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In the original paper, the characteristic oscillation frequency defined in the text, \( \omega_0 = \frac{2\pi}{\sqrt{2U_0/(md^2)}} \), differs by a factor \( \sqrt{2} \) from the one used in the numerical simulations. Therefore, the definition of \( \omega_0 \) given in the original paper should be replaced by \( \omega_0 = \frac{2\pi}{\sqrt{U_0/(md^2)}} \). As a consequence, the analytical estimates for the stability borders given in Eq. (4) of the original paper should read

\[
R_{\text{ref}} > \sqrt{2}R_{\alpha_0}, \quad R_{\text{ref}} < 0.45
\]  

(1)

and, after Eq. (6) of the original paper, \( R_{\alpha_0,\text{eff}} < R_{\text{ref}} / \sqrt{2} \).

These two theoretical borders of Eq. (1) are shown by straight black dashed lines in the top right panel of Fig. 1. We note that the lower border is in excellent agreement with the numerical data. The upper border is lower than the stability region centered around \( R_{\text{ref}} \approx 0.5 \). The reason is that the Chirikov overlap criterion gives the border for a chaotic transition between resonances while the destruction of resonances takes place at higher values (e.g., in the Chirikov standard map, the primary resonance becomes unstable at \( K \approx 2.5S^2 = 4 \) while the last invariant curve is destroyed at \( K \approx 0.9716 \)). In our case with two primary resonances, the secondary resonance at \( P = X = 0 \) becomes unstable at \( R_{\text{ref}} \approx 0.7 \) and \( R_{\alpha_0} = 0 \), as it is shown in the Poincare sections in Fig. A for \( R_{\alpha_0} = 0.65 \) (fixed point \( P = X = 0 \) is stable) and \( R_{\text{ref}} = 0.7 \) (fixed point \( P = X = 0 \) is unstable) at \( R_{\alpha_0} = 0 \). The refined upper stability border \( R_{\text{ref}} = 0.7 \) is shown in the top right panel of Fig. 1 by the upper horizontal red dashed line.

The estimation of experimental parameters given in Sec. V of the original paper are slightly affected and should read: With \( \omega_1 \approx 2\pi \times 200 \) Hz and a lattice spacing \( d \approx 532 \) nm, \( \bar{g} \approx 0.003 N/\sqrt{s} \), we have \( \bar{g} \approx 21 \) for \( N = 10^5 \) and a depth \( s = 200 \), and \( \bar{g}/Q \approx 0.55 \) for a BEC of typical size 20 \( \mu \)m.

These corrections leave all the conclusions unaffected.

FIG. 1. Poincaré sections formed by a few thousand trajectories with random initial conditions \((X(0), P(0)) \in [-2\xi : 2\xi] \times [-0.1 : 0.1] \) \((\xi = 1 \) or \( \pi \)) propagated during a timespan \( \Delta T = 400 \) for the frequency ratios \( R_{\alpha_0} = 0.075 \) and \( R_{\text{ref}} = 0.45 \) (top left), \( R_{\alpha_0} = 0.15 \) and \( R_{\text{ref}} = 0.2 \) (bottom left), and \( R_{\alpha_0} = 0.02 \) and \( R_{\text{ref}} = 0.7 \) (bottom right). (Top right) Stability region in the parameter space of frequency ratios. Color shows the largest initial momentum \( P_{\max}(0) \) for which trajectories with initial conditions \((X(0), P(0)) = (0, P(0)) \) remain stable for time \( T \in [0 : 1000] \). Crosses mark parameters of the Poincaré sections; dashed black lines show theory (1), upper red/gray dashed line shows refined theory taking into account secondary resonances (see text).
FIG. A. Poincaré sections for $R_0 = 0.65$ (left panels) and $R_0 = 0.7$ (right panels) at $R_0 = 0$ showing the loss of stability of the fixed point at $P = X = 0$. In these plots, $X$ stands for $(X \mod 2\pi) - \pi$. 