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Dynamical thermalization conjecture (DTC) paradox

Dear friends,

from the early days of our scientific research life we were fascinated by the question of emergence of statistical laws from dynamical equations of motion and related dynamical chaos. For systems with many degrees of freedom the question is turned to the emergence of thermalization. The Dynamical Thermalization Conjecture (DTC) tells us that due to dynamical chaos we should have Energy Equipartition between all degrees of Freedom (EEF) (as clearly stated in books on statistical physics, e.g. Landau, Lifshits Vol.5 (LL5)).

The verification of EEF led to the Fermi-Pasta-Ulam (FPU) problem in which EEF was not found. There are several specific features of the FPU chain of nonlinear oscillators (e.g. proximity to the integrable Toda lattice etc.). So this system is not generic and other generic systems should be studied.

Here I would like to attract you interest to a more generic situation where the unperturbed system has certain disorder (e.g. random frequencies of linear modes). The obtained results are reported in Refs.[1-3] with the early results in Ref.[4] where, however, the surprise of DTC paradox was not really realized. In principle all DTC paradox features are explained in Refs.[1-3] in all detail. These works are not so new but the recent email exchange with Boris Shapiro (which did not yet arrive to a mutual position) initiated me to write to you this letter on DTC paradox.

Below I present the main DTC points:

We take some FINITE quantum system which can be exactly diagonalized and thus reduced to certain linear oscillators. This can be 1D or 2D Anderson model with disorder (DANSE) as in [1]; it can be the Schrodinger equation in a chaotic billiard (e.g. Sinai billiard, Bunimovich stadium analyzed in [2] or Sinai oscillator [2,3]). Then we add in some way a moderate nonlinear coupling between linear modes. For DANSE [1] it is on-site nonlinear energy shift, for Bunimovich stadium [2] and Sinai oscillator [3] the nonlinear coupling appears from the Gross-Pitaevskii equation (GPU). Again we assume that the nonlinear strength is relatively weak so that it leads to dynamical chaos but it is not so strong to significantly modify the linear modes.

With this setup, as Fermi, Pasta and Ulam, we should expect energy equipartition over linear oscillator modes (EEF).

However, there is also another view point. We say that the moderate nonlinearity induces dynamical thermalization with the usual quantum Gibbs probability distribution over energy levels of the unperturbed QUANTUM system (linear modes) or to the Bose-Einstein (BE) distribution (for the GPU case of billiard/oscillator). This is DTC. Let me note that the quantum Gibbs distribution follows also from the maximization of entropy (as noted in [4]).

As in LL5, for the quantum Gibbs we have for DANSE the anzats for the probabilities ρ_m on energy levels ϵ_m :

$\rho_m = Z^{-1}\exp(-\epsilon_m/T)$, $Z = \sum_m \exp(-\epsilon_m/T)$ **[EQ.(1)]**

or for the GPU case the BE anzats:

$\rho_m = 1/[\exp[(\epsilon_m - \epsilon_g - \mu)/T] - 1]$ **[EQ.(2)]** .

Here, μ is the chemical potential, ϵ_g is the ground state energy, T is temperature appearing due to DTC.

The Eq.(1) (or/and (2)) determines the system energy $E(T) = \sum \epsilon_m \rho_m$ and its entropy $S(T) = -\sum \rho_m \ln \rho_m$. In this way from DTC and known energies ϵ_m we determine the dependence $S(E)$. The check of this dependence is very convenient for numerical simulations since both E and S are extensive variables and their fluctuations are reduced.

The numerical results reported in Refs. [1-4] confirm the validity of the quantum Gibbs and BE distribution with $S(E)$ curve being in a good agreement with the anzats (1) or (2). Let me stress again that these distributions are DRASTICALLY different from the energy equipartition between linear mode, expected by Fermi-Pasta-Ulam and the common lore of LL5. Let me also note that EEF should result in the ultraviolet catastrophe.

Of course, from a numerical view point the distributions (1) and (2) are not very different and it is not so easy to distinguish them due to numerical fluctuations. Let me note that (2) is accepted without any doubt by the cold gas community (see Phys. Rev. Lett. v.87, 210404 (2001) and C.R. Physique v.5, 107 (2004) by Y.Castin et al.). Boris Shapiro is strongly against (2). But for me the main SURPRISE is that we obtain QUANTUM distributions (1) or (2) for CLASSICAL nonlinear modes which is DRASTICALLY different from energy equipartition expected by all historical figures of statistical physics starting from Boltzmann.

In many cases any system of linear modes can be effectively reduced to the Schrodinger equation and then the nonlinear interactions between modes will lead to DTC with distributions (1) or (2) in a drustic difference from EEF. In [1] the validity of (1) was confirmed for various models of coupled oscillators including 1D Klein-Gordon model where the norm is not the integral of motion (Arkady Pikovsy was attributing all problems to existence of this integral while I am strongly disagree with this).

Of course, the numerical simulations are always done on finite time scales and it is not possible to exclude a possibility that at some VERY-VERY long times the DTC distributions (1),(2) will be washed out by the slow Arnold diffusion [5] with the transition to EEF. However, the present status of numerical studies confirmes the DTC with (1) or (2) distributions.

Finally, let me note that for the many-body quantum systems with second quantization the validity of DTC with BE (interacting bosons) and Fermi-Dirac (interacting fermions) distributions have been confirmed in [6,7,8]. However, there is no paradox for DTC in many-body quantum systems.

I hope that the DTC paradox will attract your attention and I am looking forward for your feedback. In the case of your interest I am ready to host you in Toulouse (covering your local expenses) for a few days to discuss the problem of dynamical thermalization and chaos in nonlinear systems. If several people are interested this can be a mini-workshop in Toulouse for 2-3 days, e.g. in the fall 2019.

Best regards,

Dima Shepelyansky

P.S. This letter is publicaly available at

http://www.quantware.ups-tlse.fr/dima/myrefs/myunp007.pdf

Refs:

Ref.1. L.Ermann and D.L.Shepelyansky, "Quantum Gibbs distribution from dynamical thermalization in classical nonlinear lattices", New J. Phys. v.15, p.123004 (2013)

http://www.quantware.ups-tlse.fr/dima/myrefs/my218.pdf

Ref.2. L.Ermann, E.Vergini and D.L.Shepelyansky, "Dynamical thermalization of Bose-Einstein condensate in Bunimovich stadium", Europhys. Lett. v.111, p. 50009 (2015)

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Ref.3. L.Ermann, E.Vergini and D.L.Shepelyansky, "Dynamics and thermalization a Bose-Einstein condensate in a Sinai-oscillator trap", Phys. Rev. A v.94, p.013618 (2016)

http://www.quantware.ups-tlse.fr/dima/myrefs/my241.pdf

Ref.4. M.Mulansky, K.Ahnert, A.Pikovsky and D.L.Shepelyansky, "Dynamical thermalization of disordered nonlinear lattices", Phys. Rev. E v.80, p.056212 (2009)

http://www.quantware.ups-tlse.fr/dima/myrefs/my176.pdf

Ref.5. B.V.Chirikov, "A universal instability of many-dimensional oscillator systems", Phys. Rep. v.52, p.263 (1979)

Ref.6. P.Schlageck and D.L.Shepelyansky, "Dynamical thermalization in Bose-Hubbard systems", Phys. Rev. E v.93, p.012126 (2016)

http://www.quantware.ups-tlse.fr/dima/myrefs/my237.pdf

Ref.7. A.R.Kolovsky and D.L.Shepelyansky, "Dynamical thermalization in isolated quantum dots and black holes", Europhys. Lett. v.117, p.10003 (2017)

http://www.quantware.ups-tlse.fr/dima/myrefs/my245.pdf

Ref.8. K.M.Frahm and D.L.Shepelyansky, "Dynamical decoherence of a qubit coupled to a quantum dot or the SYK black hole" Eur. Phys. J. B v.91, p.257 (2018)

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