Anderson transition in three and four effective dimensions for the frequency modulated kicked rotator

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The critical exponents for the Anderson transition in three and four effective dimensions are discussed on the basis of previous data obtained for the frequency modulated kicked rotator. Without appeal to a scaling function they are shown to be in a satisfactory agreement with the theoretical relation known for them.

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Recent experiments of the Garreau group with kicked cold atoms [1–3] renewed interest to the frequency modulated kicked rotator (FMKR), which had been introduced and studied some time ago [4–7]. The evolution of the quantum system is described by the unitary propagation operator

$$\hat{S}_1 = \exp(-iH_0(\hat{n}))\exp(-iV(\theta,t)) \qquad (1)$$

with quasiperiodic frequency modulation of kick potential $V(\theta,t) = V(\theta,\theta_1,\theta_2)$ with $\theta_{1,2} = \omega_{1,2}t$ at $d = 3$ and $V(\theta,t) = V(\theta,\theta_1,\theta_2,\theta_3)$ with $\theta_{1,2,3} = \omega_{1,2,3}t$ at $d = 4$. Here, the notations following those of [6].

The model with arctangent kick rotator (AKR) potential was considered in [5, 7], while the frequency modulated kicked rotator (FMKR) corresponding to the quantum Chirikov standard map with modulated kick amplitude had been analyzed in [4, 6].

In view of this interest I reconsider in more detail the results presented in [6] for FMKR in effective dimensions $d = 3, 4$ (data of Figs.1,9 in [6] respectively). In the case $d = 3$ we have $V(\theta,t) = k\cos\theta(1 + \epsilon\cos\omega_1 t\cos\omega_2 t)$ with fixed $\epsilon = 0.75$, irrational frequencies $\omega_{1,2}$ and random but fixed in time rotational phases $H_0(n)$ (see [6] for detailed notations). The similar choice of $V(\theta,t) = k\cos\theta(1 + \epsilon\cos\omega_1 t\cos\omega_2 t\cos\omega_3 t)$, $\epsilon = 0.9$ is done for the case of $d = 4$ (data of Fig.9 in [6]). Let me note that in the experiment [1–3] the rotational phases correspond to a free propagation with $H_0(n) = Tn^2/2$ with the classical chaos parameter $K = kT$. However, even for one modulation frequency the chaos border for destruction of two-frequency invariant torus is very low so that the quantum chaotic dynamics mimics rather well random quantum phases $H_0(n)$. Indeed, according to Fig.5 in [8] the invariant torus with two spiral mean frequencies is destroyed at $K \approx 0.3$ for $\epsilon \approx 0.75$ that is significantly smaller than the experimental values with $K \approx 6$, $T \approx 2.89$, $k \approx 2$. On the basis of these arguments it is natural to expect that the FMKR with random phases has transition approximately at the same parameters as for the FMKR model with quadratic rotational phases (e.g. with $T = 2$). This was confirmed by the first numerical simulations for the case of quadratic rotational phases performed in the proposal of Garreau experiment in 2005 [9] (done for effective $d = 3$ at $\epsilon = 0.75$, $T = 2$ with estimated critical $k_c \approx 1.8$). Hence, the both models are rather similar. The transition border found in numerical simulations [1–3] is also in agreement with this statement. Indeed, according to the data of Fig.1 in [1] one finds $k = K/T = K/\hbar = 1.88 \pm 0.035$ for $\epsilon = 0.75$ and $T = \hbar = 2.89$ that agrees with the critical value $k_c = 1.8$ given in [6] for the case with random rotation phases.

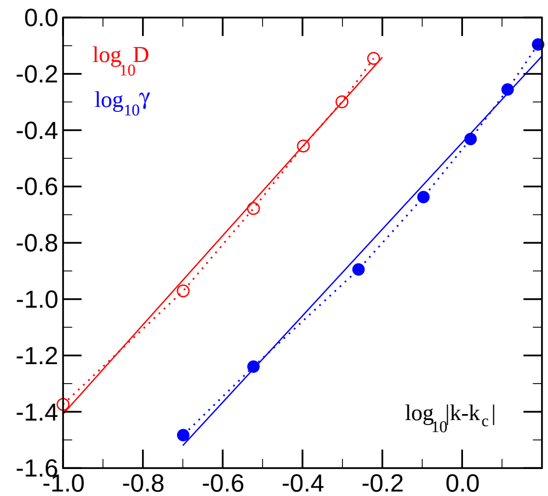

FIG. 1: (Color online) Power law dependence for the inverse localization length $\gamma = 1/l_1$ and diffusion rate D obtained at asymptotically large times t at FMKR with $\epsilon = 0.75$ (in a vicinity of critical point $t \geq 10^6$). Data are taken from Fig.1 of [6] and are plotted for the fixed value of critical parameter $k_c = 1.8$ for dimension $d = 3$ with full circles for γ and open circles for D (dotted lines are drown to adapt an eye). The straight lines show the power law fits with fixed k_c and $\nu = 1.537 \pm 0.0539$, $\log_{10}\gamma_0 = -0.444 \pm 0.0192$ and $s = 1.583 \pm 0.0511$, $\log_{10} D_0 = 0.175 \pm 0.0301$.

The data of Figs.1,9 of [6] clearly show the presence of Anderson transition at $k_c \approx 1.8$ ($d = 3$) and $k_c \approx 1.15$ ($d = 4$). These data give the inverse localization length $\gamma = 1/l_1$ extracted from a steady-state distribution at asymptotically large times $t > 1/\gamma^d$ in the localized phase $k < k_c$. In the same way in the metallic phase they give

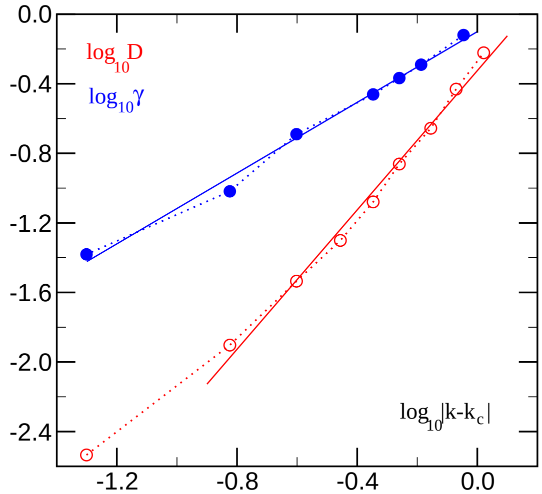

FIG. 2: (Color online) Power law dependence for the inverse localization length $\gamma = 1/l_1$ and diffusion rate D obtained at asymptotically large times t at FMKR with $\epsilon = 0.9$ (in a vicinity of critical point $t \geq 10^6$). Data are taken from Fig.9 of [6] and are plotted for the fixed value of critical parameter $k_c = 1.15$ for dimension $d = 4$ with full circles for γ and open circles for D (dotted lines are drawn to adapt an eye). The straight lines show the power law fits with fixed k_c and $\nu = 1.017 \pm 0.041$, $\log_{10} \gamma_0 = -0.100 \pm 0.0266$ and $s = 2.003 \pm 0.0740$, $\log_{10} D_0 = -0.324 \pm 0.0317$ (the lowest value of D is not taken into account in the fit since the evolution time was smaller than $1/D^4$).

the diffusion rate D computed on asymptotically large times $t_a > 1/D^d$ from a gaussian distribution over sites. The results are averaged over 100 disorder realizations for $k < k_c$ and 10 realizations for $k > k_c$ with $t \sim 10^6$. In contrast to the approach used in [1–3], the extraction of such asymptotic values used in [6] does not rely on the existence of scaling function.

The data of [6] can be used for extraction of the scaling exponents ν, s in the localized $\gamma = \gamma_0 |k_c - k|^\nu$ and metallic $D = D_0 |k_c - k|^s$ phases. For $d = 3$ the fit of data for γ with fixed critical point $k_c = 1.8$ is shown in Fig. 1. In a similar way the fit for $d = 4$ is shown in Fig. 2. The obtained values of ν and s are in a satisfactory agreement with the scaling relation $s = (d-2)\nu$ (see. e.g. [10]) both for $d = 3, 4$. For $d = 3$ the values of s, ν are compatible with those obtained in [1–3]. According to [11] D.Delande obtains for $d = 4$ the critical exponents compatible with the scaling relation and values similar to those of Fig. 2.

Even if formally the statistical errors are relatively small the actual values of ν and s remain rather sensitive to variation of k_c. Thus for $d = 3$ a variation of k_c by ± 0.05 gives variation of s by $+0.35, -0.40$ and of ν by $-0.17, +0.16$. In a similar way for $d = 4$ a variation of k_c by $\pm 2\%$ gives variation of s by $\mp 6\%$ and ν by $\pm 12\%$. At the same time the statistical accuracy remains approximately on the same level. The fit of data by 3-parameter power law gives: $D_0 = 1.626 \pm 0.086$, $k_c = 1.728 \pm 0.039$, $s = 2.074 \pm 0.177$ and $\gamma_0 = 0.135 \pm 0.0269$, $k_c = 2.220 \pm 0.0783$, $\nu = 2.626 \pm 0.145$ for $d = 3$. Respectively, such a fit for $d = 4$ data gives $D_0 = 0.397 \pm 0.090$, $k_c = 1.033 \pm 0.082$, $s = 2.697 \pm 0.263$ and $\gamma_0 = 0.821 \pm 0.028$, $k_c = 1.180 \pm 0.033$, $\nu = 1.176 \pm 0.091$. These results show that the exact computation of the critical exponents s and ν remains very hard task even if the FMKR model is much more efficient comparing to the transfer matrix techniques and other numerical methods. It is possible that the scaling methods used in [1–3] correspond to a better accuracy of the exponents. However, this approach uses extrapolation methods combined with the scaling function which are not used in the data presented here.

A separate note should be done for the AKR model. The results presented in [7] show very strong deviation from the scaling relation between s and ν for $4 \leq d \leq 11$. It is possible that at large d the critical kick parameter becomes relatively small $k_c \sim 1/d$ and thus this model becomes too close to a model with practically decoupled quasi-periodic driving in time that can make it rather specific or can require to study very small vicinity of k_c for extraction of correct critical exponents.

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