UNIVERSITÉ PAUL SABATIER
Laboratoire de PHYSIQUE THÉORIQUE
(UMR 5152 du CNRS)

118, route de Narbonne

31062 TOULOUSE Cédex 4 (France)

(33) 5 61 55 75 72; Shepelyansky 61 55 60 68

FAX (33) 5 61 55 60 65

e-mail dima@irsamc.ups-tlse.fr / http://www.quantware.ups-tlse.fr/dima

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TO: N.Cherroret,B.Vermersch,J.C.Garreau,D.Delande
Email: nicolas.cherroret@spectro.jussieu.fr
CC: S.Fishman, S.Flach, A.Pikovsky
Subject: comment on
arXiv:1401.1038[cond-mat.dis-nn]

Dear Colleagues and Authors of ArXiv:1401.1038,

I think that the main statement of ArXiv:1401.1038 is not correct.

In fact Eqs.(3) of Arxiv are derived for a disordered system with dimension $d = 3$ and they can be generalized for the case $d = 4, 5, ...$ At the critical point they give the spreading $r^2 \sim t^{2/d}$. However, your numerical results are presented for QPKNR described by Eq(5). At zero nonlinearity $g = 0$ QPKNR corresponds to the frequency modulated kicked rotator model (FMKR) introduced in [1] and numerically studied in [2,3] (see Refs. below). At two and three modulational frequencies the system correspond to Anderson transition in effective dimensions $d_{eff} = 3$ and 4. The case $d_{eff} = 3$ had been experimentally verified in impressive cold atoms experiments of Garreau group.

However, the role of nonlinear term in QPKNR is different from the nonlinear term in the real dimensional system with dimension d of Eq.(1). Indeed, a wave function which speads on a size L has a decreasing nonlinear term $|\psi|^2 \sim 1/L^d$ due to norm conservation while in QPKNR you have always $|\psi|^2 \sim 1/L$ independently of d_{eff} and number of modulational frequencies. Due to that your numerical results give a spreading exponent for QPKNR which is close to the value 2/5 found in DANSE model (see Arxiv Refs. [A15,A31,A32]) even if the exact value of the exponent α, using your notations, is under discussion (see e.g. [4,5]). However, for $d = 2$ it is well established that the value of α decreases ($\alpha \approx 0.25$) due to a more rapid decrease on the nonlinear term in higher dimensions (see [A31,6,7]). Hence, one expects the value $\alpha = 2/(3d+2) = 2/11$ for the Anderson model at $d = 3$ (see Eq.(3) in [A31]). This effect is absent in QPKNR since the nonlinear term ALWAYS decreases as in $d = 1$. Thus if you will take 3 modulational frequencies corresponding to $d_{eff} = 4$ you will still find numerically $\alpha = 2/5$ while your scaling theory (3) would predict another value for real dimension $d = 4$.

In my opinion the effects of nonlinearty involve nontrivial properties of classical chaos with many degrees of freedom (see e.g. [8]) and are not linked with the scaling theory of Anderson localization. Indeed, the subdiffusive spreading induced by nonlinearity takes place in a strongly localized regime with localization length being comparable with a few sites that is very far from the critial point of Anderson transition in dimension d.

Technical notes:

- I think you do not give the value of $\hbar$ for Eq.(5) as it was the case in a few of your previous arXiv preprints

- what is *log*-basis in Fig.2 ? are you running computations till $t = 10^{14}$? (all previous runs are done up to $t = 10^{10}$) it will be useful to give fit with error-bars values of exponents

If one of you is interested in more extended discussions I think that his short oral presentation can be accepted at the MPI Workshop "Weak Chaos and Weak Turbulence" 3-7 Feb 2014 http://www.mpipks-dresden.mpg.de/ wechat14/ (a place can be find since a couple of speakers happened to be not able to come, MPI probably can cover local expenses for a few days). All experts in the field of Anderson localization and nonlinearity will be present there. In case of interest the person should email request to Arkady Pikovsky at wech14@pks.mpg.de who will make decision on acceptence or not of such a late application.

This letter is also available on the web page

http://www.quantware.ups-tlse.fr/dima/myrefs/myunp004.pdf

Yours sincerely,
Dima Shepelyansky

Refs:

Ref.1 D.L.Shepelyansky, "Some Statistical Properties of Simple Classically Stochastic Quantum Systems", Physica v.8D, p.208 (1983)

Ref.2 F.Borgonovi, D.L.Shepelyansky, "Two interacting particles in an effective 2-3-d random potential", J. de Physique I France v.6, p.287 (1996)

Ref.3 D.L.Shepelyansky, "Anderson transition in three and four effective dimensions for the frequency modulated kicked rotator", arXiv:1102.4450[cond-mat.dis-nn] (2011)

Ref.4. J.D. Bodyfelt, T.V. Laptyeva, Ch. Skokos, D.O. Krimer, S. Flach, "Nonlinear waves in disordered chains: probing the limits of chaos and spreading ", rXiv:1103.4034 (2011)

Ref.5 M. Mulansky, A. Pikovsky, "Energy Spreading in Strongly Nonlinear Disordered Lattices", New J. Phys. 15, 053015 (2013)

Ref.6. T. V. Laptyeva, J. D. Bodyfelt, S. Flach, "Universal subdiffusion of nonlinear waves in two dimensions with disorder", arXiv:1203.3053 (2012)

Ref.7. M.Mulansky, A.Pikovsky, "Scaling properties of energy spreading in nonlinear Hamiltonian two-dimensional lattices", Phys. Rev. E 86, 056214 (2012)

Ref.8. A.Pikovsky, S.Fishman, "Scaling Properties of Weak Chaos in Nonlinear Disordered Lattices", Phys. Rev. E, v. 83, 025201(R) (2011)