Microwave ionization of hydrogen atoms

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Microwave ionization of hydrogen atoms is a process of electron ionization of excited hydrogen atoms by an electromagnetic microwave field when tens or hundreds of photons are required to ionize one electron. Even if a microwave field is relatively weak this multiphoton ionization is much more efficient than a direct one-photon ionization at high photon energies (see Fig.1). Such a rapid ionization happens due to a diffusive growth of electron energy generated by dynamical chaos in the classical system. Quantum effects can suppress this diffusion with emergence of photonic localization which is similar to the Anderson localization in disordered solid state systems. The diffusive photoeffect was first observed in experiments of Bayfield and Koch (1974) [1], which happened to be first experiments performed in a regime of quantum chaos. The quantum effects of photonic localization were first observed by the group of Koch (1988) [2].

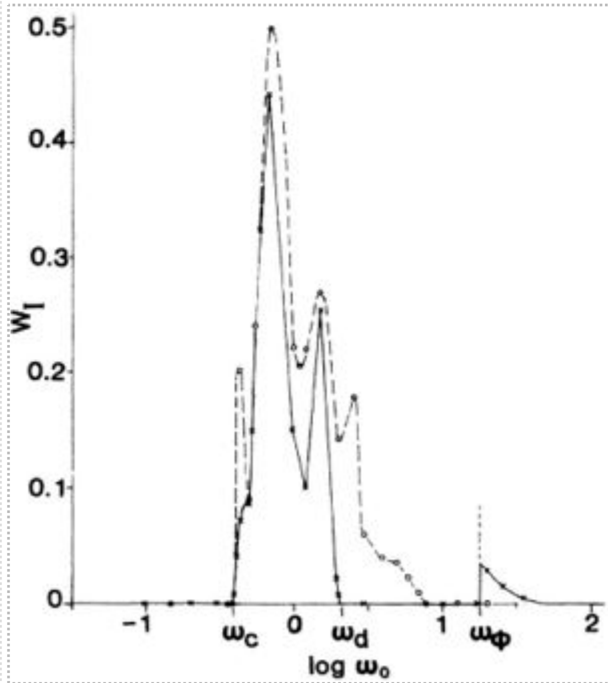


Figure 1: Fraction of ionized hydrogen atoms as a function of microwave frequency ω_0, measured in units of level spacing for initially excited level $n_0 = 66$; data are obtained by numerical simulations of classical (circles) and quantum (crosses) evolution; one-photon ionization threshold is marked by ω_ϕ (from [3])

Contents

System description

Kepler map

The classical dynamics of one-dimensional atom is well described by the Kepler map

$$\bar{N} = N + k\sin\phi, \;\; \bar{\phi} = \phi + 2\pi\omega(-2\omega\bar{N})^{-3/2} \;,$$

where $N = -1/2\omega n^2$ is a number of photons at a given electron energy, ϕ is the phase of microwave field at the perihelion of electron orbit, bars mark new values of canonical conjugate variables after one orbital period, and $k = 2.58\epsilon/\omega^{5/3}$ is dimensionless kick amplitude, corresponding to an energy change generated by a microwave field on one electron period. This symplectic map is valid for $\omega_0 \geq 1$. The second equation can be locally linearized giving $\bar{\phi} = \phi + T\bar{N}$ with $T = 6\pi\omega^2 n_0^5$. This local map is equivalent to the Chirikov standard map with the chaos parameter $K = kT = \epsilon_0/\epsilon_c$. An example of the Poincare section of the phase space is shown in Fig.2, here the electron energy $E_0 = \omega N n_0^2$, expressed via the number of photons N, is shown as the phase of microwave field $\phi = \omega t$ at the moment when electron passes via the perihelion. Locally, the dynamics in the phase space can be described by the Chirikov standard map with the global chaos and diffusion appearing at $K = \epsilon_0/\epsilon_c > 1$. As a result the chaotic diffusive ionization takes place above the chaos border [4],[5],[6]

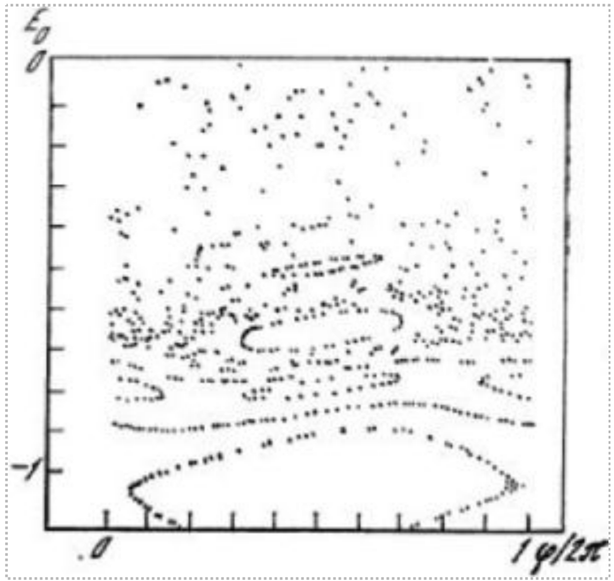


Figure 2: Poincare section of the Kepler map at $\epsilon_0 = 0.04, \omega_0 = 3$ with rescaled electron energy $E_0 = \omega N n_0^2$ and microwave phase taken at perihelion.

$$\epsilon_0 > \epsilon_c \approx 1/(49\omega_0^{1/3}).$$

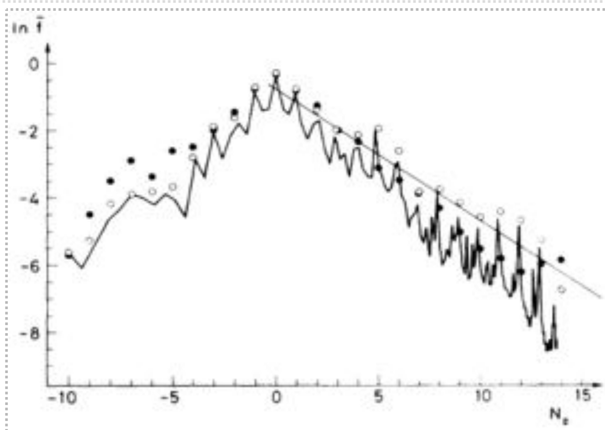
Above the chaos border the classical chaos leads to a rapid diffusive ionization which rate is significantly larger than a direct one-photon ionization, that is clearly seen in Fig.1. This diffusion process gives a sequence of absorption and emission of photons which is changes by $k\sin\phi$ at each orbital period of electron.

Chirikov localization of photonic transitions

The photonic transitions are characterized by the classical diffusion with the diffusion coefficient $D \approx k^2/2$. Under certain conditions the quantum interference effects lead to the Chirikov (or dynamical) localization of this diffusion. As for the quantum Chirikov standard map, the localization length counted in the number of photons is given by the diffusion rate [5],[6],[7]

$$\ell_\phi \approx D \approx k^2/2 = 3.33\epsilon^2/\omega^{10/3}.$$

The localization length can be also expressed in another form useful for a generic system with density levels ρ excited by a monochromatic field of strength ϵ and



one-photon transition matrix element μ:

$$\ell_\phi \approx 2\pi^2 \epsilon^2 \mu^2 \rho^2.$$

Figure 3: The Chirikov localization of photonic transitions: exponential probability distribution as a function of the number of absorbed photons $N_\phi = N_I - 1/(2\omega n^2)$ for $n_0 = 100, \epsilon_0 = 0.04, \omega_0 = 3$, open circles give probability in one-photon interval, full circles are obtained from the quantum Kepler map, the straight line shows exponential localization.

The above expressions are valid for $\rho\omega > 1, \ell_\phi > 1$. For the case of hydrogen atom we have $\rho = n^3, \mu = 0.41/(\omega^{5/3} n^3)$. This localization is similar to the Anderson localization in disordered quasi-one-dimensional systems with the important difference that here we have a purely dynamical system without any disorder. Due to localization the excitation probability on high levels drops exponentially $f_N =< |\psi|^2 > \propto \exp(-2|N - N_0|/\ell_\phi)$ as it is well seen for the results of numerical simulations shown in Fig.3. The excitation process is approximately described by the quantum Kepler map

$$\bar{\psi} = e^{-i\hat{H}_0} \hat{P} e^{-ik\cos\phi} \psi,$$

where $\hat{H}_0 = 2\pi[-2\omega(N_0 + \hat{N}_\phi)]^{-1/2}$, $N_0 = -1/(\omega n_0^2), [N_\phi, \phi] = -i$ and $\hat{P}$ is the projection operator on coupled states [6].

The quantum delocalization border takes place when the localization length becomes larger than the number of photons required to ionize one electron $\ell_\phi > N_I$ that gives

$$\epsilon_0 > \epsilon_q \approx \omega_0^{7/6}/(6.6 n_0)^{1/2} \approx 0.4\omega^{1/6}\omega_0.$$

At fixed microwave frequency the delocalization border is growing with an increase of initially excited level since $\epsilon_q \propto \omega_0 \propto n_0^3$. This theoretical prediction had been observed in the experiments of Koch group [2]. The experimental results are in good agreement with the numerical simulations based on the quantum Kepler map as it is shown in Fig.4 from [8]. Further experiments of Bayfield group with hydrogen atoms [9] and Walther group with Rydberg atoms [10] also confirmed the predictions of photonic localization theory.

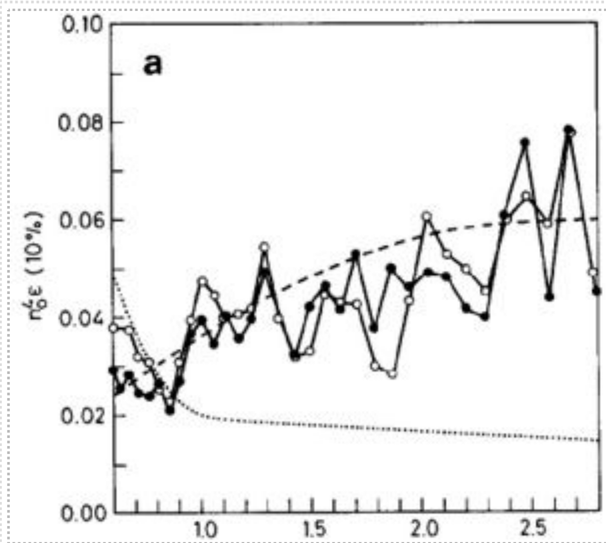


Figure 4: Scaled 10% ionization threshold filed ϵ_0 versus scaled microwave frequency $\omega_0 = \omega n_0^3$: experimental results of Koch group, taken from [2], are shown by circles, results of numerical simulations with the quantum Kepler map are shown by full circles, here $\omega/2\pi = 36.02 GHz$ and initial level changes from 45 to 80, dashed curve shows quantum delocalization border for experiment conditions, dotted curve shows classical chaos border, no fit parameters (from [8]).

Ionization of three-dimensional atoms

The above theory is based on a one-dimensional approximation of excited states of the hydrogen atom. In fact, as it is explained in [6], this approximation describes well the ionization process of real three-dimensional atoms if initial orbital number $l < (3/\omega)^{1/3}$. The physical origin of the Kepler map validity is related to the Kepler degeneracy of hydrogen atom due to which the dynamics in orbital momenta and conjugated phases is adiabatically slow and does not give significant influence on rapid dynamics in N, ϕ variables.

Chirikov localization and Anderson localization

The phenomenon of Anderson localization (1958) appears in disordered solids when a diffusive spreading in space, existing for classical trajectories, becomes exponentially localized due to quantum interference effects (a detailed description can be find in internal references and recommended reading). In systems of dynamical chaos there is no disorder but the classical diffusive spreading appears due to chaos. In a way similar to the Anderson localization this chaotic diffusion can be localized by quantum interference effects. This localization of dynamical quantum chaos is known in a literature as dynamical or Chirikov localization. This term stresses the dynamical origin of this phenomenon emerging in absence of any disorder. Here we discuss the appearence of Chirikov localization for a microwave ionization of hydrogen atoms. More examples of this phenomenon is given in the internal references below.

Stabilization in strong fields

At very strong fields the classical dynamics of three-dimensional atom becomes stable and ionization is suppressed. This stabilization regime exists for the fields in the range $10\omega/(|m|+1) < \epsilon < 20\omega^2 n_0^2/(|m|+1)^2$, where m is initial magnetic quantum number [11]. The analogies between this stabilization, the Kapitsa pendulum and channeling of charged particle beams in crystals are discussed in [11]. The stabilization theory still has not been verified experimentally.

Related physical systems

- **Ionization of chaotic Rydberg atoms:** A hydrogen atom is an integrable system and a microwave field should be relatively strong to induce chaotic diffusive ionization. However, it is possible to have atoms which are chaotic in absence of microwave field: it can be hydrogen or Rydberg atoms in a magnetic field, or Rydberg atoms in a static electric field. For such atoms a diffusive excitation can take place even when the microwave frequency is significantly smaller than the Kepler frequency so that hundreds of photons are required to ionize one electron. The properties of photonic localization for such atoms are analyzed in [12]. A similar type of photonic localization appears for microwave excitation of noninteracting electrons in metallic quantum dots of micron size, which can be viewed as artificial Rydberg atoms, the theory of photonic localization in such systems is described in [13].
- **Chaotic autoionization of molecular Rydberg states:** For Rydberg states of a molecule there is a coupling between rotations of dipole moment of charged molecular core and electron excited states. This coupling can be described in the frame of the Kepler map which under certain conditions leads to a diffusive autoionization of molecular Rydberg states as described in [14].
- **Capture of dark matter by the Solar System:** The capture of dark matter particles scattering on the Sun with a rotating planet is a process inverse to ioinization. This capture process is also described by a simple map which is similar to the Kepler map. The energies of particles which can be captured and the capture cross-section are analytically determined in [15]. According to these results the cross-section diverges as an inverse particle energy being much larger than the planetary orbit area.

Historical notes

The striking experiments of Bayfield and Koch [1] done at Yale in 1974 remained for a long time as a theoretical puzzle. In 1978 Delone, Zon and Krainov [16] proposed to describe the excitation process by a diffusion equation in energy using random phase approximation and assuming that this approach is valid at $\epsilon > 1/n^5$, when the field perturbation becomes larger than the level spacing. This gave a correct estimate of the diffusion rate and ionization time scale but a wrong ionization border which goes to zero in the semiclassical limit. Independently, in 1978 Leopold and Percival [17] performed numerical simulations of classical dynamics showing that such an approach gives a fraction of ionized atoms being close to the experimental values.

However, the discussion of dynamical chaos as the origin of strong ionization appears only in the work of Meerson, Oks and Sasorov in 1979 [18], where they give a correct estimate of the ionization threshold on the basis of the Chirikov criterion of resonance overlap for the case of $\omega_0 \sim 1$. The dependence of the chaos border on frequency was found in [4]. The first signatures of quantum suppression of classical diffusive excitations have been found in [19] and later in advanced studies [20]. The numerical codes for quantum evolution developed in [19] were used in [3],[5],[6],[9],[20],[21]. The analytical theory of photonic localization was developed in [5],[6],[20],[21] and it was confirmed by extensive numerical simulations performed in these works. The first experimental confirmations of this theory have been obtained by the Koch group [2] and later by the groups of Bayfield [9] and Walther [10]. The description of ionization by the classical and quantum Kepler map was developed in [6],[21]. The process of microwave ionization was studied also by Jensen [22], Blumel and Smilansky [23]. The classical Kepler map for the hydrogen ionization was also obtained in [24], for comet dynamics in the Solar system this map was derived by Petrosky [25]; map description of the dynamics of the comet Halley was developed by Chirikov and Vecheslavov [26]. Additional references and results for microwave ionization of excited atoms can be find in the reviews [27],[28],[29]. More recent experiments on microwave ionization of Rydberg atoms are presented in [30]. The term Chirikov localization was introduced in [31] to honor the pioneering contribution of Boris Chirikov in the discovery and investigation of this phenomenon.

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