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Contagion effects in the world network of economic activities

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Abstract. Using the new data from the OECD-WTO world network of economic activities we construct the Google matrix G of this directed network and perform its detailed analysis. The network contains 58 countries and 37 activity sectors for years 1995, 2000, 2005, 2008, 2009. The construction of G, based on Markov chain transitions, treats all countries on equal democratic grounds while the contribution of activity sectors is proportional to their exchange monetary volume. The Google matrix analysis allows to obtain reliable ranking of countries and activity sectors and to determine the sensitivity of CheiRank-PageRank commercial balance of countries in respect to price variations and labor cost in various countries. We demonstrate that the developed approach takes into account multiplicity of network links with economy interactions between countries and activity sectors thus being more efficient compared to the usual export-import analysis. Our results highlight the striking increase of the influence of German economic activity on other countries during the period 1995 to 2009 while the influence of Eurozone decreases during the same period. We compare our results with the similar analysis of the world trade network from the UN COMTRADE database. We argue that the knowledge of network structure allows to analyze the effects of economic influence and contagion propagation over the world economy.

PACS. 89.75.Fb Structures and organization in complex systems – 89.65.Gh Econophysics – 89.75.Hc Networks and genealogical trees – 89.20.Hh World Wide Web, Internet

1 Introduction

The matrix analysis of Input-Out transactions had been pushed forward in the fundamental works of Leontief [1,2] becoming nowadays at the heart of modern treatment of economic relations (see e.g. [3]). The recent reports of the Organisation for Economic Co-operation and Development (OECD) [4] and of the World Trade Organization (WTO) [5] demonstrate the growing complexity of global manufactoring activities, exchange and trade in the modern world. Thus the advanced matrix methods are highly desirable for the analysis of these complex systems.

In parallel, during the last decade the modern society generated enormous communication and social networks including the World Wide Web (WWW), Wikipedia, Twitter and other directed networks (see e.g. [6]). The concept of Markov chains provides a powerful mathematical approach for analysis of such networks. In particular, the PageRank algorithm, developed by Brin and Page in 1998 [7] for the WWW information retrieval, became at the mathematical foundation of the Google search engine (see e.g. [8]). This algorithm constructs the Google matrix G of Markov chain transitions between network nodes and allows to rank billions of web pages of the WWW. The spectral and other properties of the Google matrix are analyzed in [9]. The history of the development of Google matrix methods and their links with research in social sciences and works of Leontief in economy is reviewed in in [10,11].

The results presented in [12,13] for the World Trade Network (WTN), constructed from the United Nations COMTRADE database [14], show that the Google matrix analysis is well adapted to the ranking of world countries and trade products and to determination of the sensitivity of trade to price variations of various products. The new element of such an approach is an equal ("democratic") treatment of all countries independently of their richness thus being rather different from the usual Import and Export ranking. At the same time the contributions of various products are considered being proportional to their trade value (volume) contribution in the exchange flows.

In this work we use the Google matrix methods for analysis of the contagion effects on the World Network of Economic Activities (WNEA). We use the new database of the OECD-WTO with the network of 58 countries and 37 activity sectors. At the difference of the World Trade matrix which report trade in goods between countries, the WNEA maps the imports and exports of intermediate goods and services between industries. Those globalised inter-industrial exchanges of intermediate inputs are one of the characteristics of the International Supply Chains where the production of final goods results from the combination of various industrial tasks that are internationally outsourced. The first results of the Google matrix analysis of this database have been reported in [15,16] for years 1995 and 2008 while here we analyze the time evolution for years 1995, 2000, 2005, 2008, 2009. We show that our approach gives the results being different from the usual import-export flows for individual countries represented in Fig. 1 for year 2005 (world map of countries is available at [17]). The main reason of this difference is due to the fact that the Google matrix analysis takes into account the multiplicity of links between various nodes of the network while the import-export approach provides only local information without taking into account the complex link relations between nodes.

The new element of the OECD-WTO database is that it includes the transactions between different activity sectors while the COMTRADE database for multiproduct trade has no transitions between different products (even if they exist in reality, e.g. metal and plastic are used for production of cars).

We note that there has been a number of other investigations of the WTN reported in [18,19,20,21,22,23,24]. However, in this work we have the new important elements, developed in [12,13,15]: the analysis of PageRank and CheiRank probabilities corresponding to direct and inverted network flows and related to Import and Export; democratic treatment of countries combined with the contributions of sectors (or products) being proportional to their commercial exchange fractions. We point out that the OECD-WTO TiVA database of economic activities between world countries and activity sectors has been created very recently (2013) and thus this work represents new studies of the WNEA data evolving in time, extending the results reported recently in [15].

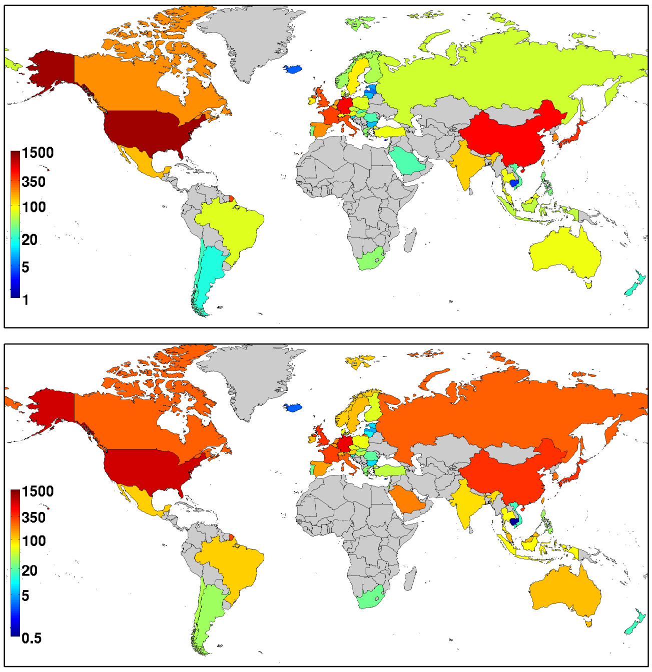

Fig. 1. World map of countries with color showing country import (top panel) and export (bottom panel) with economic activity (trade) value (volume) expressed in billions of USD and given by numbers at color bars; the gray color marks countries attributed to the ROW group (rest of the world) with exchange values 387 (Import) and 547 (Export) in billions of USD. The data are shown for year 2005 with $N_c = 57+1$ countries (with ROW) for the economic activities in all $N_s = 37$ sectors. Country names can be found in Table 1 and in the world map of countries [17].

2 Methods and data description

The list of $N_c = 58$ countries (57 plus 1 for the Rest Of the World (ROW) is given in Table 1 with their flags. Following [12] we use for countries ISO 3166-1 alpha-3 code available at Wikipedia. The list of sector activities with their names is given in Table 2 . The sectors are classified according to the International Standard Industrial Classification of All Economic Activities (ISIC) Rev.3 [25].

For a given year, the TiVA data extend OECD Input/Output tables of economic activity expressed in terms of USD for a given year. These data are tentative and had been released in 2013. A next version is expected to be available in 2015. From these data we construct the matrix $M_{cc',ss'}$ of money transfer between nodes expressed in USD:

$$M_{cc',ss'} = \text{transfer from country } c', \text{ sector } s' \text{ to } c, s \quad (1)$$

Here the country indexes are $c, c' = 1, \ldots, N_c$ and activity sector indexes are $s, s' = 1, \ldots, N_s$ with $N_c = 58$ and $N_s = 37$. The whole matrix size is $N = N_c \times N_s = 2146$. Here each node represents a pair of country and activity sector, a link gives a transfer from a sector of one country to another sector of another country. We construct the matrix $M_{cc',ss'}$ from the OECD-WTO TiVA Input/Output tables using the transposed representation so that the volume of products or sectors flows in a column from line to line. In the construction of $M_{cc',ss'}$ we exclude exchanges inside a given country in order to highlight the trade exchange flows between countries (elements inside country are zeros). The method of construction of the Google matrix from the matrix $M_{cc',ss'}$ are described in [15] (see [13] for the COMTRADE database) but for convenience of a reader we repeat this description here.

We define the value of imports V_{cs} and exports V^*_{cs} for a given country c and sector s as

$$V_{cs} = \sum_{c',s'} M_{cc',ss'} \, , \;\; V^*_{cs} = \sum_{c',s'} M_{c'c,s's}. \quad (2)$$

The import $V_c = \sum_s V_{cs}$ and export $V^*_c = \sum_s V^*_{cs}$ values for countries c are shown on the world map of countries in Fig. 1 for year 2005. We note that often one uses the notion of volume of export or import (see. e.g. [13]) but from the economic view point it more correct to speak about value of export or import.

In order to compare later with the PageRank and CheiRank probabilities used below we define exchange value ranks in the whole matrix space of dimension $N = N_c \times N_s$. Thus the ImportRank ($\hat{P}$) and ExportRank ($\hat{P}^*$) probabilities are given by the normalized import and export values

$$\hat{P}_i = V_{cs}/V \, , \;\; \hat{P}^*_i = V^*_{cs}/V \, , \qquad (3)$$

where $i = s + (c-1)N_s$, $i = 1,\ldots,N$ and the total exchange value is $V = \sum_{c,c',s,s'} M_{cc',ss'} = \sum_{c,s} V_{cs} = \sum_{cs} V^*_{cs}$.

The Google matrices G and G^* are defined as $N \times N$ real matrices with non-negative elements:

$$G_{ij} = \alpha S_{ij} + (1-\alpha) v_i e_j \, , \;\; {G^*}_{ij} = \alpha {S^*}_{ij} + (1-\alpha) v^*_i e_j \, , \qquad (4)$$

where $N = N_c \times N_s$, $\alpha \in (0,1]$ is the damping factor ($0 < \alpha < 1$), e_j is the row vector of unit elements ($e_j = 1$), and v_i is a positive column vector called a *personalization vector* with $\sum_i v_i = 1$ [8,13]. We note that the usual Google matrix corresponds to a personalization vector $v_i = e_i/N$ with $e_i = 1$. In this work, following [12,13], we fix $\alpha = 0.5$ noting that a variation of α in a range $(0.5, 0.9)$ does not significantly affect the probability distributions of PageRank and CheiRank vectors [8,9,12]. The choice of the personalization vector is specified below. Following [13] we call this approach the Google Personalized Vector Method (GPVM).

The matrices S and S^* are built from money matrices $M_{cc',ss'}$ as

$$S_{i,i'} = \begin{cases} M_{cc',ss'}/V_{c's'} & \text{if } V_{c's'} \neq 0 \\ 1/N & \text{if } V_{c's'} = 0 \end{cases}$$

$$S^*_{i,i'} = \begin{cases} M_{c'c,s's}/V^*_{c's'} & \text{if } V^*_{c's'} \neq 0 \\ 1/N & \text{if } V^*_{c's'} = 0 \end{cases} \qquad (5)$$

where $c, c' = 1,\ldots,N_c$; $s, s' = 1,\ldots,N_s$; $i = s + (c-1)N_s$; $i' = s' + (c'-1)N_s$; and therefore $i, i' = 1,\ldots,N$. Here $V_{c's'} = \sum_{cs} M_{cc',ss'}$. The sum of elements of each column of S and S^* is normalized to unity and hence the matrices G, G^*, S, S^* belong to the class of Google matrices and Markov chains. Thus S, G look at the import perspective and S^*, G^* at the export side of transactions.

PageRank and CheiRank (P and P^*) are the right eigenvectors of G and G^* matrices respectively at eigenvalue $\lambda = 1$. The equation for right eigenvectors have the form

$$\sum_j G_{ij}\psi_j = \lambda\psi_i \, , \;\; \sum_j {G^*}_{ij}{\psi^*}_j = \lambda{\psi^*}_j \; . \qquad (6)$$

For the eigenstate at $\lambda = 1$ we use the notation $P_i = \psi_i, P^* = {\psi^*}_i$ with the normalization $\sum P_i = \sum_i {P^*}_i = 1$. For other eigenstates we use the normalization $\sum_i |\psi_i|^2 = \sum_i |\psi^*_i|^2 = 1$. The eigenvalues and eigenstates of G, G^* are obtained by a direct numerical diagonalization using the standard numerical packages.

The components of P_i, ${P^*}_i$ are positive. In the WWW context they have a meaning of probabilities to find a random surfer on a given WWW node in the limit of large number of surfer jumps over network links [8]. In WNEA context nodes can be viewed and markets with a random trader transitions between them. We will use in the following notation of network nodes. We define the PageRank K and CheiRank K^* indexes ordering probabilities P and P^* in a decreasing order as $P(K) \geq P(K+1)$ and $P^*(K) \geq P^*(K^*+1)$ with $K, K^* = 1,\ldots,N$.

We note that the pair of PageRank and CheiRank vectors is very natural for economy and trade networks corresponding to Import and Export flows. For the directed networks the statistical properties of the pair of such ranking vectors have been introduced and studied in [26,27,12].

We compute the reduced PageRank and CheiRank probabilities of countries tracing probabilities over all sectors and getting $P_c = \sum_s P_{cs} = \sum_s P\left(s + (c-1)N_s\right)$ and $P^*_c = \sum_s P^*_{cs} = \sum_s P^*\left(s + (c-1)N_s\right)$ with the corresponding K_c and K^*_c indexes. In a similar way we obtain the reduced PageRank and CheiRank probabilities for sectors tracing over all countries and getting
$P_s = \sum_c P\left(s + (c-1)N_s\right) = \sum_c P_{cs}$ and
$P^*_s = \sum_c P^*\left(s + (c-1)N_s\right) = \sum_c P^*_{cs}$ with their corresponding sector indexes K_s and K^*_s. A similar procedure has been used for the multiproduct WTN data [13].

In summary we have $K_s, K^*_s = 1,\ldots,N_s$ and $K_c, K^*_c = 1,\ldots,N_c$. A similar definition of ranks from import and export exchange value can be done in a straightforward way via probabilities $\hat{P}_s, \hat{P}^*_s, \hat{P}_c, \hat{P}^*_c, \hat{P}_{cs}, \hat{P}^*_{cs}$ and corresponding indexes $\hat{K}_s, \hat{K}^*_s, \hat{K}_c, \hat{K}^*_c, \hat{K}, \hat{K}^*$.

To compute the PageRank and CheiRank probabilities from G and G^*, keeping a "democratic", or equal, treatment of countries (independently of their richness) and at the same time keeping the proportionality of activity sectors to their exchange value, we use the Google Personalized Vector Method (GPVM) developed in [13] with a personalized vector v_i in (4). At the first iteration of Google matrix we take into account the relative product value per country using the following personalization vectors for G and G^*:

$$v_i = \frac{V_{cs}}{N_c \sum_{s'} V_{cs'}} \, , \;\; v^*_i = \frac{V^*_{cs}}{N_c \sum_{s'} V^*_{cs'}} \, , \qquad (7)$$

using the definitions (2) and the relation $i = s + (c-1)N_s$. This personalized vector depends both on sector and country indexes. As for the multiproduct WTN in [13] we define the second iteration vector being proportional to the reduced PageRank and CheiRank vectors in sectors, obtained from the GPVM Google matrix of the first iteration:

$$v'(i) = \frac{P_s}{N_c} \, , \;\; v'^*(i) = \frac{P^*_s}{N_c} \, . \qquad (8)$$

In this way we keep democracy in countries but keep contribution of sectors proportional to their exchange value. This second iteration personalized vectors are used in the following computations and operations with G and G^* giving us the PageRank and CheiRank vectors. This procedure with two iterations forms our GPVM approach. The difference between results obtained from the first and second iterations is not very large, but the personalized vector for the second iteration gives a reduction of fluctuations

[15]. Below, in all Figures we show the GPVM results after the second iteration.

As for the WTN it is convenient to analyze the distribution of nodes on the PageRank-CheiRank plane (K, K^*). In addition to two ranking indexes K, K^* we use also 2DRank index K_2 which describes the combined contribution of two ranks as described in [27]. The ranking list $K_2(i)$ is constructed by increasing $K \to K+1$ and increasing 2DRank index $K_2(i)$ by one if a new entry is present in the list of first $K^* < K$ entries of CheiRank, then the one unit step is done in K^* and K_2 is increased by one if the new entry is present in the list of first $K < K^*$ entries of CheiRank. More formally, 2DRank $K_2(i)$ gives the ordering of the sequence of nodes, that appear inside the squares $[1, 1;\ K = k, K^* = k;\ ...]$ when one runs progressively from $k = 1$ to N. Additionally, we analyze the distribution of nodes for reduced indexes (K_c, K_c^*), (K_s, K_s^*).

3 Results

We apply the GPVM approach to the data sets of OECD-WTO TiVA of WNEA and present the obtained results below.

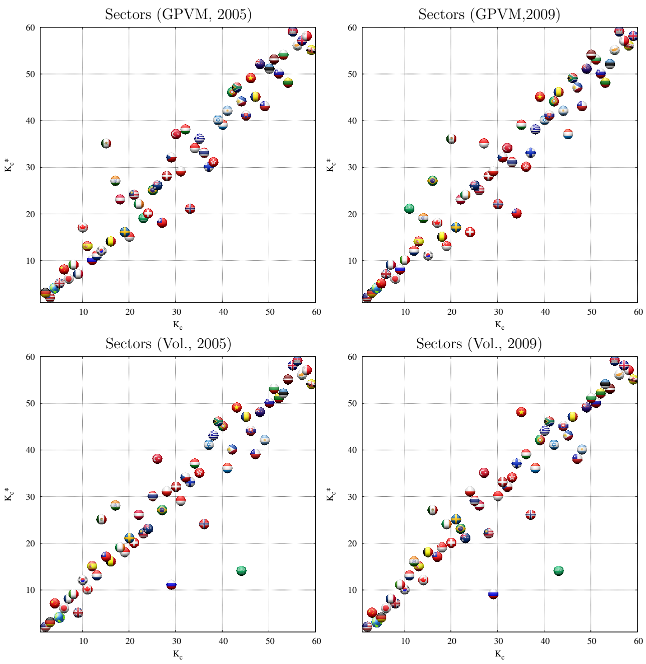

Fig. 2. Country positions on PageRank-CheiRank plane (K_c, K_c^*) obtained for the WNEA by the GPVM analysis (top panels), ImportRank-ExportRank of exchange value/volume (bottom panels). Left (right) panels show year 2005 (2009).

3.1 Ranking of countries and sectors

After tracing the probabilities $P(K), P^*(K^*)$ over sectors we obtain the distribution of world countries on the PageRank-CheiRank plane (K_c, K_c^*) presented in Fig. 2 for WNEA in years 2005, 2009. In the same figure we present the rank distributions obtained from ImportRank-ExportRank probabilities of exchange value. For the GPVM data we see the global features already discussed in [12,15]: the countries are distributed in a vicinity of diagonal $K_c = K_c^*$ since for each country the size of imports is correlated with the size of exports, even if trade is never exactly balanced and some countries can sustain significant trade surplus or deficit. The top 20 list of top K_2 countries recover 13 of 19 countries of $G20$ major world economies (EU is the number 20) thus obtaining 58% (2005) and 63% (2009) of the whole list. This is close to the percentage 68% obtained in [15] for year 2008. The Google ranking for WNEA (top panels in Fig. 2) gives different positions for specific countries. Thus Russia and Saudi Arabia go to top K_c index values in PageRank comparing to ImportRank showing that their economies are highly sensitive to activity of petroleum sector. Similar features for these two countries are visible in 1995, 2008 [15].

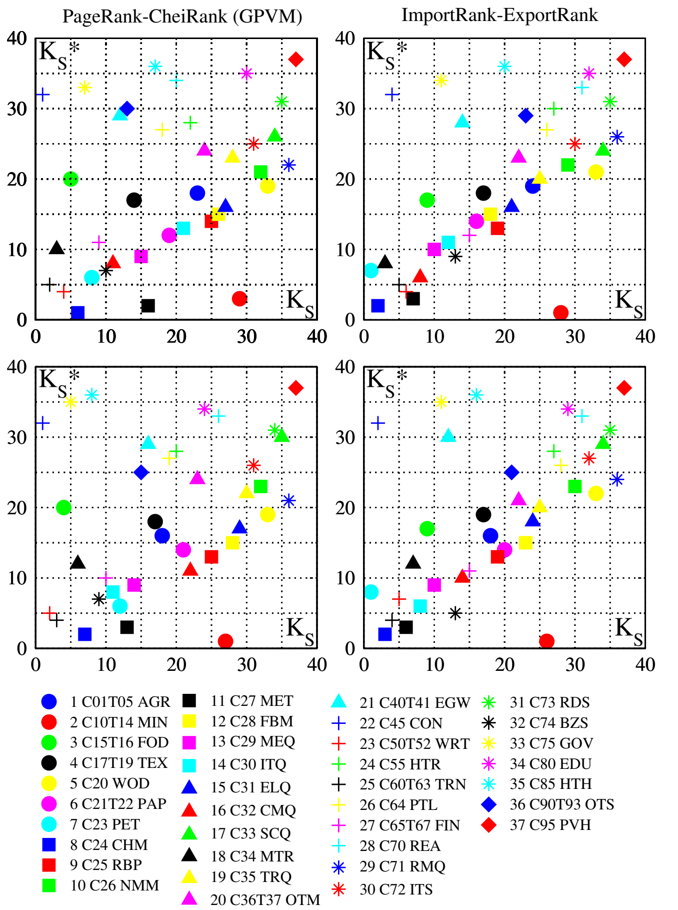

Fig. 3. Two-dimensional ranking of sectors on the (K_s, K_s^*) plane using the GPVM approach for PageRank and CheiRank (left panels) and ImportRank-ExportRank (right panels). Each sector is represented by its specific combination of color and symbol. The list of all 37 sectors are given in Table 2. Top panels are for year 2005 and bottom panels are for year 2009.

After tracing over countries we obtain the PageRank-CheiRank plane ($K_s, K_s{}^*$ of activity sectors shown in Fig. 3. As in [15] we find that some sectors are export oriented (e.g. $s = 2$ *C10T14 Mining* at $K_s^* = 1$ in 2009) others are import oriented (e.g. $s = 22$ *C45 CON Construction* at $K_s = 1$ in 2009). The ImportRanking gives a rather different import leader $s = 7$ *C23 Manufacture of coke, refined petroleum products etc.* with $K_s = 1$ in 2009. Thus the Google ranking highlights highly connected network nodes while Import-Export gives preference to high value neglecting existing network relations between various countries and activity sectors.

3.2 Price shocks and trade balance sensitivity

On the basis of the obtained WNEA Google matrix we can now analyze the trade balance in various activity sectors for all world countries. Usually economists consider the export and import of a given country as it is shown in Fig. 1. Then the trade balance of a given country c can be defined making summation over all sectors:

$$B_c = \sum_s (P^*_{cs} - P_{cs}) / \sum_s (P^*_{cs} + P_{cs}) = (P^*_c - P_c)/(P^*_c + P_c). \quad (9)$$

In economy, P_c, P^*_c are defined via the probabilities of trade value $\hat{P}_{cs}, \hat{P}^*_{cs}$ from (3). In our matrix approach, we define P_{cs}, P^*_{cs} as PageRank and CheiRank probabilities. In contrast to the Import-Export value our approach takes into account the multiple network links between nodes.

The comparison of CheiRank-PageRank balance with Export-Import balance for the world countries is shown in Fig. 4 for year 2009. Each country is shown by color which is proportional to the country balance B_c (9) with the color bar given on the figure. For Export-Import balance we see the dominance of petroleum producing countries Saudi Arabia, Russia, Norway with the largest values in 2009 and 2008 (see Fig.13 in [15]). The CheiRank-PageRank balance highlights new features placing on the top Russia, Norway, Germany, China in 2008 [15]. In fact in 2008, USA has now a slightly positive balance in top panel of Fig.13 while it was negative before in bottom panel of same figure. In 2009 after the world crisis there is a significant change for CheiRank-PageRank balance in the top panel of Fig. 4 in 2009: USA takes the leading position while Saudi Arabia becomes even negative. The variation of CheiRank-PageRank balance $\Delta B_c = B_c(2009) - B_c(2008)$ from 2008 to 2009 is shown in Fig. 5. The strongest positive variation is obtained by Ireland, USA and Japan, the strongest negative variation is for Saudi Arabia and Norway. We see that the broad network of economic activity relations and links makes the economies of the above countries more important in the world economy while Saudi Arabia, with the largest positive Export-Import balance, looses its leading position.

To analyze the sensitivity of price variation in a certain activity sector s we increase from 1 to $1 + \delta_s$ the money transfer in the sector s in $M_{cc\,ss'}$ in (1), where δ_s is a dimensionless fraction variation of price in this sector.

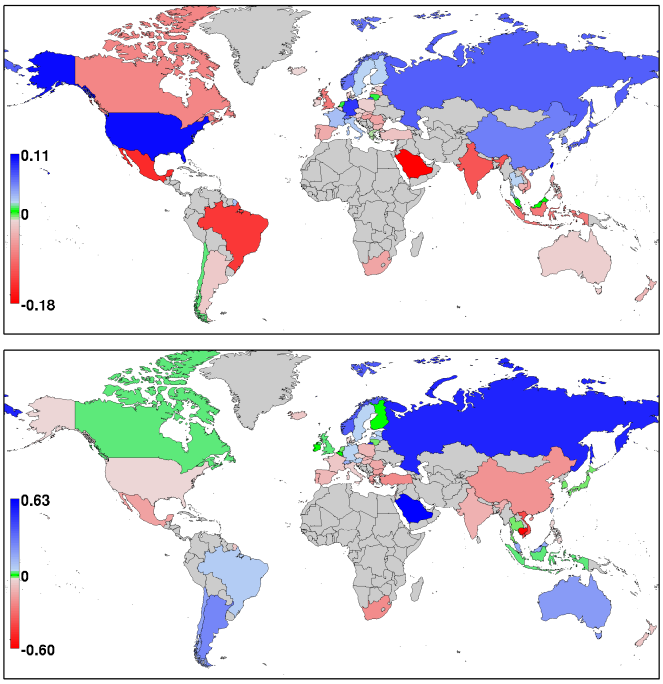

Fig. 4. World map of CheiRank-PageRank balance $B_c = (P^*_c - P_c)/(P^*_c + P_c)$ determined for all $N_c = 58$ countries in year 2009. Top panel shows the probabilities P and P^* given by PageRank and CheiRank vectors; the value of ROW group is $B_{c=58} = -0.0202$. Bottom panel shows the probabilities P and P^* computed from the Export and Import value; the value of ROW group is $B_{c=58} = 0.0637$. Names of the countries are given in Table 1 and in the world map of countries [17].

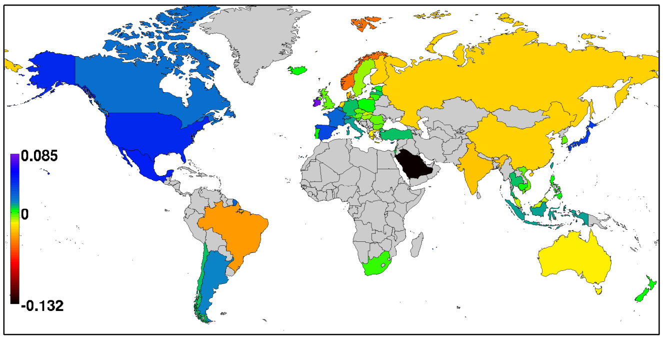

Fig. 5. Difference of CheiRank-PageRank balance $\Delta B_c = B_c(2009) - B_c(2008)$ between years 2009 and 2008 shown by color for the world countries; for the ROW group we have $\Delta B_c = -0.043$ (gray). Names of the countries can be found in Table 1 and in the world map of countries [17].

After that the matrices G, G^* are recomputed in the usual way described above and their rank probabilities P, P^* are determined. Then we compute the derivatives of probabilities balance $dB_c/d\delta_s$ over a price variation δ_s in a specific sector s. Of course, the computation is done at small values of δ_s when the derivative is independent of δ_s and all price variations are kept sufficiently small.

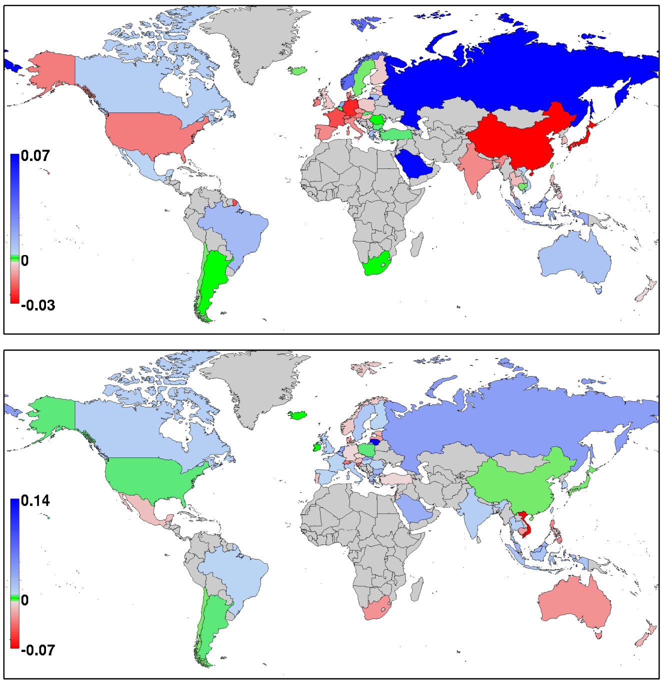

Fig. 6. Derivative of probabilities balance $dB_c/d\delta_7$ over price of sector $s = 7$ C23PET for year 2009. Top panel shows the case when B_c is determined by CheiRank and PageRank vectors as in the top panel of Fig.4; the value of ROW group is $dB_{58}/d\delta_7 = 0.0414$. Bottom panel shows the case when B_c is computed from the Export-Import value as in the bottom panel of Fig.4; the value of ROW group is $dB_{58}/d\delta_7 = -0.0637$. Names of the countries can be found in Table 1 and in the world map of countries [17].

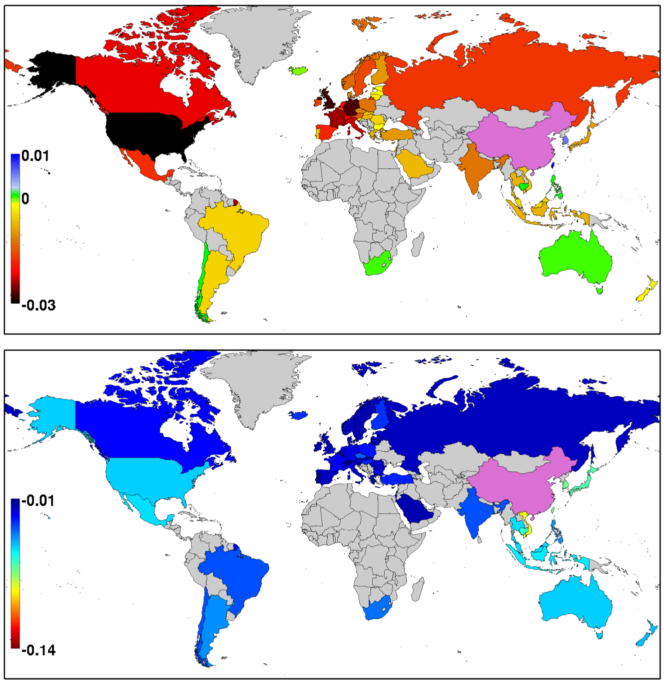

Fig. 7. Derivative of probabilities balance $dB_c/d\sigma_{c'}$ over labor cost of China $c' = 37$ for year 2009. Top panel shows the case when B_c is determined by CheiRank and PageRank vectors; here the special values are $dB_{58}/d\sigma_{37} = -0.0163$ for ROW group (gray) and $dB_{37}/d\sigma_{37} = 0.3253$ for China (magenta). Bottom panel shows the case when B_c is computed from the Export-Import value; the special values are $dB_{58}/d\sigma_{37} = -0.0381$ for ROW group (gray) and $dB_{37}/d\sigma_{37} = 0.4732$ for China (magenta). Names of the countries can be found in Table 1 and in the world map of countries [17].

The sensitivity of country balance $dB_c/d\delta_7$ to price variation of sector $s = 7$ *Manufacture of coke, refined petroleum products and nuclear fuel* is shown in Fig. 6. For Export-Import in bottom panel the most sensitive countries are Lithuania (positive) and Vietnam (negative). Lithuania does not produce petroleum, but in fact in 2008 there was a large oil refinery company there which had a large exportation value (see e.g. http://en.wikipedia.org/wiki/Economy_of_Lithuania). The Export-Import approach shows that Russia is slightly positive, even less positive is Saudi Arabia, China and Germany are close to zero change, USA is only very slightly positive. The results of CheiRank-PageRank sensitivity (top panel) are significantly different showing strongly positive sensitivity for Saudi Arabia, Russia and strongly negative sensitivity for China, Germany and Japan; USA goes from slightly positive side in bottom panel to moderate negative one in top panel. The CheiRank-PageRank balance demonstrates much higher sensitivity of Russia, Saudi Arabia and China to price variations of $s = 7$ sector comparing to the case of Export-Import value analysis. The economies of Germany, China and Japan are also very sensitive to petroleum prices that is correctly captured by our analysis. We consider that the CheiRank-PageRank approach describes the economic reality from a new complementary angle and that provides new useful information about complex trade systems. We also note that the highly negative sensitivity of China to petroleum prices has been also obtained on the basis of Google matrix analysis of COMTRADE data (see Fig.21 in [13]).

It is also possible to determine the partial balance for a given sector s and given country c and to study its sensitivity to price variations in a sector s'. We do not discuss these characteristics here but an interested reader can find these results for year 2008 in [15].

Of course, the above derivatives over price of activity sector and labor country cost give only an approximate consideration of effects of price variations which is a very complex phenomenon. For an economic discussion of the effect of price shocks on international production networks we address a reader to the research performed in [28]. We will see below that our approach gives results being in a good agreement with economic realities thus opening complementary possibilities of economic activity analysis based on the underlying network relations between countries and activity sectors which are absent in the usual Import-Export consideration. We present the results on sensitivity to sector prices and labor cost in next subsections.

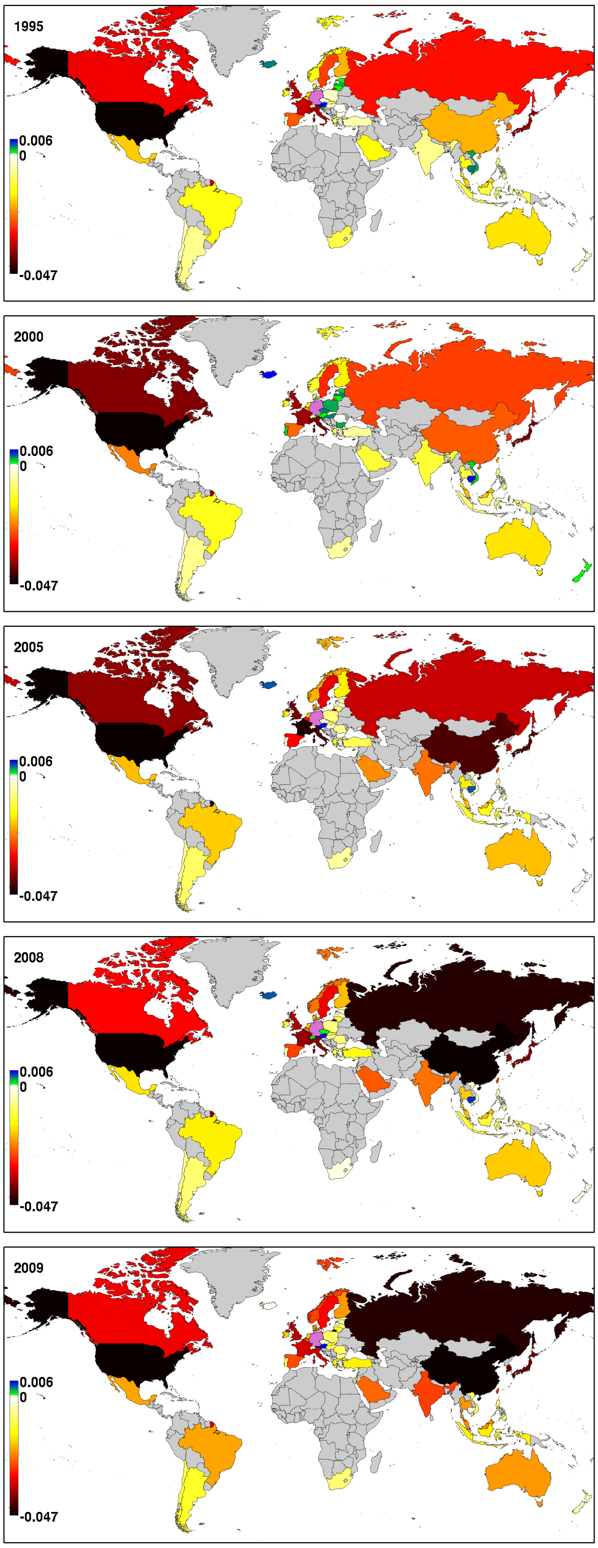

Fig. 8. Time evolution of the derivative $dB_c/d\sigma_{c'}$ over the labor cost $c' = 11$ of Germany for years 1995, 2000, 2005, 2008, 2009. For these years the special values are respectively: $dB_{58}/d\sigma_{11} = -0.0402, -0.0307, -0.0351, -0.0367, -0.0388$ for ROW group (gray); $dB_{11}/d\sigma_{11} = 0.33, 0.3274, 0.3290, 0.3248, 0.3760$ for Germany (magenta). Names of the countries can be found in Table 1 and in the world map of countries [17].

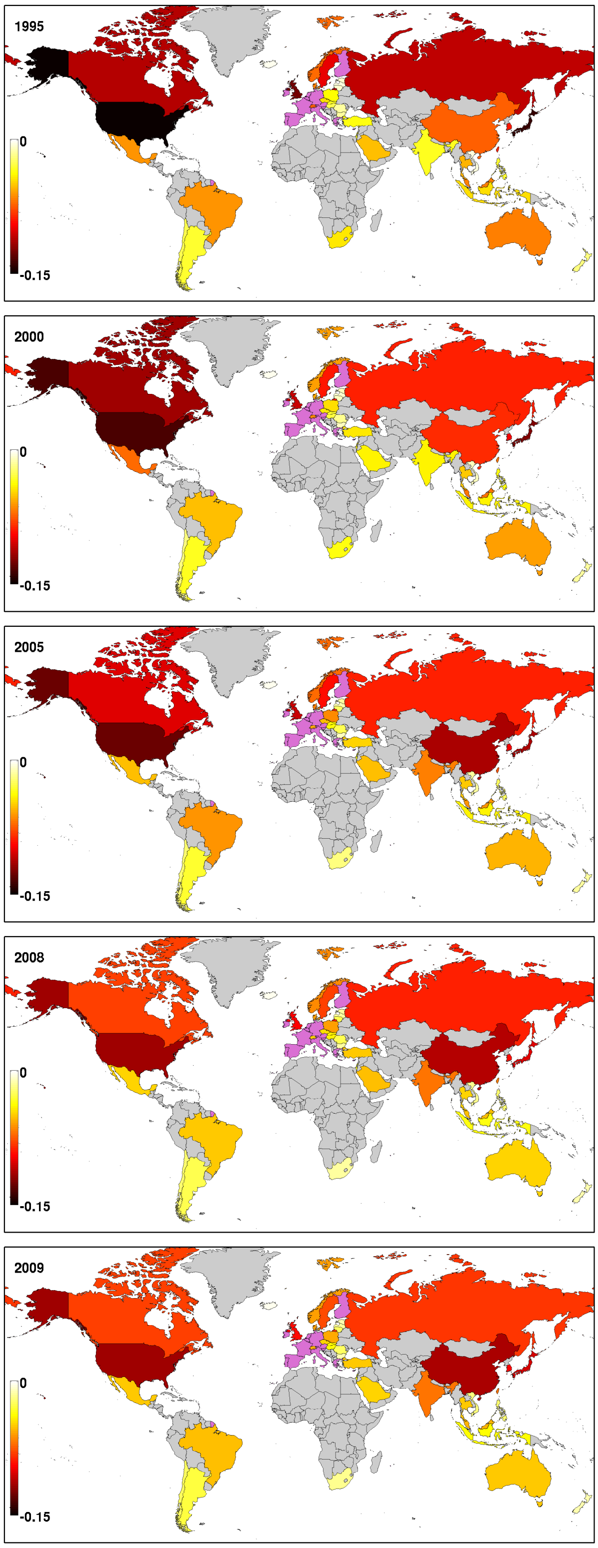

Fig. 9. Time evolution of the derivative $dB_c/d\sigma_{c'}$ over the labor cost of Eurozone monetary union (state in 2008 composed of 15 countries) for years 1995, 2000, 2005, 2008, 2009. For these years the special values are respectively: $dB_{58}/d\sigma_{ez} = -0.1259, -0.0957, -0.0992, -0.0908, -0.0921$ for ROW group (gray); and $dB_{ez}/d\sigma_{ez} = 1.8422, 1.9235, 1.9394, 1.9418, 1.9508$ for Eurozone (magenta). Eurozone is composed from 15 countries of its state in 2008: Austria ($c = 2$), Belgium (3), Finland (9), France (10), Germany (11), Greece (12), Ireland (15), Italy (17), Luxembourg (20), Netherlands (22), Portugal (26), Slovenia (28), Spain (29), Malta (56). Names of the countries can be found in Table 1 and in the world map of countries [17].

3.3 World map of sensitivity to labor cost

The new element of the WNEA, compared to the multi-product WTN, is existence of transfers between sectors of the same economy. This allows us to consider the sensitivity not only to sectoral prices but also the sensitivity to labor cost in a given country c (e.g. price shock affecting all industries in the same country). This can be taken into account by the introduction of the dimensionless labor cost change in a given country c by replacing the related monetary flows from coefficient 1 to $1 + \sigma_c$ in $M_{cc',ss;}$ (1) for a selected country c.

Using the established structure of WNEA we can study the sensitivity of country balance $dB_c/d\sigma'_c$ to the labor cost in different countries. At the difference of sectoral shocks on one product, here the price shock affects all industries in a country. As before, the change in price has to be small enough for the resulting simulation to remain in a neighbourhood of the original data. In fact we compute numerically the derivate $dB_c/d\sigma'_c$ corresponding only to small values of σ'_c. Indeed, larger shocks would trigger a series of substitution effects diverting trade to other partners. The modelling in the case of large shock variations is a very difficult task (see discussion and analysis of such situations at [29]).

The derivative $dB_c/d\sigma'_c$ is computed numerically as described above. The world sensitivity to the labor cost of China is shown for year 2009 in Fig. 7. Of course, the largest derivative is found for China itself ($dB_c/d\sigma_c$ at $c = 37$ from Table 1). The effect on other countries is given by non-diagonal derivatives at $c \neq c' = 37$. From the CheiRank-PageRank balance we find that the most strong negative effect (minimal negative $dB_c/d\sigma_{c'}$) is obtained for USA, Germany, UK; a positive derivative is visible only for Chinese Taipei ($s = 38$) and S.Korea ($s = 19$). For the Export-Import balance the results are rather different: at first all derivatives at $c \neq c'$ are negative; among the most negative values are such countries as Hong Kong (most negative with dark red color but hardly visible due to its small size), Chinese Taipei, S.Korea, Vietnam. This sensitivity map for year 2008 is given in [15], it has rather similar features. Thus the Google matrix approach brings a new perspective for analysis of complex of economic relations between countries and sectors.

Another results for the time evolution of effects of labor cost in Germany are shown in Fig. 8 for all available years from 1995 to 2009. Here we present results only for CheiRank-PageRank balance since the results of Export-Import balance, presented for year 2008 in [15] do not capture efficiently the multiple link relations. This time evolution demonstrates a spectacular increase of German influence from 1995 to 2009. Indeed, in 1995 the most strong negative sensitivity to German labor cost is visible mainly for USA but with time the influence of German economic activity extends to Russia and China capturing the large fraction of the whole world. Inside EU this influence is maximally negative in 2005 but it is slightly reduced in 2008 and 2009.

For comparison we present the same time evolution of the sensitivity to the labor cost in Eurozone in Fig. 9. Here, Eurozone is composed by 15 countries present in the Euro monetary union in 2008 and the labor variation $\sigma_{c'}$ is taken to be the same in all these countries when computing the derivative $dB_c/d\sigma_{c'}$ for all available years. It is striking to see that the evolution of Eurozone influence is opposite to Germany. Thus in 1995 we see a strong influence on USA which however decreases significantly in 2008 and 2009. The Eurozone influence on Russia (even if not so strong as for USA) also decreases with time. There is only a certain influence increase on China which however remains steady for years 2005 - 2009.

4 Discussion

In this work we have developed the Google matrix analysis of the world network of economic activities from the OECD-WTO TiVA database. The PageRank and CheiRank probabilities allowed to obtain ranking of world countries independently of their richness being mainly determined by the efficiency of their economic relations. The developed approach demonstrated the asymmetry in the economic activity sectors some of which are export oriented and others are import oriented.

The CheiRank-PageRank balance B_c allows to determine economically rising countries with robust network of economic relations. The sensitivity of this B_c to price variations and labor cost in various countries determines the hidden relations between world economies being not visible via usual Export-Import exchange analysis.

Our Google matrix analysis highlights the striking increase of the influence of German economic activity on the economy of world countries during the period 1995 to 2009. At the same time the influence of Eurozone decreases significantly.

The knowledge of network connections in WNEA allows to investigate contagion propagation over the whole world. Indeed, a significant increase of petroleum prices can produce a shock wave which will propagate over the most sensitive links highlighted in our studies. We note that we consider the case of price contagion effect. We plan to develop and study the models of such shock contagion propagation in the future works.

The comparison with the multiproduct world trade network from UN COMTRADE shows certain similarities between the two networks of WNEA and WTN. At the same time the WNEA data provides new elements for interactions of activity sectors while there are no direct interactions of products in COMTRADE database. From this viewpoint the OECD-WTO data captures the economic reality on a deeper level. But at the same time the OECD-WTO network is less developed compared to COMTRADE (less countries, years, sectors). Thus it is highly desirable to extend the OECD-WTO database.

We think that the Google matrix analysis developed here and in [12,13] captures better the new reality of multifunctional directed tensor interactions and that the universal features of this approach can be also extended to multifunctional financial network flows which now attract an active interest of researchers [30,31]. Unfortunately, the data on financial flows have much less accessibility compared to the networks discussed here.

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	country name	country code	country flag		country name	country code	country flag
1	Australia	AUS		30	Sweden	SWE	
2	Austria	AUT		31	Switzerland	CHE	
3	Belgium	BEL		32	Turkey	TUR	
4	Canada	CAN		33	United Kingdom	GBR	
5	Chile	CHL		34	United States	USA	
6	Czech Republic	CZE		35	Argentina	ARG	
7	Denmark	DNK		36	Brazil	BRA	
8	Estonia	EST		37	China	CHN	
9	Finland	FIN		38	Chinese Taipei	TWN	
10	France	FRA		39	India	IND	
11	Germany	DEU		40	Indonesia	IDN	
12	Greece	GRC		41	Russia	RUS	
13	Hungary	HUN		42	Singapore	SGP	
14	Iceland	ISL		43	South Africa	ZAF	
15	Ireland	IRL		44	Hong Kong	HKG	
16	Israel	ISR		45	Malaysia	MYS	
17	Italy	ITA		46	Phillippines	PHL	
18	Japan	JPN		47	Thailand	THA	
19	Korea	KOR		48	Romania	ROU	
20	Luxembourg	LUX		49	Vietnam	VNM	
21	Mexico	MEX		50	Saudi Arabia	SAU	
22	Netherlands	NLD		51	Brunei Darussalam	BRN	
23	New Zealand	NZL		52	Bulgaria	BGR	
24	Norway	NOR		53	Cyprus	CYP	
25	Poland	POL		54	Latvia	LVA	
26	Portugal	PRT		55	Lithuania	LTU	
27	Slovak Republic	SVK		56	Malta	MLT	
28	Slovenia	SVN		57	Cambodia	KHM	
29	Spain	ESP		58	Rest of the World	ROW	

Table 1. List of $N_c = 58$ countries (with rest of the world ROW) with country name, code and flag.

	OECD ICIO Category	ISIC Rev. 3 correspondence
1	C01T05 AGR	01 - Agriculture, hunting and related service activities 02 - Forestry, logging and related service activities 05 - Fishing, operation of fish hatcheries and fish farms; service activities incidental to fishing
2	C10T14 MIN	10 - Mining of coal and lignite; extraction of peat 11 - Extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction excluding surveying 12 - Mining of uranium and thorium ores 13 - Mining of metal ores 14 - Other mining and quarrying
3	C15T16 FOD	15 - Manufacture of food products and beverages 16 - Manufacture of tobacco products
4	C17T19 TEX	17 - Manufacture of textiles 18 - Manufacture of wearing apparel; dressing and dyeing of fur 19 - Tanning and dressing of leather; manufacture of luggage, handbags, saddlery, harness and footwear
5	C20 WOD	20 - Manufacture of wood and of products of wood and cork, except furniture; Manufacture of articles of straw and plaiting materials
6	C21T22 PAP	21 - Manufacture of paper and paper products 22 - Publishing, printing and reproduction of recorded media
7	C23 PET	23 - Manufacture of coke, refined petroleum products and nuclear fuel
8	C24 CHM	24 - Manufacture of chemicals and chemical products
9	C25 RBP	25 - Manufacture of rubber and plastics products
10	C26 NMM	26 - Manufacture of other non-metallic mineral products
11	C27 MET	27 - Manufacture of basic metals
12	C28 FBM	28 - Manufacture of fabricated metal products, except machinery and equipment
13	C29 MEQ	29 - Manufacture of machinery and equipment n.e.c.
14	C30 ITQ	30 - Manufacture of office, accounting and computing machinery
15	C31 ELQ	31 - Manufacture of electrical machinery and apparatus n.e.c.
16	C32 CMQ	32 - Manufacture of radio, television and communication equipment and apparatus
17	C33 SCQ	33 - Manufacture of medical, precision and optical instruments, watches and clocks
18	C34 MTR	34 - Manufacture of motor vehicles, trailers and semi-trailers
19	C35 TRQ	35 - Manufacture of other transport equipment
20	C36T37 OTM	36 - Manufacture of furniture; manufacturing n.e.c. 37 - Recycling
21	C40T41 EGW	40 - Electricity, gas, steam and hot water supply 41 - Collection, purification and distribution of water
22	C45 CON	45 - Construction
23	C50T52 WRT	50 - Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of automotive fuel 51 - Wholesale trade and commission trade, except of motor vehicles and motorcycles 52 - Retail trade, except of motor vehicles and motorcycles; repair of personal and household goods
24	C55 HTR	55 - Hotels and restaurants
25	C60T63 TRN	60 - Land transport; transport via pipelines 61 - Water transport 62 - Air transport 63 - Supporting and auxiliary transport activities; activities of travel agencies
26	C64 PTL	64 - Post and telecommunications
27	C65T67 FIN	65 - Financial intermediation, except insurance and pension funding 66 - Insurance and pension funding, except compulsory social security 67 - Activities auxiliary to financial intermediation
28	C70 REA	70 - Real estate activities
29	C71 RMQ	71 - Renting of machinery and equipment without operator and of personal and household goods
30	C72 ITS	72 - Computer and related activities
31	C73 RDS	73 - Research and development
32	C74 BZS	74 - Other business activities
33	C75 GOV	75 - Public administration and defense; compulsory social security
34	C80 EDU	80 - Education
35	C85 HTH	85 - Health and social work
36	C90T93 OTS	90 - Sewage and refuse disposal, sanitation and similar activities 91 - Activities of membership organizations n.e.c. 92 - Recreational, cultural and sporting activities 93 - Other service activities
37	C95 PVH	95 - Private households with employed persons

Table 2. List of sectors considered by Input/Output matrices from OECD database, their correspondence to the ISIC classification is also given.