Chirikov standard map

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The **Chirikov standard map** [1], [2] is an area-preserving map for two canonical dynamical variables, e.g. momentum and coordinate (p,x). It is described by the equations:

$$\bar{p} = p + K \sin x \\ \bar{x} = x + \bar{p} \tag{1}$$

where bars mark the new values of variables after one map iteration and K is a dimensionless parameter that influences the degree of chaos. Due to the periodicity of $\sin x$ the dynamics can be considered on a cylinder (by taking $x \mod 2\pi$) or on a torus (by taking both $x,p \mod 2\pi$). The map is generated by the time dependent Hamiltonian $H(p,x,t) = p^2/2 + K\cos x \delta_1(t)$, where $\delta_1(t)$ is a periodic $\delta-$function with period 1 in time t. The dynamics is given by a sequence of free propagations interleaved with periodic kicks.

Examples of the Poincare sections of the standard map on a torus are shown in the following Figs. 1,2,3.

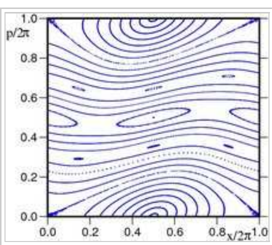

Figure 1: K=0.5

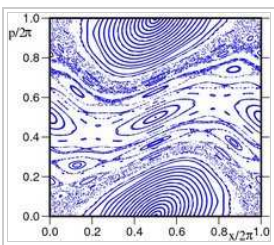

Figure 2: K=0.971635

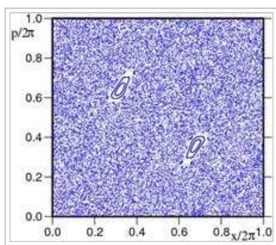

Figure 3: K=5

Below the critical parameter $K < K_c$ (Fig.1) the invariant Kolmogorov-Arnold-Moser (KAM) curves restrict the variation of momentum p to be bounded. The golden KAM curve with the rotation number $r =< p/2\pi >= r_g = (\sqrt{5}-1)/2 = 0.618033...$ is destroyed at $K = K_g = 0.971635...$ [3], [4] (Fig.2). This Fig. shows a generic phase space structure typical for various area-preserving maps with smooth generating functions: stability islands are embedded in a chaotic sea, similar structure appears on smaller and smaller scales. In a vicinity of a critical invariant curve, with a golden tail in a continued fraction expansion of r, the phase space structure is universal for all smooth maps [4]. Above the critical value $K > K_c$ (see Fig.3 showing a chaotic component and visible islands of stability) the variation of p becomes unbounded and is characterized by a diffusive growth $< p^2 >= D_0 t$ with number of map iterations t. Here D_0 is a diffusion rate with $D_0 \approx (K - K_c)^3/3$ for $K_c < K < 4$ and $D_0 \approx D_{ql} = K^2/2$ for $4 < K$ [2], [5]. There are strong arguments in favor of the equality $K_c = K_g$ but rigorously it is only proven that there are no KAM curves for $K > 63/64 = 0.984375...$ [6]. With the numerical results [3], [4] this implies inequality for the global chaos border: $K_g \le K_c < 63/64$.

A simple analytical criterion proposed in 1959 and now known as the Chirikov resonance-overlap criterion [7] gives the chaos border $K_c = \pi^2/4$ [1] and after some improvements leads to $K_c \approx 1.2$ [2],[8]. This accuracy is not so impressive compared to modern numerical methods but still up to now this criterion remains the only simple analytical tool for determining the chaos border in various Hamiltonian dynamical systems.

The Kolmogorov-Sinai entropy of the map is well described by relation $h \approx \ln(K/2)$ valid for $K > 4$ [1], [2].

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Universality and Applications

The map (1) describes a situation when nonlinear resonances are equidistant in phase space that corresponds to a local description of dynamical chaos. Due to this property various dynamical systems and maps can be locally reduced to the standard map and due to this reason the term *standard map* was coined in [2]. Thus, the standard map describes a universal, generic behavior of area-preserving maps with divided phase space when integrable islands of stability are surrounded by a chaotic component. A short list of systems reducible to the standard map is given below:

- chaotic layer around separatrix of a nonlinear resonance induced by a monochromatic force (the whisker map) [2]
- charged particle confinement in mirror magnetic traps [1], [2], [7], [9]
- fast crossing of nonlinear resonance [1], [10]
- particle dynamics in accelerators [11]
- comet dynamics in solar system [12] with a rather similar map for the comet Halley [13]
- microwave ionization of Rydberg atoms (linked to the Kepler map) [14] and autoionization of molecular Rydberg states [15]
- electron magnetotransport in a resonant tunneling diode [16]

Open Problems

- In spite of fundamental advances in ergodic theory [17], a rigorous proof of the existence of a set of positive measure of orbits with positive entropy is still missing, even for specific values of K (see e.g. [18]).
- What are the fractal properties of critical chaos parameter $K_c(r)$ as a function of arithmetic properties of the rotation number r of KAM curve? do local maxima correspond only to a golden tail of continuous fraction expansion [3], [4] or they may have tails with Markov numbers as it is conjectured in [19]? (see also [20])

- Due to trajectory sticking around stability islands the statistics of Poincare recurrences in Hamiltonian systems with divided phase space (see e.g. Fig.2 with a critical golden KAM curve) is characterized by an algebraic decay $P(\tau) \propto 1/\tau^{\alpha}$ with $\alpha \approx 1.5$ while a theory based on the universality in a vicinity of critical golden curve gives $\alpha \approx 3$; this difference persists up to 10^{13} map iterations; as a result correlation functions decay rather slowly $C(\tau) \sim \tau P(\tau) \propto 1/\tau^{\alpha-1}$ that can lead to a divergence of diffusion rate $D \sim \tau C(\tau)$ (see [21] and Refs. therein)

Quantum Map

The quantization of the standard map is obtained by considering variables in (1) as the Heisenberg operators with the commutation relation $[p,x] = -i\hbar$, where $\hbar$ is an effective dimensionless Planck constant. In a same way it is possible to use the Schrodinger equation with the Hamiltonian $H(\hat{p},\hat{x},t)$ given above and $\hat{p} = -i\hbar\partial/\partial x$. Integration on one period gives the quantum map for the wave function ψ :

$$\bar{\psi} = \hat{U}\psi = e^{-i\hat{p}^2/2\hbar} e^{-iK/\hbar \cos\hat{x}} \psi \qquad (2)$$

where bar marks the new value of ψ after one map iteration. Due to space periodicity of the Hamiltonian the momentum can be presented in the form $p = \hbar(n+\beta)$, where n is an integer and β is a quasimomentum preserved by the evolution operator $\hat{U}$. The case with $\beta = 0$ corresponds to a periodic boundary conditions with $\psi(x+2\pi) = \psi(x)$ and is known as the kicked rotator introduced in [22].

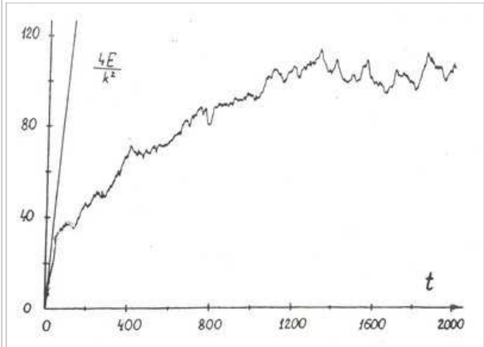

Figure 4: Dependence of rescaled rotator energy $E/(k^2/4)$ on time t for $K = kT = 5, \hbar = 0.25 (k = 20, T = 0.25)$; the full curve shows numerical data and the straight line gives the diffusive energy growth in the classical case (from [23]).

Other notations with $\hbar \to T$, $K/\hbar \to k$ are also used to mark the dependence on the period T between kicks, then $K = kT$. The diffusion rate over quantum levels n is $D = D_0/\hbar^2 = n^2/t \approx K^2/2\hbar^2 = k^2/2$, thus the rotator energy $E = <n^2>/2$ grows linearly with time. Quantum interference effects lead to a suppression of this semiclassical diffusion [22] on the diffusive time scale t_D so that the quantum probability spreads effectively only on a finite number of states $\Delta n \sim \sqrt{Dt_D}$ (Fig.4). According to the analytical estimates obtained in [23]:

$$t_D \sim \Delta n \sim D \sim k^2 \sim D_0/\hbar^2. \qquad (3)$$

This diffusive time scale is much larger than the Ehrenfest time scale [23], [24] $t_E \sim \ln(1/\hbar)/2h$ after which a minimal coherent wave packet spreads over the whole phase space due to the exponential instability of classical dynamics. For $t < t_E$ a quantum wave packet follows the chaotic dynamics of a classical trajectory as it is guaranteed by the Ehrenfest theorem [23]. For the case of Fig.4 the Kolmogorov-Sinai entropy $h \approx 1$ and the Ehrenfest time $t_E \sim 1$ is extremely short comparing to the diffusive time $t_D \sim D \sim 200$. The quantum suppression of chaotic diffusion is similar to the Anderson localization in disordered systems if to consider the level number as an effective site number in a disordered lattice, such an analogy has been established in [25]. However, in contrast to a disordered potential for the case of Anderson localization, in the quantum map (2) diffusion and chaos have a pure deterministic origin appearing as a result of dynamical chaos in the classical limit.

Due to that this phenomenon is called the dynamical localization. The eigenstates of the unitary evolution operator $\hat{U}$ are exponentially localized over momentum states $\psi_m(n) \sim \exp(-|n-m|/l)/\sqrt{l}$ with the localization length $l \sim \Delta n \sim t_D$ given by the relation [26], [27]

$$l = D(K)/2 = D_0(K)/2\hbar^2, \tag{4}$$

where D is the semiclassical diffusion expressed via a square number of levels per period of perturbation. For $\hbar = T > 1$ the chaos parameter K in the dependence $D(K)$ should be replaced by its quantum value $K \to K_q = 2k \sin T/2$ [27]. The quantum localization length l repeats the characteristic oscillations of the classical diffusion as it is shown in Fig.5. The relation (4) assumes that $T/4\pi$ is a typical irrational number while for rational values of this ratio the phenomenon of quantum resonance takes place and the energy grows quadratically with time for rational values of quasimomentum [28]. The derivations of the relation (4) based on the field theory methods applied to dynamical systems with chaotic diffusion can be find in [29], [30] (see also Refs. therein).

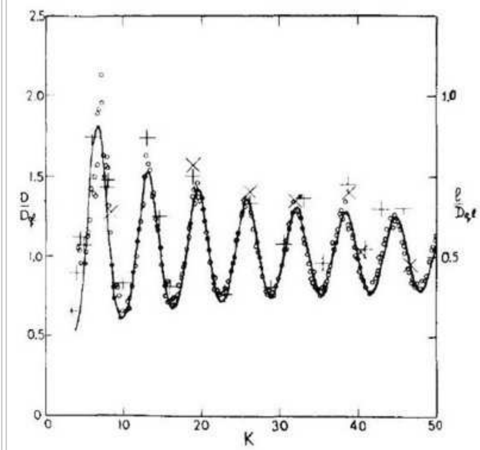

Figure 5: Dependence of the localization length l on the quantum parameter of chaos $K \to K_q = 2k \sin T/2$. The circles and the curve are, respectively, the numerical data and the theory for the classical diffusion $D(K)$ (see [8]). The quantum data for l are shown by + (for $0 < T < \pi$) and by $\times$ (for $\pi < T < 2\pi$); here $k = 30; D_{ql} = k^2/2$ (from [27]).

If the quantum map (2) is taken on a torus with N levels then the level spacing statistics is described by the Poisson law for $N \gg l$ and by the Wigner-Dyson law of the random matrix theory for $N \ll l$ [24],[31]. In the later case the quantum eigenstates are ergodic on a torus in agreement with the Shnirelman theorem and the level spacing statistics agrees with the Bohigas-Giannoni-Schmit conjecture (see books on quantum chaos in Recommended Reading).

The quantum map (2) was built up experimentally with cold atoms in a kicked optical lattice by the group of M.Raizen [32]. Such a case corresponds to a particle in an infinite periodic lattice with averaging over many various β. The quantum resonances at $\beta \approx 0$ were also experimentally observed with the Bose-Einstein condensate (BEC) in [33]. Quantum accelerator modes for kicked atoms falling in the gravitational field were found and analyzed in [34].

Extensions and Related Quantum Systems

Due to universal properties of the standard map its quantum version also finds applications for various systems and various physical effects:

- dynamical localization for ionization of excited hydrogen atoms in a microwave field

was theoretically predicted in [35] and was experimentally observed by the group of P.Koch [36] (see more details in [14],[37],[38])

- quantum particle in a triangular well and monochromatic field with a quantum delocalization transition [39]
- the kicked Harper model where in contrast to the relation (4) the quantum delocalization can take place due to quasi-periodicity of unperturbed spectrum (see [40], [41] and Refs. therein)
- 3D Anderson transition in kicked rotator with modulated kick strength and quantum transport in mesoscopic conductors (see [42] and Refs. therein)
- dissipative quantum chaos [43]
- fractal Weyl law for the quantum standard map with absorption (see [44] and Refs. therein)

Time Reversibility and Boltzmann - Loschmidt Dispute

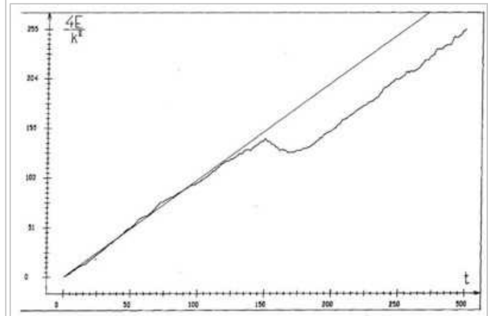

Figure 6: Dependence of rescaled energy $E/(k^2/4)$ on time in the classical map (1) at $K = 5$; time reversal is performed at $t = 150$; numerical simulations are done on BESM-6 with relative accuracy $\epsilon \approx 10^{-12}$ (from [46]).

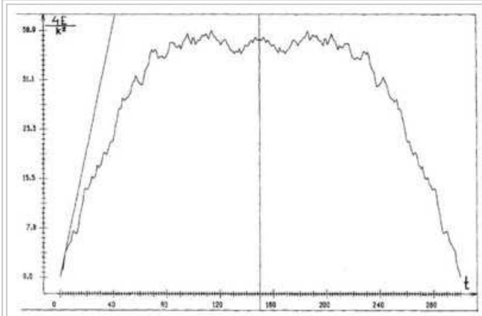

Figure 7: Same as in Fig.6 but for the quantum map (2) with $K = 5, \hbar = 0.25$, the straight line shows the classical diffusion; time reversal is performed at the moment $t = 150$ marked by the vertical line, numerical simulations are done on the same computer BESM-6, in addition random quantum phases $0 < \Delta\phi < 0.1$ are added for quantum amplitudes in momentum representation at the moment of time reversal (from [46]).

The statistical theory of gases developed by Boltzmann leads to macroscopic irreversibility and entropy growth even if dynamical equations of motion are time reversible. This contradiction was pointed out by Loschmidt and is now known as the Loschmidt paradox. The reply of Boltzmann relied on the technical difficulty of velocity reversal for material particles: a story tells that he simply said "then go and do it" [45]. The modern resolution of this famous dispute, which took place around 1876 in Wien, came with the development of the theory of dynamical chaos (see e.g. [8], [17]). Indeed, for chaotic dynamics the Kolmogorov-Sinai entropy is positive and small perturbations grow exponentially with time, making the motion practically irreversible. This fact is convenient to illustrate on the example of the standard map which dynamics is time reversible, e.g. by inverting all velocities at the middle of free propagation between two kicks (see Fig.6). This explanation is valid for classical dynamics, while the case of quantum dynamics requires special consideration. Indeed, in the quantum case the exponential growth takes place only during the rather short Ehrenfest time, and the quantum evolution remains stable and reversible in presence of small perturbations [46] (see Fig.7). Quantum reversibility in presence of various perturbations has been actively studied in recent years and is now described through the Loschmidt echo (see [47] and Refs. therein). A method of approximate time reversal of matter waves for ultracold atoms in the regime of quantum chaos, like those in [32], [33], is proposed in [48]. In this method a large fraction of the atoms returns back even if the time reversal is not perfect. This fraction of the atoms exhibits Loschmidt cooling which can decrease their temperature by several orders of magnitude. At the same time a kicked BEC of attractive atoms (soliton) described by the Gross-Pitaevskii equation demonstrates a truly chaotic dynamics for which the exponential instability breaks the time reversibility [49]. However, since a number of atoms in BEC is finite and since BEC is a really quantum object one should expect that the Ehrenfest time is still very short and hence the time reversibility should be preserved in presence of small errors if the second quantization is taken into account.

Links to Other Physical Topics

Frenkel-Kontorova Model

The Frenkel-Kontorova model describes a one-dimensional chain of atoms/particles with harmonic couplings placed in a periodic potential [50]. This model was introduced with the aim to study crystal dislocations but it also successfully applies for the description of commensurate-incommensurate phase transitions, epitaxial monolayers on the crystal surface, ionic conductors, glassy materials, charge-density waves and dry friction [51]. The Hamiltonian of the model is $H = \sum_i (\frac{P_i^2}{2} + \frac{(x_i - x_{i-1})^2}{2} - K \cos x_i)$, where P_i, x_i are momentum and position of atom i. At the equilibrium the momenta $P_i = 0$ and $\partial H/\partial x_i = 0$ so that the positions of atoms are described by the map (1) with $p_{i+1} = x_{i+1} - x_i$, $p_{i+1} = p_i + K \sin x_i$. The density of atoms corresponds to the rotation number r of an invariant KAM curve. For the golden density with $r = r_g$ the chain slides in the periodic potential for $K < K_g$ (KAM curve regime) while for $K > K_g$ the transition by the breaking of analyticity, or

Aubry transition, takes place, the chain becomes pinned and atoms form an invariant Cantor set called cantorus (see [52] and Aubry-Mather theory). In this regime the phonon spectrum has a gap so that the phonon excitations are suppressed at low temperature. The mathematical Aubry-Mather theory guarantees that the ground state of the chain exists and is unique. However there exist exponentially many static equilibrium configurations which are exponentially close to the energy of the ground state. The energies of these configurations form a fractal quasi-degenerate band structure and become mixed at any physically realistic temperature. Thus, such configurations can be viewed as a dynamical spin glass. For a case of Coulomb interactions between particles (e.g. ions or electrons) one obtains a problem of Wigner crystal in a periodic potential which again is locally described by the Frenkel-Kontorova model since the map (1) gives the local description of the dynamics. For the quantum Frenkel-Kontorova model the dynamics of atoms (ions) in the chain is quantum. In this case the quantum vacuum fluctuations and instanton tunneling lead to a quantum melting of pinned phase: above a certain effective Planck constant a quantum phase transition takes place from pinned instanton glass to sliding phonon gas (see [53] and Refs. therein).

Quantum Computing

One iteration of maps (1) and (2) can be simulated on a quantum computer in a polynomial number of quantum gates for an exponentially large vector representing a Liouville density distribution or a quantum state. The quantum algorithm of such a quantum computation is described in [54], effects of quantum errors are analyzed in [55] (see also Refs. therein).

Historical Notes

The standard map (1) in a form of recursive relation for atoms in a periodic potential appears already in the works of Kontorova and Frenkel [50]. As a dynamical map it first appeared as a description of electron dynamics in a new relativistic accelerator proposed by V.I.Veksler (Dokl. Akad. Nauk SSSR 43: 346 (1944)). The regime of a stable regular accelaration was studied later also by A.A.Kolomensky (Zh. Tekh. Fiz. 30: 1347 (1960)) and S.P.Kapitsa, V.N.Melekhin ("Microtron", Nauka, Moscow (1969) in Russian). Among the early researchers of model (1) was also British physicist J.B.Taylor (unpublished reports). The description of chaos in map (1) and its main properties, including chaos border, diffusion rate and positive entropy, was given in [1]. The term "standard map" appeared in [2], "Chirikov-Taylor map" [8] and "Chirikov standard map" [16] are also used, the quantum standard map or kicked rotator was first considered in [22]. Appearance of other terms: Kolmogorov-Arnold-Moser theory [1], Arnold diffusion [1], Kolmogorov-Sinai entropy [2], Ehrenfest time [24].

Recommended Reading

B.V.Chirikov, "Research concerning the theory of nonlinear resonance and stochasticity", Preprint N 267, Institute of Nuclear Physics, Novosibirsk (1969), (Engl. Trans., CERN Trans. 71-40 (1971))

B.V.Chirikov, "A universal instability of many-dimensional oscillator systems", Phys. Rep. 52: 263 (1979).

B.V.Chirikov, "Time-dependent quantum systems" in "Chaos and quantum mechanics", Les Houches Lecture Series, Vol. 52, pp.443-545, Eds. M.-J.Giannoni, A.Voros, J.Zinn-Justin, Elsevier Sci. Publ., Amsterdam (1991)

A.J.Lichtenberg, M.A.Lieberman, "Regular and chaotic dynamics", Springer, Berlin (1992).

F.Haake, "Quantum signatures of chaos", Springer, Berlin (2001).

L.E.Reichl, "The Transition to chaos in conservative classical systems and quantum manifestations", Springer, Berlin (2004).

Internal references

- James Meiss (2007) Hamiltonian systems. Scholarpedia, 2(8):1943.

- Martin Gutzwiller (2007) Quantum chaos. Scholarpedia, 2(12):3146.

External Links

Selected publications of Boris Chirikov [1] (http://www.quantware.ups-tlse.fr/chirikov/)

Sputnik of Chaos [2] (http://www.quantware.ups-tlse.fr/dima/chirikov.html)

Some web sites can be found by making Google search on "standard map"

References

[1] B.V.Chirikov, "Research concerning the theory of nonlinear resonance and stochasticity", Preprint N 267, Institute of (http://www.quantware.ups-tlse.fr/chirikov/refs/chi1969e.pdf) .

[2] B.V.Chirikov, "A universal instability of many-dimensional oscillator systems", Phys. Rep. 52: 263 (1979) [5] (http://www.sciencedirect.com/science?_ob=ArticleURL&_udi=B6TVP-46SPHBD-5V&_user=10&_rdoc=1&_fmt=

[3] J.M.Greene, "Method for determining a stochastic transition", J. Math. Phys. 20(6): 1183 (1979).

[4] R.S.MacKay, "A renormalization approach to invariant circles in area-preserving maps", Physica D 7(1-3): 283 (1983

[5] R.S.MacKay, J.D.Meiss, I.C.Percival, "Transport in Hamiltonian systems", Physica D 13(1-2): 55 (1984).

[6] R.S.MacKay, I.C.Percival, "Converse KAM - theory and practice", Comm. Math. Phys. 94(4): 469 (1985).

[7] B.V.Chirikov, "Resonance processes in magnetic traps", At. Energ. 6: 630 (1959) (in Russian [6] (http://www.quantw (http://www.quantware.ups-tlse.fr/chirikov/refs/chi1959e.pdf)).

[8] A.J.Lichtenberg, M.A.Lieberman, "Regular and chaotic dynamics", Springer, Berlin (1992).

[9] B.V.Chirikov, "Particle confinement and adiabatic invariance", Proc. R. Soc. Lond. A 413: 145 (1987) [8] (http://jour

[10] B.V.Chirikov, D.L.Shepelyanskii, "Diffusion during multiple passage through a nonlinear resonance", Sov. Phys. Tec (http://www.quantware.ups-tlse.fr/dima/myrefs/my009b.pdf))

[11] F.M.Izraelev, "Nearly linear mappings and their applications", Physica D 1(3): 243 (1980).

[12] T.Y.Petrowsky, "Chaos and cometary clouds in the solar system", Phys. Lett. A 117(7): 328 (1986).

[13] B.V.Chirikov, V.V.Vecheslavov, "Chaotic dynamics of comet Halley", Astron. Astrophys. 221: 146 (1989) [11] (http

[14] G.Casati, I.Guarneri, D.L.Shepelyansky, "Hydrogen atom in monochromatic field: chaos and dynamical photonic loca

[15] F.Benvenuto, G.Casati, D.L.Shepelyansky, "Chaotic autoionization of molecular Rydberg states", Phys. Rev. Lett. 72

[16] D.L.Shepelyansky, A.D.Stone, "Chaotic Landau level mixing in classical and quantum wells", Phys. Rev. Lett. 74: 20

[17] I.P.Cornfeld, S.V.Fomin, Ya.G.Sinai, "Ergodic theory", Springer, Berlin (1982).

[18] A.Giorgilli, V.F.Lazutkin, "Some remarks on the problem of ergodicity of the standard map", Phys. Lett. A 272: 359 (

[19] B.V.Chirikov, D.L.Shepelyansky, "Chaos border and statisical anomalies", Eds. D.V.Shirkov, D.I.Kazakov and A.A.V

[20] J.M.Greene, R.S.MacKay, J.Stark, "Boundary circles for area-preserving maps", Physica D 21(2-3): 267 (1986).

[21] B.V.Chirikov, D.L.Shepelyansky, "Asymptotic statistics of Poincare recurrences in Hamiltonian systems with Divided (http://www.quantware.ups-tlse.fr/dima/myrefs/my101b.pdf) .

[22] F.M.Izrailev, G.Casati, J.Ford, B.V.Chirikov, "Stochastic behavior of a quantum pendulum under a periodic perturbat (http://www.quantware.ups-tlse.fr/chirikov/refs/chi1978.pdf) ; G.Casati, B.V.Chirikov, F.M.Izrailev, J.Ford, Lecture N

[23] B.V.Chirikov, F.M.Izrailev, D.L.Shepelyansky, "Dynamical stochasticity in classical and quantum mechanics", Sov. S (http://www.quantware.ups-tlse.fr/dima/myrefs/my004.pdf) .

[24] B.V.Chirikov, F.M.Izrailev, D.L.Shepelyansky, "Quantum chaos: localization vs. ergodicity", Physica D 33: 77 (1988

[25] S.Fishman, D.R.Grempel, R.E.Prange, "Chaos, quantum recurrences, and Anderson localization", Phys. Rev. Lett. 49

[26] B.V.Chirikov, D.L.Shepelyanskii, "Localization of dynamical chaos in quantum systems", Izv. Vyssh. Ucheb. Zaved. (http://www.quantware.ups-tlse.fr/dima/myrefs/my024b.pdf)).

[27] D.L.Shepelyansky, "Localization of diffusive excitation in multi-level systems", Physica D 28: 103 (1987).

[28] F.M.Izrailev, D.L.Shepelyanskii, "Quantum resonance for a rotator in a nonlinear periodic field", Theor. Math. Phys. 4

[29] K.M.Frahm, "Localization in a rough billiard: a sigma model formulation", Phys. Rev. B 55: 8626(R) (1997).

[30] C.Tian, A.Kamenev, A.Larkin, "Weak dynamical localization in periodically kicked cold atomic gases", Phys. Rev. L

[31] F.M.Izrailev,, "Simple models of quantum chaos: spectrum and eigenfunctions", Phys. Rep. 196: 299 (1990).

[32] F.L.Moore, J.C.Robinson, C.F.Bharucha, B.Sundaram, M.G.Raizen, "Atom optics realization of the quantum δ-kick

[33] C.Ryu, M.F.Anderen, A.Vaziri, M.B.d'Arcy, J.M.Grossman, K.Helmerson, W.D.Phillips, "High-order quantum reson

[34] A.Buchleitner, M.B.d'Arcy, S.Fishman, S.A.Gardiner, I.Guarneri, Z.-Y.Ma, L.Rebuzzini, G.S.Summy, "Quantum acc

[35] G.Casati, B.V.Chirikov, D.L.Shepelyansky, ""Quantum limitations for chaotic excitation of the hydrogen atom in a m

[36] E.J.Galvez, B.E.Sauer, L.Moorman, P.M.Koch, D.Richards, "Microwave ionization of H atoms: breakdown of classic

[37] G.Casati, B.V.Chirikov, D.L.Shepelyansky, I.Guarneri, ""Relevance of classical chaos in quantum mechanics: the hy (http://www.sciencedirect.com/science?_ob=ArticleURL&_udi=B6TVP-46P3XHF-9W&_user=3521029&_rdoc=1& .

[38] P.M.Koch, K.A.H. van Leeuwen, "The importance of resonances in microwave “ionization” of excited hydrogen aton

[39] F.Benvenuto, G.Casati, I.Guarneri, D.L.Shepelyansky, "A quantum transition from localized to extended states in a cl

[40] R.Lima, D.L.Shepelyansky, "Fast delocalization in a model of quantum kicked rotator", Phys. Rev. Lett. 67: 1377 (19

[41] T.Prosen, I.I.Saija, N.Shah, "Dimer decimation and intricately nested localized-ballistic phases of a kicked Harper mo

[42] F.Borgonovi, D.L.Shepelyansky, "Two interacting particles in an effective 2-3-d random potential", J. de Physique I F 72: 1463 (1994).

[43] G.G.Carlo, G.Benenti, D.L.Shepelyansky, "Dissipative quantum chaos: transition from wave packet collapse to explo

[44] D.L.Shepelyansky, "Fractal Weyl law for quantum fractal eigenstates", Phys. Rev. E 77: 015202(R) (2008).

[45] J.E.Mayer, M.Goppert-Mayer, "Statistical mechanics", John Wiley & Sons, N.Y. (1977).

[46] D.L.Shepelyansky, "Some statistical properties of simple classically stochastic quantum systems", Physica D 8: 208 (

[47] T.Gorin, T.Prosen, T.H.Seligman, M.Znidaric, "Dynamics of Loschmidt echos and fidelity decay", Phys. Rep. 435: 3

[48] J.Martin, B.Georgeot, D.L.Shepelyansky, "Loschmidt cooling by time reversal of atomic matter waves", arxiv:0710.4

[49] F.Benvenuto, G.Casati, A.S.Pikovsky, D.L.Shepelyansky, "Manifestations of classical and quantum chaos in nonlinea

[50] T.A.Kontorova, Ya.I.Frenkel , "On theory of plastic deformation and doubling", Zh. Eksp. Teor. Fiz 8: 89 (1938); 8: 1

[51] O.M. Braun, Yu.S. Kivshar, "The Frenkel-Kontorova Model: Concepts, Methods, and Applications", Springer, Berlin

[52] S.Aubry, "The twist map, the extended Frenkel-Kontorova model and the devil's staircase", Physica D 7(1-3): 240 (19

[53] I.Garcia-Mata, O.V.Zhirov, D.L.Shepelyansky, "Frenkel-Kontorova model with cold trapped ions", Eur. Phys. J. D 41

[54] B.Georgeot, D.L.Shepelyansky, "Exponential gain in quantum computing of quantum chaos and localization", Phys. I

[55] K.M.Frahm, R.Fleckinger, D.L.Shepelyansky, "Quantum chaos and random matrix theory for fidelity decay in quantu

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Hamiltonian systems, Mapping, Chaos, Kolmogorov-Arnold-Moser Theory, Kolmogorov-Sinai entropy, Aubry-Mather theory, Quantum chaos

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