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Capture cross-section for the 3-body problem

Dear Professor Mauri Valtonen,

following our discussion at the day of defense of Guillaume Rollin 2 November 2015 at Universite Franche-Compte, Besancon, France, I give here a comparative analysis of yours and mine (and my collaborators I.B.Khriplovich and J.Lages) results for the capture cross-section. The aim is to obtain the estimate for the capture cross-section \(\sigma\) of a particle of mass \(m_s\) coming from infinity with a velocity \(u_s\) and scattering on a binary with masses \(m_a \leq m_b\), \(m_B = (m_a + m_b)\). The masses \(m_a, m_b\) have a relative velocity approximately being \(v_0\) and binary distance \(a_B\). These are your notations from your book (Ref.1). The corresponding notations of Ref.2 are: \(v \rightarrow u_s, M = m_b \approx m_B, \ m_p = m_a, v_p \rightarrow v_0, r_p \rightarrow a_B\) (we assume \(m_b \geq m_a\)). We are considering the restricted 3-body problem with \(m_s \ll m_a \leq m_b\).

The results obtained in Refs.2,3 give the capture cross-section \(\sigma\):
\[
\sigma \sim a_B^2(v_0/u_s)^2 \quad [\text{EQ.}(1)]
\]
(see Eq.(9) in Ref.2 and Fig.2 (left panel) in Ref.3; this Eq. in notations of Ref.2 reads as \(\sigma \sim r_p^2(v_p/v)^2\). This result is obtained on the basis of result of Petrosky who gave correct expression for the energy change of mass \(m_s\) after a passage in a vicinity on the binary. The velocity change \(u_s\) is approximately \(\Delta(u_s)^2 \sim (m_a/m_B)v_0^2\) for the perihelion distance \(q \sim a_B\) and it drops exponentially with \(q\) for \(q \gg a_B\). The validity of the result EQ.(1) is confirmed by extensive numerical simulations presented
in Ref.3 (Fig.2 (left panel)). Our numerical simulations of Ref.3 also show that for 
\( u_s \gg v_0\sqrt{m_a/m_b} \) the cross-section \( \sigma \) starts to drop with a larger power of \( u_s \) since the 
capture in such a case is possible only due to close encounters when the Petrosky result 
becomes not valid.

At the same time the result of your book (Eq. (6.43) p.154) gives:

\[
\sigma \sim a_B^2 (m_a/m_B)^2 (v_0/u_s)^2 \quad \text{[EQ.(2)]}
\]

The results EQ.(1) and EQ.(2) are drastically different for \( m_a \ll m_b \). Thus for the case 
of Sun and Jupiter your value of \( \sigma \) is in million times smaller compared to the result 
of EQ.(1).

There is a certain remark in your book in paragraphs 6.1-6.3 that the masses \( m_a \) and 
\( m_b \) are not very different (\( m_a \sim m_b \)). In such a case indeed EQ.(1) and EQ.(2) give 
comparable values of \( \sigma \) for \( u_s < v_0 \) (our result is not valid for \( u_s \gg v_0 \)).

However, you directly consider the case of significantly different masses \( m_a \) and \( m_b \) in 
pp.161-162 of Ref.1 considering the case of Sun and Jupiter directly corresponding to 
the case of Refs.2,3. Here you give the cross-section

\[
\sigma \approx 25\pi a_B^2 (1km/s/u_s)^7 \approx a_B^2 (v_0/u_s)^7 / 10^6 \quad \text{[EQ.(3)]}
\]

In EQ.(3) it is taken into account that for Jupiter \( v_0 = 13 \text{km/s} \). This result is very 
different from the expression \( \sigma \sim a_B^2 (v_0/u_s)^2 \) of EQ.(1).

The numerical simulations performed in Ref.3 confirm very well the analytic expression 
EQ.(1) and without doubts it describes the correct expression for the capture cross-
section in the 3-body problem. Thus, I think that your expressions EQ.(2) and EQ.(3) 
are not correct. The reason is related to the fact that you follow the expressions of 
Gould (1991) which are missing the correct consideration of Petrosky, on which our 
results are based (the results similar to Petrosky were also independently obtained by 
me in the problem of microwave ionization of Rydberg atoms, see discussion at Ref.4).

I am sorry that in my emails addressed to you at 5 Nov 2015 I made some uncertain 
statements related to that I did not understand your notations correctly. Now I consider 
the above arguments as the firm ones.

I hope that the correct expressions for the capture cross-section in the 3-body problem 
will be taken into account in the next edition of your fundamental book.
Since the considered problem is a fundamental one I send the copy of this letter to other jury members of the defense of 2 Nov 2015 (J.-M.Petit, M.Guzzo, J.Lages, I.Shevchenko). This letter is also available on my web page

http://www.quantware.ups-tlse.fr/dima/myrefs/myump005.pdf

Best regards,

Dima Shepelyansky

Refs:


Ref.3. J.Lages and D.L.Shepelyansky, ”Dark matter chaos in the Solar System”, Mon. Not. Royal Ast. Soc. Lett. v.430, p.L25 (2013); relevant p.L26, Fig.2 (left panel)


http://www.scholarpedia.org/article/Microwave_ionization_of_hydrogen_atoms