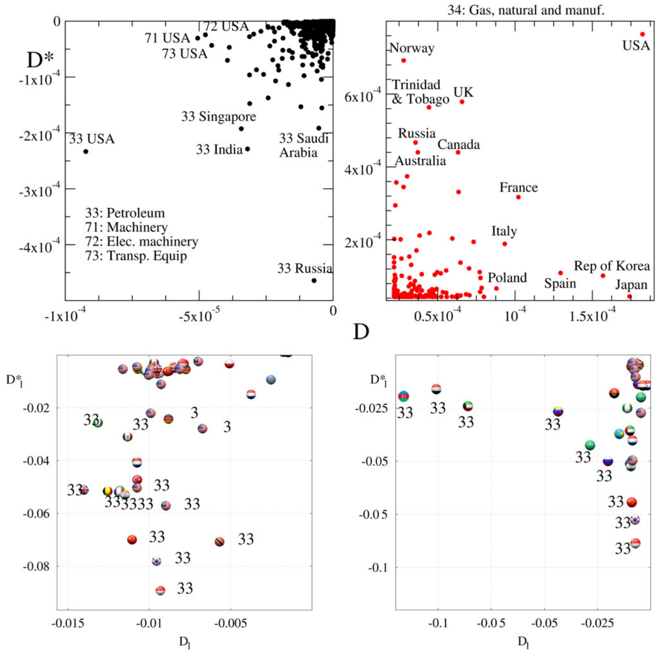

Fig. 19. Derivative of P and P^* (D and D^* respectively) for a price variation of *34 Gas, natural and manufactured* for 2008. Top left and right panels show the cases of negative and positive sectors of D and D^* respectively, with some products and countries labeled by their 2 digit code and names (in top right panel all points correspond to product 34). Bottom panels show the cases of the logarithmic derivatives D_l and D_l^* for countries and products with $K_2 \leq 50$ (bottom left panel) and $K, K^* \leq 25$ (bottom right panel); flags and 2 digit codes for countries and products are shown. In bottom right panel ($K, K^* \leq 25$) we do not show the case of Sudan (*73 Transport equipment*) which has values of $(D_l, D_l^*) = (2 \times 10^{-4}, 1.75 \times 10^{-2})$. Codes are described in Table 1.

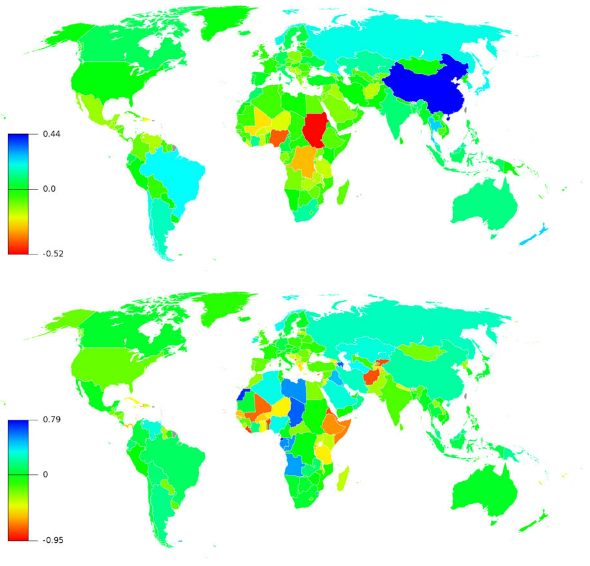

Fig. 20. World map of probabilities balance $B_c = (P_c^* - P_c)/(P_c^* + P_c)$ determined for each of $N_c = 227$ countries in year 2008. Top panel: probabilities P_c^*, P_c are given by CheiRank and PageRank vectors; bottom panel: probabilities are computed from the trade volume of Export-Import (3). Names of countries can be found at [10].

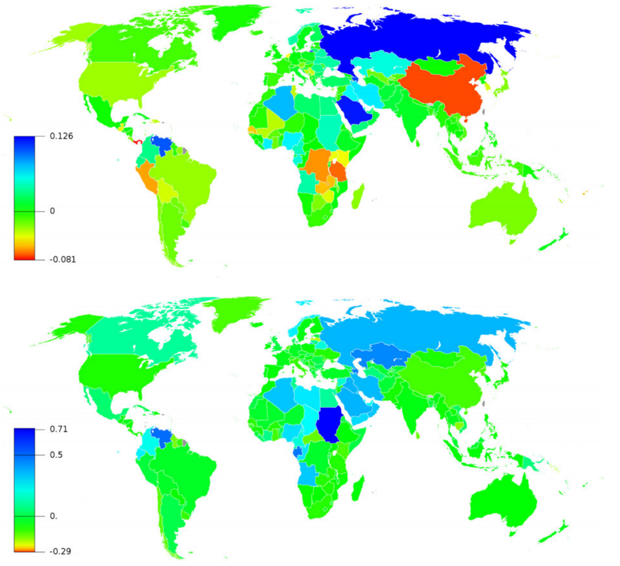

Fig. 21. Derivative of probabilities balance $dB_c/d\delta_{33}$ over petroleum price δ_{33} for year 2008. Top panel: balance of countries B_c is determined from CheiRank and PageRank vectors as in the top panel of Figure 20; bottom panel: B_c values are computed from the trade volume as in the bottom panel of Figure 20. Names of countries can be found at [10].

the usual trade volume balance highlights Western Sahara and Afghanistan at large positive and negative trade balance in 2008.

We can also determine the sensitivity of trade balance to price variation of a certain product p computing the balance derivative $dB_c/d\delta_p$. The world map sensitivity in respect to price of petroleum $p = 33$ is shown in Figure 21 for the above two methods of definition of probabilities P_c, P_c^* in (13). For the CheiRank-PageRank balance we see that the derivative $dB_c/d\delta_{33}$ is positive for countries producing petroleum (Russia, Saudi Arabia, Venezuela) while the highest negative derivative appears for China which economy is happened to be very sensitive to petroleum price. The results from the trade volume computation of $dB_c/d\delta_p$, shown in Figure 21, give rather different distribution of derivatives over countries with maximum for Sudan and minimum for the Republic of Nauru (this country has very small area and is not visible in the bottom panel of Fig. 21), while for China the balance looks to be not very sensitive to δ_{33} (in contrast to the CheiRank-PageRank method). This happens due to absence of links between nodes in the trade volume computations while the CheiRank-PageRank approach takes links into account and recover hidden trade relations between products and countries.

This absence of links in the trade volume approach becomes also evident if we consider the derivative of the partial trade balance for a given product p defined as:

$$\begin{aligned} B_{cp} &= (P_{cp}^* - P_{cp})/\sum_p (P_{cp}^* + P_{cp}) \\ &= (P_{cp}^* - P_{cp})/(P_c^* + P_c), \end{aligned} \tag{14}$$

so that the global country balance is $B_c = \sum_p B_{cp}$. Then the sensitivity of partial balance of a given product p in respect to a price variation of a product p' is given by the derivative $dB_{cp}/d\delta_{p'}$. The sensitivity for balance of product $p = 72$ (*72 Electrical machinery ...*) in respect to petroleum $p' = 33$ price variation δ_{33} is shown for the CheiRank-PageRank balance in Figure 22 (top panel) indicating sensitivity of trade balance of product $p = 72$ at the petroleum $p' = 33$ price variation. We see that China has a negative derivative for this partial balance. In contrast, the computations based on the trade volume (Fig. 22 bottom panel) give a rather different distribution of derivatives $dB_{cp}/d\delta_{p'}$ over countries. In the trade volume approach the derivative $dB_{cp}/d\delta_{p'}$ appears due to the renormalization of total trade volume and nonlinearity coming from the ratio of probabilities. We argue that the CheiRank-PageRank approach treats the trade relations between products and countries on a significantly more advanced level taking into account all the complexity of links in the multiproduct world trade.

Using the CheiRank-PageRank approach we determine the sensitivity of partial balance of all 61 products in respect to petroleum price variation δ_{33} for China, Russia and USA, as shown in Figure 23 (top panel). We see that the diagonal derivative $dB_{c33}/d\delta_{33}$ is positive for Russia but is negative for China and USA. Even if USA produce petroleum its sensitivity is negative due to a significant import of petroleum to USA. For non-diagonal derivatives over δ_{33} we find positive sensitivity of Russia and USA for products $p = 71, 72, 73$ while for China it is negative. Other product partial balances sensitive to petroleum are e.g. *84 Clothing* for China for which expensive petroleum

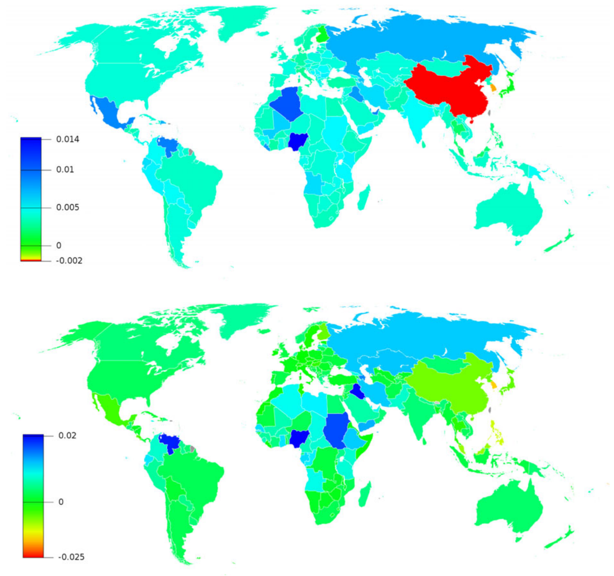

Fig. 22. Derivative of partial probability balance of product p defined as $dB_{cp}/d\delta_{33}$ over petroleum price δ_{33} for year 2008; here $B_{cp} = (P^*_{cp} - P_{cp})/(P^*_c + P_c)$ and $p = 72$ (*72 Electrical machinery ...* from Tab. 1); the product balance of countries B_{cp} is determined from CheiRank and PageRank vectors (top panel) and from the trade volume of Export-Import (3) (bottom panel). Names of countries can be found at [10].

gives an increase of transportation costs; negative derivative of balance in metal products $p = 67, 68$ for Russia due to fuel price increase; positive derivative for *93 Special transact ...* of USA.

The sensitivity of country balance B_c to price variation $\delta_{p'}$ for all products is shown in Figure 23 for China (middle panel) and USA (bottom panel). We find that the balance of China is very sensitive to $p' = 33, 84$ and indeed, these products play an important role in its economy with negative and positive derivatives, respectively. For USA the trade balance is also very sensitive to these two products $p' = 33, 84$ but the derivative is negative in both cases. We also present the derivative of balance without diagonal term $(d(B_c - B_{cp'})/d\delta_{p'})$ for China and USA. This quantity shows that for USA all other products give a positive derivative for $p' = 33$ but the contribution of petroleum import gives the global negative derivative of the total USA balance. In a similar way for China for $p' = 84$ all products, except the diagonal one $p' = 84$, give a negative sensitivity for balance but the diagonal contribution of $p' = 84$ gives the final positive derivative of China total balance in respect to δ_{84}.

The CheiRank-PageRank approach allows to determine cross-product sensitivity of partial trade balance computing the derivative $dB_{cp}/d\delta_{p'}$ shown in Figure 24 for China and USA. The derivatives are very different for two countries showing a structural difference of their economies. Thus for China the cross-derivative (at $p \neq p'$) are mainly negative (except a few lines around $p = 33$) but the diagonal terms $dB_{cp}/d\delta_p$ are mainly positive.

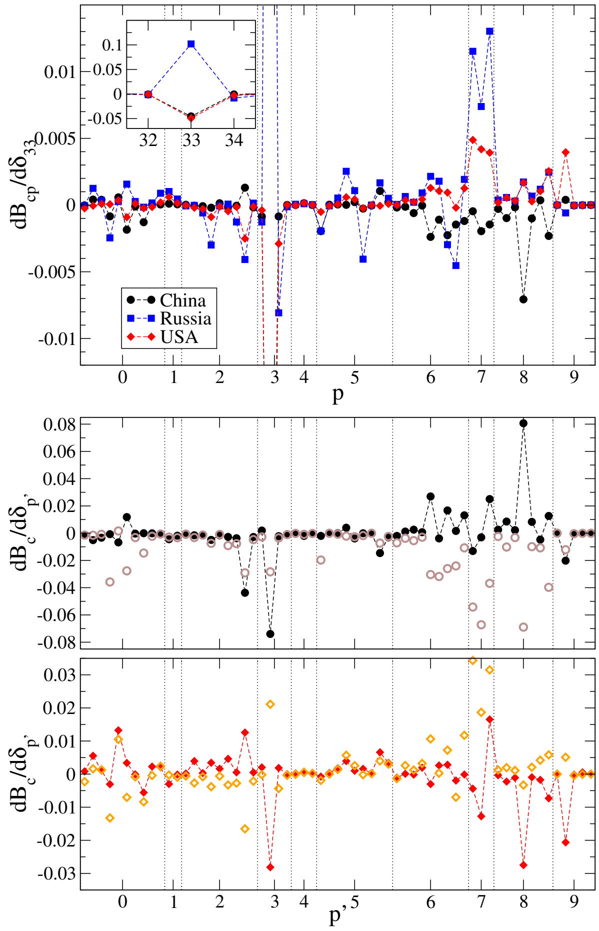

Fig. 23. Top panel: derivative $dB_{cp}/d\delta_{33}$ of partial probability balance B_{cp} of product p over petroleum price δ_{33} for year 2008 and countries: China (black circles), Russia (blue squares) and USA (red diamonds); inset panel shows the products of digit 3 including the diagonal term $p = 33$ being out of scale in the main panel; here $B_{cp} = (P^*_{cp} - P_{cp})/(P^*_c + P_c)$ (14). Center (China) and bottom (USA) panels show derivative $dB_c/d\delta_{p'}$ of country total probability balance B_c over price $\delta_{p'}$ of product p' for year 2008; derivatives of balance without diagonal term $(dB_c/d\delta_{p'} - dB_{cp'}/d\delta_{p'})$ are represented by open circles and open diamonds for China and USA, respectively. The product balance of countries B_{cp} and B_c are determined from CheiRank and PageRank vectors. The vertical dotted lines mark the first digit of product index p or p' from Table 1.

In contrast, for USA the situation is almost the opposite. We attribute this to the leading role of China in export and the leading role of USA in import. However, a detailed analysis of these cross-products derivatives and correlations require further more detailed analysis. We think that the presented cross-product sensitivity plays and important role in the multiproduct trade network that are highlighted by the Google matrix analysis developed here. This analysis allows to determine efficiently the sensitivity of multiproduct trade in respect to price variations of various products.

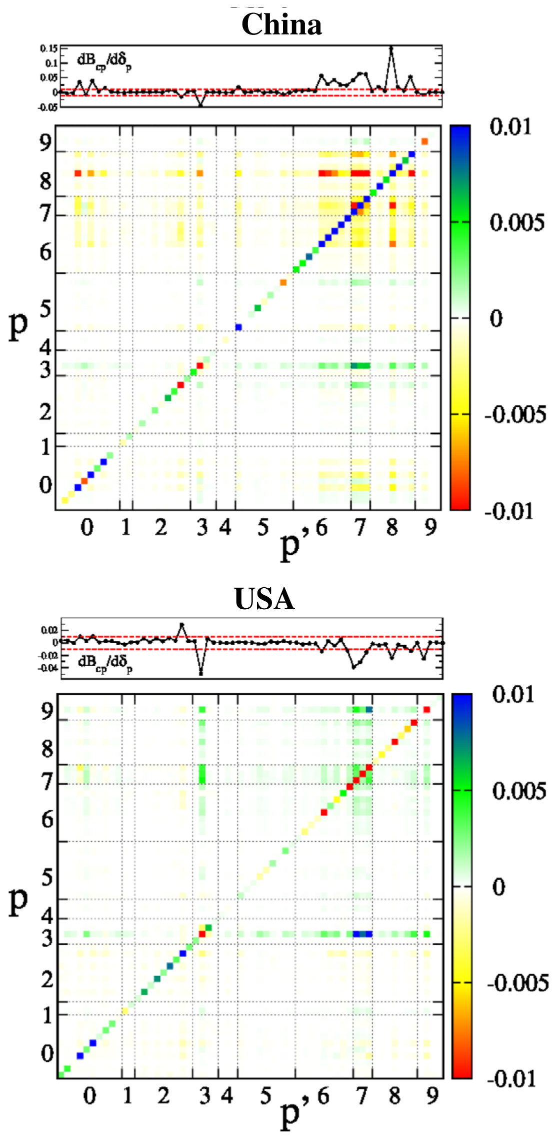

Fig. 24. China (top) and USA (bottom) examples of derivative $dB_{cp}/d\delta_{p'}$ of partial probability balance B_{cp} of product p over price $\delta_{p'}$ of product p' for year 2008. Diagonal terms, given by $dB_{cp}/d\delta_p$ vs. $p = p'$, are shown on the top panels of each example. Products p' and p are shown in x-axis and y-axis respectively (indexed as in Tab. 1), while $dB_{cp}/d\delta_{p'}$ is represented by colors with a threshold value given by -0.01 and 0.01 for negative and positive values respectively, also shown in red dashed lines on top panels with diagonal terms. Dotted lines mark the first digit of Table 1. Here B_{cp} are defined by CheiRank and PageRank probabilities.

5 Discussion

In this work we have developed the Google matrix analysis of the multiproduct world trade network. Our approach allows to treat all world countries on equal democratic grounds independently of their richness keeping the contributions of trade products proportional to their fractions in the world trade. As a result of this approach we have obtained a reliable ranking of world countries and products for years 1962−2010. The Google analysis captures the years with crises and also shows that after averaging over all world countries some products are export oriented while others are import oriented. This feature is absent in the usual Import-Export analysis based on trade volume which gives a symmetric orientation of products after such an averaging.

The WTN matrix analysis determines the trade balance for each country not only in trade volume but also in CheiRank-PageRank probabilities which take into account multiple trade links between countries which are absent in the usual Export-Import considerations. The CheiRank-PageRank balance highlights in a clear manner the leading WTN role of new rising economies of China and other BRICS countries. This analysis also allows to determine the sensitivity of trade network to price variations of various products that opens new possibilities for analysis of cross-product price influence via network links absent in the standard Export-Import analysis.

We think that this work makes only first steps in the development of WTN matrix analysis of multiproduct world trade. Indeed, the global properties of the Google matrix of multiproduct WTN should be studied in more detail since the statistical properties of matrix elements of G, shown in Figure 25 for year 2008, are still not well understood (e.g. visible patterns present in the coarse-grained representation of G in Fig. 25).

Even if the UN COMTRADE database contains a lot of information there are still open questions if all essential economic aspects are completely captured in this database. Indeed, the COMTRADE data for trade exchange are diagonal in products since there are no interactions (trade) between products. However, this feature may be a weak point of collected data since in a real economy there is a transformation of some products into some other products (e.g. metal and plastic are transferred to cars and machinery). It is possible that additional data should be collected to take into account the existing interactions between products. There are also some other aspects of services and various other activities which are not present in the COMTRADE database and which can affect the world economy. At the same time our results show that the existing COMTRADE data allow to obtain reliable results using the Google matrix analysis: thus the ranking of countries and products are reasonable being in correspondence with results of other methods. Also sensitivity to price variations is correct from the economy view point (e.g Fig. 22 showing a high sensitivity of China economy to petroleum price). We think that additional inter-product links will not modify significantly the results presented here but we expect that they will allow to characterize in a better way how one product is transfered to others in the result of the multiproduct world trade.

One of the important missing element of COMTRADE are financial flows between countries. Indeed, the product *93 Special trans ...* (see Tabs. 1 and 2) partially takes into account the financial flows but it is clear that the interbank flows are not completely reported in the database. In fact the Wold Bank Web (WBW) really exists

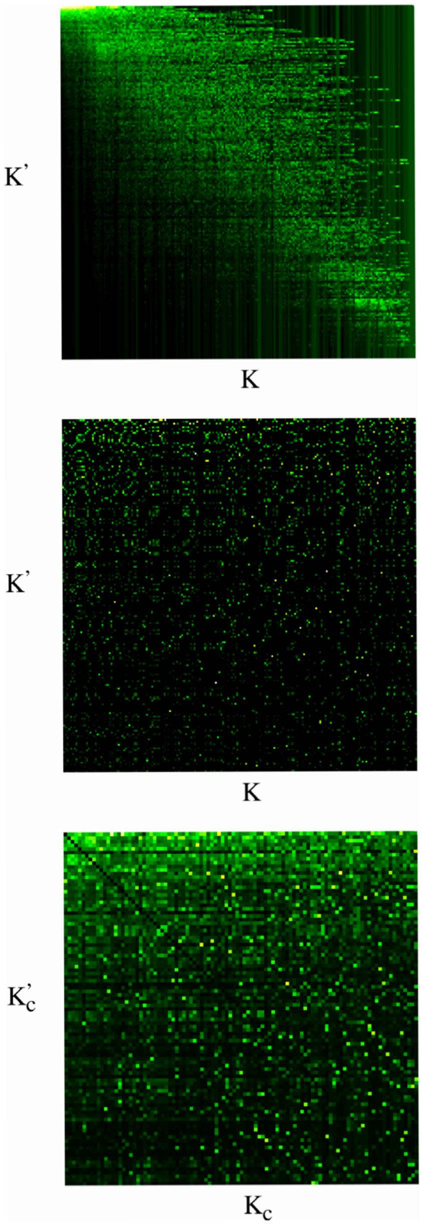

Fig. 25. Google matrix $G_{KK'}$ representation for 2008 with $\alpha = 0.5$ ordered by PageRank index Kvalue (where $K = K' = 1$ is on top left corner). Top panel shows the whole Google matrix ($N = N_c \times N_p = 227 \times 61 = 13\,847$) with coarse-graining of $N \times N$ elements down to 200×200 shown cells. Center panel represents the top corner of the full Google matrix with $K, K' \leq 200$. Bottom panel shows the coarse-grained Google matrix for countries for the top 100 countries ($K_c, K'_c \leq 100$). Color changes from black at minimal matrix element to white at maximal element, $\alpha = 0.5$.

(e.g. a private person can transfer money from his bank account to another person account using SWIFT code) but the flows on the WBW remain completely hidden and not available for scientific analysis. The size on interbank networks are relatively small (e.g. the whole Federal Reserve of USA has only $N \approx 6600$ bank nodes [27] and there are only about $N \approx 2000$ bank nodes in Germany [28]). Thus the WBW size of the whole world is about a few tens of thousands of nodes and the Google matrix analysis should be well adapted for WBW. We consider that there are many similarities between the multiproduct WTN and the WBW, where financial transfers are performed with various financial products so that the above WTN analysis should be well suited for the WBW. The network approach to the WBW flows is now at the initial development stage (see e.g. [27–29]) but hopefully the security aspects will be handled in an efficient manner opening possibilities for the Google matrix analysis of the WBW. The joint analysis of trade and financial flows between world countries would allow to reach a scientific understanding of peculiarities of such network flows and to control in an efficient way financial and petroleum crises.

The developed Google matrix analysis of multiproduct world trade allows to establish hidden dependencies between various products and countries and opens new prospects for further studies of this interesting complex system of world importance.

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