UNIVERSITÉ PAUL SABATIER

Laboratoire de PHYSIQUE THÉORIQUE

(UMR 5152 du CNRS)

118, route de Narbonne

31062 TOULOUSE Cédex 4 (France)

(33) 5 61 55 75 72; Shepelyansky 61 55 60 68

FAX (33) 5 61 55 60 65

e-mail dima@irsamc.ups-tlse.fr / http://www.lpt.ups-tlse.fr/dima

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TO: tkottos@wesleyan.edu,
boris@physics.technion.ac.il
CC: a.politi@abdn.ac.uk,
angelo.vulpiani@roma1.infn.it,
benettin@math.unipd.it, Roberto.Livi@fi.infn.it,
stefano.ruffo@sissa.it, leoermann@gmail.com,
pikovsky@uni-potsdam.de,
Antonio.Picozzi@u-bourgogne.fr,
frahm@irsamc.ups-tlse.fr

Reply of D.Shepelyansky
to the Comment of
Tsampikos Kottos and Boris Shapiro
dated 31 January 2024

Dear Tsampikos Kottos and Boris Shapiro, and friends and colleagues in CC,

at first I thank Boris and Tsampikos for their Comment in continuation of our dispute on dynamical thermalization in classical oscillator systems with weak or moderate nonlinearity started in May 2019. All exchange letters are available at my web page

https://www.quantware.ups-tlse.fr/dima/publications.html

(see point 007unpub).

Let me note that after the appearance of Frahm and DS arXiv dated 22 Dec 2022 all dispute participants of my letter at May 2019 had been informed/emailed that the paradox between quantum Bose-Einstein (BE) thermal distribution and classical thermal equipartition (EQ or Rayleigh-Jeans(RJ)) had been resolved in the arXiv in favor of EQ-RJ. Here I attach this email of 23 Dec 2022 since my impression is that TK and BS missed it (it is also available on the web page from 1 Feb 2024). This arXiv, with certain additional notes and refs, had been published in PRL 2023 (see Ref.1 below).

Recently with Klaus Frahm we extended this model as a random matrix model of Kolmogorov-Zakharov turbulence (see Ref.2 below).

In PRL of Ref.1 there is a bunch of Refs. on multimode fiber experiments where EQ-RJ thermalization had been observed (even negative temperature was demonstrated, see Ref.52 of our PRL). Let me note that the discussion of EQ-RJ distribution and experimental reports are published in year 2011-2023 well before PRX 10, 031024 (2020) by TK and BS.

In fact we may say that the derivation of EQ-RJ is well available in the Landau-Lifshits volume "Statistical physics" (1976) cited in our PRL. Indeed, EQ-RJ is simply a result of BE at large temperature, and chemical potential μ appears due to conservation of number of bosons or norm in nonlinear Schrodinger equation. The distribution EQ-RJ is also given in the book of Zakharov, L'vov, Falkovich "Kolmogorov Spectra of Turbulence I" Springer (1992) (see Eq. (2.2.14) there; this book is cited in PRL of Ref.1 as Ref.16). I also agree that Boris was a great defender of EQ-RJ distribution (see e.g. Ref.6 in the Comment (cited here as Ref.3) and the dispute at web page given above). But in my opinion it is not enough to say that there should be EQ-RJ distribution in any oscillator system with weak or moderate nonlinearity. Without numerical results such a statement, known already from Landau-Lifshits, has no weight, especially if certain numerical results give indications in favor of quantum BE distribution.

Let me trace a parallel with the famous Fermi-Pasta-Ulam problem (1955): there would be not much attention to their work if without any numerical results they would simply published one-page letter writing their Hamiltonian of nonlinear oscillators and stating that according to statistical mechanics there should be energy equipartition over oscillator modes (which in fact was not found).

In my opinion a similar situation is with our PRL of Ref.1. Here, for a first time extensive numerical simulations showed that there is clearly EQ-RJ dynamical thermalization (being different from quantum BE distribution) if nonlinearity is above certain chaos border for nonlinearity. Below this border one have certain quasi-integrable dynamics (without any thermalization) in a spirit of Kolmogorov-Arnold-Moser (KAM) theory. The dependence of Lyapunov exponent on system parameters was also numerically obtained for large number of degrees of freedom, even if a detailed dependence of chaos border and Lyapunov exponents still require further studies (see Ref.1 and especially SupMat of PRL).

In relation to experiments and certain numerical simulations with multimode optical fibers I should point that these fiber systems have a rather specific feature: their linear Hamiltonian is a two-dimensions oscillator with equal frequencies. Thus there is a degeneracy of quantum levels and KAM theory cannot be applied to this system (see discussions in SupMat of Ref.1 and Ref.2). It is still a question how dynamical thermalization appears in this system and if it is generic (e.g. fluctuations along a fiber may play an important role as pointed in a private note of Antonio Picozzi in 2023). A case of a fiber with random linear modes/energies like for RMT is still waiting its investigation (we propose that such kind of regime can be studied in a D-shape fiber section where linear system is in a regime of quantum chaos similar to RMT, see discussions in SupMat of Ref.1 and Ref.2).

My true story with PRX of TK-BS: Boris was always saying that there should be EQ-RJ, he gave me a reference on their PRX (or may be it was arXiv version) pointing that there is a note which states that there is EQ-RJ distribution contrary to our BE like results with Ermann in NJP; I looked on PRX of TK-BS and saw that it is about spin networks, there were no statements about dynamical thermalization; so I did not look on this paper in detail in 2020 considering that it is a usual EQ-RJ statement of Boris. I am sorry for that. Thus only after the Comment at 31 Jan 2024 I noted that in PRX there is the RMT model with nonllinearity (which is not the main feature of PRX) and certain numerics with N=16 oscillators presented in PRX Fig.5 stating that numerics confirms EQ-RJ distribution. BUT THERE IS NO COMPARISON WITH QUANTUM BE DISTRIBUTION. Thus, I am sure that if one plots the BE distribution for the conditions of Fig.5 there will no significant difference between EQ-RJ and BE theories and numerics. In any case in PRX there is no comparisons between EQ-RJ and BE theories, no discussion of chaos border and KAM theory, no computations of Lyapunov exponents.

In view of that I consider that the paradox between EQ-RJ and BE theories was resolved only in Ref.1 where a detailed comparison had been performed and the firm conclusion in the favor of EQ-RJ theory had been obtained on the basis of extensive numerical simulations. All colleagues of my letter of May 2019 had been informed about the paradox closed by email of 23 Dec 2022 (it is attached). A link to Ref.1 (arXiv) was given on the above web page same day. Now the web page contains all materials of dipute till today.

With best chaotic regards,

Dima Shepelyansky

REFEREES:

Ref.1) K.M.Frahm and D.L.Shepelyansky, “Nonlinear perturbation of Rendom Matrix Theory”, submitted to arXiv and PRL on 22 Dec (2022), arXiv:2212.11955 [cond-mat.stat-mech] v1, update 17 May 2023 v2; Phys. Rev. Lett. v.131. p.077201 (2023) https://www.quantware.ups-tlse.fr/dima/myrefs/my293.pdf

Ref.2) K.M.Frahm and D.L.Shepelyansky, ”Random matrix model of Kolmogorov-Zakharov turbulence”, submitted to Phys. Rev. E January 21 (2024) arXiv:2401.11545 [cond-mat.stat-mech]

Ref.3) T.Kottos and B.Shapiro, ‘”Comment on the Phys. Rev. Lett. by Frahm and Shepelyansky”, dated 31 January 2024, available at https://www.quantware.ups-tlse.fr/dima/myrefs/myunp007shapiro3.pdf (also attached to this email)