UNIVERSITÉ PAUL SABATIER
Laboratoire de PHYSIQUE THÉORIQUE
(UMR 5152 du CNRS)

118, route de Narbonne
31062 TOULOUSE Cédex 4 (France)
(33) 5 61 55 75 72; Shepelyansky 61 55 60 68
FAX (33) 5 61 55 60 65
e-mail dima@irsamc.ups-tlse.fr / http://www.lpt.ups-tlse.fr/dima

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Professor Mauri Valtonen
Tuorla Observatory
University of Turku
Finland
email: mvaltonen2001@yahoo.com

Capture cross-section for the 3-body problem

Dear Professor Mauri Valtonen,

following our discussion at the day of defense of Guillaume Rollin 2 November 2015 at Universite Franche-Compte, Besancon, France, I give here a comparative analysis of yours and mine (and my collaborators I.B.Khriplovich and J.Lages) results for the capture cross-section. The aim is to obtain the estimate for the capture cross-section σ of a particle of mass m_s coming from infinity with a velocity u_s and scattering on a binary with masses $m_a \leq m_b$, $m_B = (m_a + m_b)$. The masses m_a, m_b have a relative velocity approximately being v_0 and binary distance a_B. These are your notations from your book (Ref.1). The corresponding notations of Ref.2 are: $v \to u_s$, $M = m_b \approx m_B$, $m_p = m_a$, $v_p \to v_0$, $r_p \to a_B$ (we assume $m_b \geq m_a$). We are considering the restricted 3-body problem with $m_s \ll m_a \leq m_b$.

The results obtained in Refs.2,3 give the capture cross-section σ:

$\sigma \sim a_B^2 (v_0/u_s)^2$ **[EQ.(1)]**

(see Eq.(9) in Ref.2 and Fig.2 (left panel) in Ref.3; this Eq. in notations of Ref.2 reads as $\sigma \sim r_p^2 (v_p/v)^2$. This result is obtained on the basis of result of Petrosky who gave correct expression for the energy change of mass m_s after a passage in a vicinity on the binary. The velocity change u_s is approximately $\Delta(u_s)^2 \sim (m_a/m_B) v_0^2$ for the perihelion distance $q \sim a_B$ and it drops exponentially with q for $q \gg a_B$. The validity of the result EQ.(1) is confirmed by extensive numerical simulations presented in Ref.3 (Fig.2 (left panel)). Our numerical simulations of Ref.3 also show that for $u_s \gg v_0\sqrt{m_a/m_b}$ the cross-section σ starts to drop with a larger power of u_s since the capture in such a case is possible only due to close encounters when the Petrosky result becomes not valid.

At the same time the result of your book (Eq. (6.43) p.154) gives:

$\sigma \sim a_B^2 (m_a/m_B)^2 (v_0/u_s)^2$ **[EQ.(2)]**

The results EQ.(1) and EQ.(2) are drastically different for $m_a \ll m_b$. Thus for the case of Sun and Jupiter your value of σ is in million times smaller compared to the result of EQ.(1).

There is a certain remark in your book in paragraphs 6.1-6.3 that the masses m_a and m_b are not very different ($m_a \sim m_b$). In such a case indeed EQ.(1) and EQ.(2) give comparable values of σ for $u_s < v_0$ (our result is not valid for $u_s \gg v_0$).

However, you directly consider the case of significantly different masses m_a and m_b in pp.161-162 of Ref.1 considering the case of Sun and Jupiter directly corresponding to the case of Refs.2,3. Here you give the cross-section

$\sigma \approx 25\pi a_B^2 (1km/s/u_s)^7 \approx a_B^2 (v_0/u_s)^7/10^6$ **[EQ.(3)]**

In EQ.(3) it is taken into account that for Jupiter $v_0 = 13km/s$. This result is very different from the expression $\sigma \sim a_B^2 (v_0/u_s)^2$ of EQ.(1).

The numerical simulations performed in Ref.3 confirm very well the analytic expression EQ.(1) and without doubts it describes the correct expression for the capture cross-section in the 3-body problem. Thus, I think that your expressions EQ.(2) and EQ.(3) are not correct. The reason is related to the fact that you follow the expressions of Gould (1991) which are missing the correct consideration of Petrosky, on which our results are based (the results similar to Petrosky were also independently obtained by me in the problem of microwave ionization of Rydberg atoms, see discussion at Ref.4). I am sorry that in my emails addressed to you at 5 Nov 2015 I made some uncertain statements related to that I did not understand your notations correctly. Now I consider the above arguments as the firm ones.

I hope that the correct expressions for the capture cross-section in the 3-body problem will be taken into account in the next edition of your fundamental book.

Since the considered problem is a fundamental one I send the copy of this letter to other jury members of the defense of 2 Nov 2015 (J.-M.Petit, M.Guzzo, J.Lages, I.Shevchenko). This letter is also available on my web page

http://www.quantware.ups-tlse.fr/dima/myrefs/myunp005.pdf

Best regards,

Dima Shepelyansky

Refs:

Ref.1. Mauri Valtonen and Hannu Karttunen, “The three-body problem”, Cambridge Univ. Press (2005); relevant pages pp.152-154, 161-162.

Ref.2. I.B.Khriplovich and D.L.Shepelyansky, ”Capture of dark matter by the Solar System”, International Journal of Modern Physics D (IJMPD) v.18(12), pp.1903-1912 (2009); relevant p.1907, Eq. (9)

http://www.quantware.ups-tlse.fr/dima/myrefs/my182.pdf

Ref.3. J.Lages and D.L.Shepelyansky, ”Dark matter chaos in the Solar System”, Mon. Not. Royal Ast. Soc. Lett. v.430, p.L25 (2013); relevant p.L26, Fig.2 (left panel)

http://www.quantware.ups-tlse.fr/dima/myrefs/my209.pdf

Ref.4. D.Shepelyansky, “Microwave ionization of hydrogen atoms”, Scholarpedia v.7(1), p. 9795 (2012)

http://www.scholarpedia.org/article/Microwave_ionization_of_hydrogen_atoms